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Large amplitude solitary waves in a warm magnetoplasma with kappa distributed electrons

S. A. El-Tantawy,[1,a)] N. A. El-Bedwehy,[2,b)] H. N. Abd El-Razek,[3,c)] and S. Mahmood[4,5,d)]
[1]*Department of Physics, Faculty of Science, Port Said University, Port Said 42521, Egypt*
[2]*Department of Mathematics, Faculty of Science, Damietta University, New Damietta 34517, Egypt*
[3]*Department of Physics, Faculty of Science, Damietta University, New Damietta 34517, Egypt*
[4]*Theoretical Plasma Physics Division, PINSTECH, P.O. Nilore, Islamabad 44000, Pakistan*
[5]*National Centre for Physics, Quaid-i-Azam University Campus, Shahdra Valley Road, Islamabad 44000, Pakistan*

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The large amplitude nonlinear ion acoustic solitary wave propagating obliquely to an external magnetic field in a magnetized plasma with kappa distributed electrons and warm ions is investigated through deriving energy-balance-like expression involving a Sagdeev potential. Analytical and numerical calculations of the values of Mach number reveal that both of subsonic and supersonic electrostatic solitary structures can exist in this system. The influence on the soliton characteristics of relevant physical parameters such as the Mach number, the superthermal parameter, the directional cosine, the ratio of ion-to-electron temperature, and the ion gyrofrequency has been investigated. © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4793446]

I. INTRODUCTION

In a collisionless plasma, particle distribution in velocity space can depart considerably from a Maxwellian one. Numerous observations indicate that the non-Maxwellian (power-law) high-energy particles can exist in various astrophysical plasmas, such as solar wind, magnetosphere, interstellar medium, and auroral zone plasma.[1–3] The appropriate distribution functions to describe such particles are the generalized Lorentzian distributions,[4] also known as the kappa distribution. The three dimensional form of kappa velocity distribution is given by $f_\kappa(v) = A_\kappa(1 + v^2/\kappa\beta^2)^{-(\kappa+1)}$, where A_κ is a normalization constant, $\nu = (\nu_x, \nu_y, \nu_z)$, $\beta = \sqrt{(\kappa - 3/2)/\kappa} v_{th}$ is an effective thermal speed, $v_{th} = \sqrt{2k_B T/m}$ is the thermal velocity of a particle with temperature T and mass m, and κ is the spectral index. The spectral index κ is a measure of the slope of the energy spectrum of the suprathermal particles forming the tail of the velocity distribution function. Kappa distributions approach the Maxwellian as $\kappa \to \infty$. The distributions have a high velocity tail decreasing with ν as a power law $[f_\kappa(v) \propto \nu^{-2(\kappa+1)}]$ for $\nu \gg \beta$, and when the velocity ν is smaller than or comparable to β, the kappa velocity distribution function is close to the Maxwellian having the same thermal speed v_{th}.[5]

Most astrophysical and space plasmas are observed to have non-Maxwellian high energy tail. Spacecraft measurements of electron energy spectra have been successfully modeled with kappa distribution.[6] Importantly, Kappa distribution has been used to analyze and interpret spacecraft data on the Earth's magneto-spheric plasma sheet, the solar wind, Jupiter, Saturn, and planetary magnetospheres.[7] Typically, space plasmas are observed to possess a spectral index κ in the range 2–6.[8] Some examples are as follows: in the earth's foreshock, Feldman *et al.*[9] fitted the electrons with $3 < \kappa_e < 6$, while Lemaire's group[10,11] developed a Lorentzian ion exosphere model and associated solar wind model with $\kappa-$distributed coronal electrons, using, typically, $2 < \kappa < 6$. The kappa distribution with $\kappa_e = 4$ yielding good agreement with electron distributions observed in the solar wind.[12]

