

Oscillation of some nonlinear difference equations

H.A. El-Morshedy^{a,*} and S.R. Grace^b

^a Department of Mathematics, Damietta Faculty of Science, New Damietta 34517, Egypt

^b Department of Engineering Mathematics, Faculty of Engineering, Cairo University, Orman, Giza 12000, Egypt

Received 14 July 2001

Submitted by D. O'Regan

Abstract

We investigate the oscillatory behavior of all solutions of a new class of first order nonlinear neutral difference equations. Several explicit oscillation criteria are established. Our main results are supported by illustrative examples.

© 2003 Elsevier Science (USA). All rights reserved.

Keywords: Oscillation; Nonlinear equations; Difference equations

1. Introduction

In this paper we are concerned with the oscillation of the solutions of nonlinear neutral difference equation of the form

$$\Delta g(x_n + p_n x_{\sigma_n}) + f(n, x_{\tau_n}) = 0, \quad (\text{E})$$

in which $g: R \rightarrow R$, $f: Z^+ \times R \rightarrow R$ are continuous functions, $\{p_n\}_{n \geq n_0}$ is a sequence of nonnegative reals, and $\{\sigma_n\}_{n \geq n_0}$, $\{\tau_n\}_{n \geq n_0}$ are sequences of integers such that $\lim_{n \rightarrow \infty} \sigma_n = \lim_{n \rightarrow \infty} \tau_n = \infty$ and $\sigma_{n+1} > \sigma_n$ for all $n \geq n_0 \geq 0$. The function f satisfies the condition

$$\frac{f(n, x)}{h(x)} \geq q_n, \quad x \neq 0, \quad n \geq n_0, \quad (1)$$

* Corresponding author.

E-mail address: elmorshedy@hotmail.com (H.A. El-Morshedy).