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The distribution of zeros of all solutions of first order neutral differential equations

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ABSTRACT

This paper is concerned with the distribution of zeros of all solutions of the first-order neutral differential equation

$$[x(t)+p(t)x(t-\tau)]'+Q(t)x(t-\sigma)=0,\quad t\geqslant t_0,$$

where

$$p\in C([t_0,\infty),[0,\infty)),\quad Q\in C([t_0,\infty),(0,\infty)) \text{ and } \tau,\sigma\in R^+.$$

New estimations for the distance between adjacent zeros of this neutral equation are obtained via comparison with a corresponding differential inequality. These results extend some known results from the non-neutral to the neutral case and improve other published results as well.

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1. Introduction

The oscillation theory of neutral differential equations has been investigated extensively in the last decades and increasingly attract much interest due to the valuable applications of these type of equations in many fields; see [5–8]. This theory, generally, investigates the problem of existence of an infinite number of zeros of all solutions; see [1–3,6] for the recent advances in oscillation theory. A crucial question in the theory is to determine the zeros locations for each solution of a given equation. This problem, for functional differential equations, did not receive the deserved interest, although it gives more insight into the properties of the solutions which means better understanding for some phenomenon that can be modeled by these equations.

This work is devoted to study the distribution of zeros of all solutions of the neutral equation

$$[x(t)+p(t)x(t-\tau)]'+Q(t)x(t-\sigma)=0,\quad t\geqslant t_0, \tag{1.1}$$

where $p\in C([t_0,\infty),[0,\infty)), Q\in C([t_0,\infty),(0,\infty))$ and $\tau,\sigma\in R^+$ such that $\sigma>\tau$. As far as these authors know; it seems that [9,10,12,13] are the known published papers that deal with the distribution of zeros of neutral equations of the form (1.1).

The usual utilized technique is to relate the distance between adjacent zeros of any solution $x(t)$ of (1.1) to a positivity problem of certain solution of a first order delay differential inequality

$$x'(t)+P(t)x(t-r)\leqslant 0, \tag{1.2}$$

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