Ion-acoustic waves (IAWs) in plasmas began to be studied long ago, and a lot of research works have already been published on this fundamental mode in both magnetized and unmagnetized cases. Nonlinear IAWs in plasmas have been investigated by many authors.[13–18] Most of these studies are restricted to the cold ion limit.[15–18] Ion-acoustic solitary waves (IASWs) in the presence of hot ions in unmagnetized electron-ion plasmas have also been investigated.[19–23] Mahmood and Saleem[24] studied the electrostatic solitary structures in unmagnetized plasmas in the presence of cold and adiabatically heated ions. Kalita and Bujarbarua[25] studied the large-amplitude ion acoustic solitons in a warm ions magnetoplasma with isothermal electrons. Most of the works in this area are concerned with the case of electron-ion plasma systems in unmagnetized plasmas,[26] cold ion magnetized plasma with isothermal electrons[27] and superthermal electrons,[8] and hot ion magnetized plasma with isothermal electrons.[24] Gogoi *et al.*[28] investigated the dust ion-acoustic solitary waves in an unmagnetized plasma with inertial warm ions and electrons following the kappa velocity distribution. However, to the authors knowledge, the possible nonlinear ion-acoustic solitary waves in hot ion magnetized plasmas with superthermal kappa distributed electrons are not investigated before, which is the goal of the present work. Arbitrary amplitude solitary waves are investigated by deriving energy-integral equation (involving a Sagdeev-like pseudopotential).

This paper is organized in the following fashion: In Sec. II, we present the governing model equations. An

[a)]E-mail: samireltantawy@yahoo.com.
[b)]E-mail: nab_elbedwehy@yahoo.com.
[c)]E-mail: hosam.abdelrazek@yahoo.com.
[d)]shahzadm100@gmail.com.

energy integral equation/the Sagdeev-like potential is derived in Sec. III. The energy integral equation is numerically solved and analyzed to study the profiles of the Sagdeev-like potential with fully nonlinear electrostatic solitary wave properties. Finally, a brief discussion is given in Sec. IV.

II. SET OF GOVERNING EQUATIONS

Consider an ideal homogeneous magnetized two component plasma consisting of warm ions and superthermal kappa distributed electrons. The ambient magnetic field $B = B_0\hat{z}$ is taken along the z-axis (where $\hat{z}$ is the unit vector along the z-axis.) while the wave vector $\vec{k}$ lies in the $x-z$ plane making an angle θ with the uniform magnetic field. The phase velocity of the IAWs is assumed to be larger than the ion thermal and much less than the electron thermal velocities for the consistency of the fluid model. We neglect any transport properties such as viscosity and heat conduction. Moreover, we assume that the ion motion is three dimensional and neglect the variation of all quantities in the y-direction.[25] Under these conditions, the nonlinear dynamics of the low frequency electrostatic wave in a two component plasma are governed by the following set of normalized equations for adiabatic process:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_{ix})}{\partial x} + \frac{\partial (n_i u_{iz})}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u_{ix}}{\partial t} + \left(u_{ix}\frac{\partial}{\partial x} + u_{iz}\frac{\partial}{\partial z}\right)u_{ix} = -\frac{\partial \phi}{\partial x} + \Omega u_{iy} - \frac{5}{3}\sigma_i n_i^{-\frac{1}{3}}\frac{\partial n_i}{\partial x}, \tag{2}$$

$$\frac{\partial u_{iy}}{\partial t} + \left(u_{ix}\frac{\partial}{\partial x} + u_{iz}\frac{\partial}{\partial z}\right)u_{iy} = -\Omega u_{ix}, \tag{3}$$

$$\frac{\partial u_{iz}}{\partial t} + \left(u_{ix}\frac{\partial}{\partial x} + u_{iz}\frac{\partial}{\partial z}\right)u_{iz} = -\frac{\partial \phi}{\partial z} - \frac{5}{3}\sigma_i n_i^{-\frac{1}{3}}\frac{\partial n_i}{\partial z}, \tag{4}$$

where n_e, n_i are the densities of electrons and ions, respectively, u_i is the ion fluid velocity, ϕ is an electrostatic potential and $\sigma_i = T_i/T_e$ is the ratio of ion-to-electron temperature. In equilibrium, we have $\mu = 1$, where $\mu = n_e^{(0)}/n_i^{(0)}$ denotes the unperturbed density ratio of electrons-to-ions. Here, the density of the kappa distributed electrons is given by[8]

$$n_e = \mu\left(1 - \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa+\frac{1}{2}}, \tag{5}$$

where $\kappa(>\frac{3}{2})$ is a real parameter measuring the deviation from Maxwellian equilibrium. The density expression given above is only valid for $\kappa > 3/2$, and it reduces to the usual Maxwellian form $n_e = \mu\exp(\phi)$ when $\kappa \to \infty$.

