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Summation averaging technique for the oscillation of second order linear difference equations

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Abstract. The present paper is concerned with the oscillation of the second order linear difference equation $\Delta(c_{n-1}\Delta u_{n-1}) + q_n u_n = 0$, $n \geq 1$, where Δ is the usual forward difference operator $\Delta u_n = u_{n+1} - u_n$, $\{c_n\}$ and $\{q_n\}$ are sequences of real numbers with $c_n > 0$. Using summation averaging technique, some new oscillation criteria are obtained and the discrete analogue of known results due to Kamenev and Philos for the corresponding differential equations are established.

1. Introduction

Consider the second order difference equation

$$\text{(E)} \qquad \Delta(c_{n-1}\Delta u_{n-1}) + q_n u_n = 0, \quad n \geq 1,$$

where Δ denotes the forward difference operator $\Delta u_n = u_{n+1} - u_n$, $\{c_n\}$ and $\{q_n\}$ are sequences of real numbers with $c_n > 0$ for all integers $n \geq 0$.

By a solution of equation (E) we mean a real sequence $\{u_n\}$, $n = 0, 1, \dots$ satisfying equation (E).

A nontrivial solution $\{u_n\}$ of equation (E) is said to be nonoscillatory if there exists $N \geq 0$ such that $u_{n+1}u_n > 0$ for all $n \geq N$ and oscillatory otherwise. Equation (E) is called oscillatory if all its solutions are oscillatory. It is known that if equation (E) has an oscillatory solution, then all its solutions are oscillatory (see [5, pp. 153]).

The oscillatory, nonoscillatory and asymptotic properties of equation (E) have been considered extensively by many authors. Among the papers

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