As we consider, only uniform magnetic field, we will deal with electrostatic case. Furthermore, we assume that the length scale of the solitary wave is greater than the Debye length. Under this assumption, we can use the charge neutrality condition instead of Poisson's equation as

$$n_i \approx n_e = n. \tag{6}$$

This implies the physical assumption that spatial variation in the electric potential is slow, essentially occurring on a scale far beyond the Debye sphere.[8]

The variables appearing in Eqs. (1)–(6) have been normalized as follows: $n_{i,e}$ is normalized by the unperturbed ion density $n_i^{(0)}$, u_i by the ion-acoustic speed $C_{si}(=(k_B T_e/m_i)^{1/2})$, the electrostatic potential ϕ by $k_B T_e/e$, the space and time variables are in units of the ion Debye length λ_{Di} $(=(k_B T_e/4\pi n_i^{(0)} e^2)^{1/2})$, and the ion plasma period $\omega_{pi}^{-1}(=(4\pi n_i^{(0)} e^2/m_i)^{-1/2})$, respectively. Moreover, we introduce the following notations: $\Omega = \omega_{ci}/\sqrt{4\pi n_i^{(0)} e^2/m_i}$, and $\omega_{ci} = eB_0/m_i c$, where ω_{ci} is the ion gyrofrequency, e is the magnitude of the electron charge, m_i is the ion mass, and c is the velocity of the light in vacuum.

The dispersion relation of electrostatic excitation in our plasma system is obtained by solving Eqs. (1)–(6) algebraically in the linear limit, which yields

$$\omega_{\pm}^2 = \frac{1}{2}b \pm \frac{1}{2}(b^2 - 4c)^{\frac{1}{2}}, \tag{7}$$

where

$$b = \left(\Omega^2 + k^2\left(\frac{1}{k^2+F} + \frac{5}{3}\sigma_i\right)\right),$$

$$c = k_z^2\Omega^2\left(\frac{1}{k^2+F} + \frac{5}{3}\sigma_i\right),$$

$$F = \left(\frac{\kappa - \frac{1}{2}}{\kappa - \frac{3}{2}}\right).$$

It can be seen from Eq. (7) that in the cold ions limit ($\sigma_i \to 0$), the dispersion relation leads to a similar expression as obtained by Sultana *et al.*[8]

In the low frequency limit ($\omega \ll \Omega$), the above dispersion relation reduces to

$$\omega^2 = k_z^2\delta\left(k^2 + F + \frac{k_x^2}{\Omega^2}\delta\right)^{-1}, \tag{8}$$

where

$$\delta = \left(1 + \frac{5}{3}\sigma_i(k^2 + F)\right).$$

In the above equations, ω is the angular frequency, k_x and k_z are the wave numbers along x– and z–direction, respectively.

III. SAGDEEV POTENTIAL AND DISCUSSION

To obtain the localized solutions, we consider a co-moving frame of single variable $\eta = l_x x + l_z z - Mt$, with velocity M normalized by C_{si}, where l_x and l_z are the direction cosines of the wave vector along with x- and z-axes such that $l_x^2 + l_z^2 = 1$. Using Eq. (6) and combining Eqs. (1)–(4), we obtain the following equation:

$$\frac{d^2}{d\eta^2}\left[\frac{M^2}{2n^2}+\phi+\frac{5}{2}\sigma_i n^{\frac{2}{3}}+1\right]=\Omega^2\left\{n\left[1+\frac{l_z^2}{M^2}\left(1+\sigma_i-\sigma_i n^{\frac{5}{3}}-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa+\frac{3}{2}}\right)\right]-1\right\}. \quad (9)$$

The quasi-neutrality condition (6), gives

$$n=\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa+\frac{1}{2}}. \quad (10)$$

Making use of Eq. (10) in Eq. (9) and using the following boundary conditions for localized waves, $\phi\to 0$, $\frac{d\phi}{d\eta}\to 0$ and $\frac{d^2\phi}{d\eta^2}\to 0$ as $|\eta|\to\infty$, we obtain an energy-balance-like expression in terms of a single independent variable ϕ described as follows:

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2+S(\phi)=0, \quad (11)$$

where the Sagdeev potential $S(\phi)$ for our purpose reads

$$S(\phi)=-\frac{\Omega^2}{\left[1+F\left(\frac{5}{3}\sigma_i n^{-\frac{1}{3}}-M^2 n^{-3}\right)\right]^2}[S_1(\phi)+\sigma_i S_2(\phi)]. \quad (12)$$

Here $S_1(\phi)$ and $S_2(\phi)$ are defined as

$$S_1(\phi)=\frac{1}{n}\left[M^2\left(1-\frac{1}{2n}\right)+l_z^2(1-Q)\right]+\frac{l_z^2}{M^2}\left(Q-\frac{1}{2}Q^2-\frac{1}{2}\right)+Q-\phi l_x^2-\frac{M^2}{2}-1,$$

$$S_2(\phi)=\left[l_z^2\left(\frac{1}{n}+\frac{3}{2l_z^2}-\frac{5}{2}\right)+\frac{n^{\frac{2}{3}}}{2}(3l_z^2-5)+n^{\frac{5}{3}}\left(1+\frac{l_z^2}{M^2}\left(1+\sigma_i-Q-\frac{\sigma_i}{2}n^{\frac{5}{3}}\right)\right)+\frac{l_z^2}{M^2}\left(Q-\frac{\sigma_i}{2}-1\right)\right],$$

where

$$F=\frac{\left(\kappa-\frac{1}{2}\right)}{\left(\kappa-\frac{3}{2}\right)}\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa-\frac{1}{2}},$$

$$Q=\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa+\frac{3}{2}}.$$

Equation (11) is regarded as an "energy integral" equation of an oscillating particle of unit mass moving with velocity $d\phi/d\eta$ in a potential well $S(\phi)$. In cold ion magnetoplasma, Eq. (11) is reduced to Eq. (24) in Ref. 8.

The conditions for the existence of localized solution of Eq. (11) require:[3,29] (i) $S(\phi)|_{\phi=0}=dS(\phi)/d\phi|_{\phi=0}=S(\phi_m)=0$, (ii) $d^2S(\phi)/d\phi^2|_{\phi=0}\le 0$, (i.e., the fixed point at the origin is unstable), and (iii) $S(\phi)<0$ when ϕ lies between 0 and ϕ_m. Here ϕ_m is the maximum amplitude of the solitary wave. From the second condition (i.e., $d^2S(\phi)/d\phi^2|_{\phi=0}\le 0$), it is found that the limits of the Mach number must be

$$\sqrt{\left[\frac{(2\kappa-3)}{(2\kappa-1)}+\frac{5}{3}\sigma_i\right]}l_z<M<\sqrt{\left[\frac{(2\kappa-3)}{(2\kappa-1)}+\frac{5}{3}\sigma_i\right]}. \quad (13)$$

It is clear from Eq. (13) that supersonic ($M>1$) as well as subsonic ($M<1$) solitons can propagate in this magnetoplasma model as a result of taking ion temperature effect into account "see Fig. 1(b)." This indicates that the ion temperature plays a key role in the nature of the ion-acoustic solitons.

The variation of the lower and upper limits of the Mach number against the direction cosine l_z for different values of ions to electrons temperature ratios and at constant spectral index κ is shown in Fig. 1(a), and it is clear that both of the lower and upper limits increase by increasing the value of σ_i. In Fig. 1(b), the variation of the upper and lower values of the Mach number is plotted with the variation of spectral index κ at constant σ_i for different values of l_z. It is obvious that the values of M limits increase with the increase of the values of the spectral index κ. Also, it is seen that l_z will have effects only on the lower limit of M as predicted from Eq. (13).

The behavior of the solitary waves can be easily determined from the behavior of the Sagdeev potential $S(\phi)$ profiles. The increase of the depth of $S(\phi)$ profile means that the width of the solitary wave decreases, whereas the increase of the width of $S(\phi)$ profile means that the amplitude of the solitary wave increases too (cf. Figs. 2(a) and 2(b)). So, we have numerically solved the energy equation (11) for various sets of parameters values. It is clear from Figs. 2(a) and 2(b) that the potential profile becomes spiky (i.e., taller amplitude and narrower width) with the increase

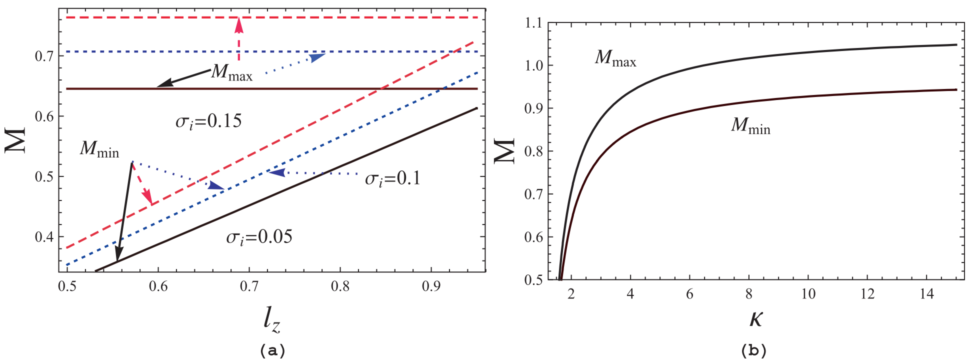

FIG. 1. The Mach number M versus (a) l_z for different values of σ_i with $\kappa = 2$ and (b) κ with $\sigma_i = 0.1$ and $l_z = 0.9$.

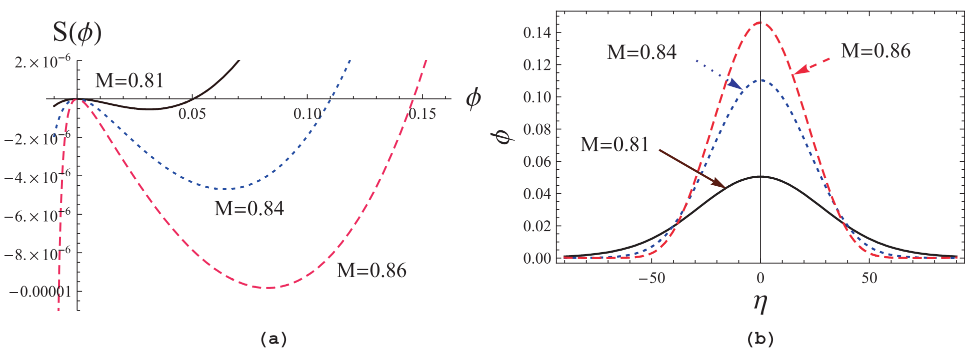

FIG. 2. (a) Behavior of $S(\phi)$ profile is shown for different values of M, with $\kappa = 3, \sigma_i = \Omega = 0.1$, and $l_z = 0.9$. (b) The corresponding electrostatic potential perturbations against η.

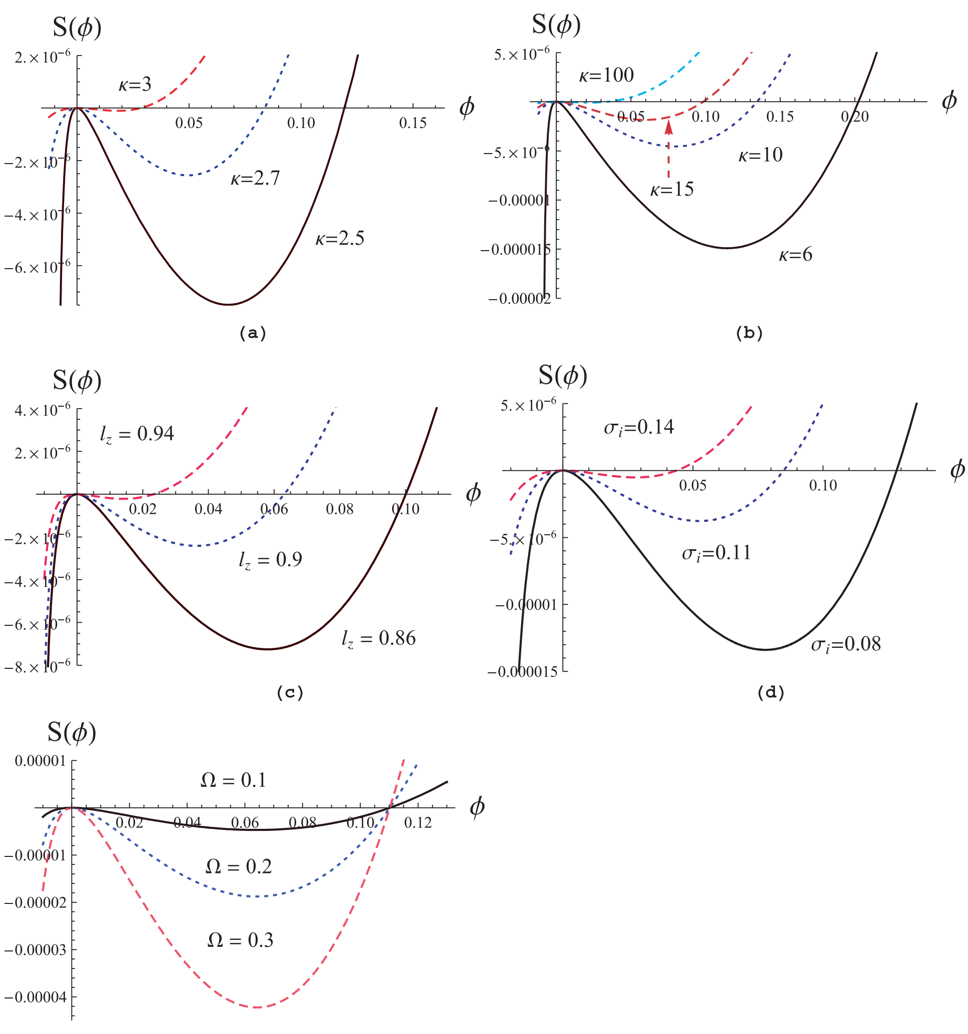

FIG. 3. Behavior of $S(\phi)$ profile is shown for different values of (a) lower values of κ, with $M = 0.8$, $\sigma_i = \Omega = 0.1$, and $l_z = 0.9$, (b) higher values of κ, with $M = 0.96$, $\sigma_i = \Omega = 0.1$, and $l_z = 0.9$, (c) l_z, with $\kappa = 2, \sigma_i = \Omega = 0.1$, and $M = 0.68$, (d) σ_i, with $M = 0.63$, $\kappa = 2$, $\Omega = 0.1$, and $l_z = 0.8$, and (e) Ω, with $\kappa = 3, M = 0.84, \sigma_i = 0.1$, and $l_z = 0.9$.

of Mach number M, while slower ones will be shorter and wider, in agreement with the soliton phenomenology. This is in agreement with the earlier result of Ref. 29 for magnetized plasma with nonextensive particles distribution. The dependence of the ion acoustic solitary pulses on the plasma parameters namely, κ, l_z, and σ_i is depicted in Figs. 3(a)–3(d). It is found that the soliton amplitude decreases with increasing κ, which agrees with the results of Ref. 8. Also, the soliton profile becomes steeper for a greater excess of superthermal electrons. Moreover, it is obvious that the potential profile becomes shorter and wider by increasing l_z and σ_i. The dependence of the solitary pulses on the ion gyrofrequency Ω is depicted in Fig. 1(e). From this figure, it is observed that the amplitude remains constant while the width decreases as Ω increases. These considerations are in full agreement with Refs. 8 and 29.

IV. SUMMARY

The large amplitude nonlinear ion acoustic solitary wave propagating obliquely to an external magnetic field in a magnetized plasma with kappa distributed electrons and warm ions is investigated. By employing the two-fluid equations and making use of the plasma approximation, we have derived the energy integral with a new Sagdeev potential. Analytical and numerical calculations reveal that subsonic and supersonic ion-acoustic waves may exist. The dependence of the solitary excitation on the Mach number, the superthermal parameter, the directional cosine, the ratio of ion-to-electron temperature, and the ion gyrofrequency has been investigated.

Finally, the present results on obliquely propagating ion-acoustic waves in magnetized plasma can contribute to understanding localized electrostatic waves in space plasmas as well as in laboratory plasmas where a kappa distributed electron component is observed.[3,7,8,30,31]

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