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OSCILLATION AND NONOSCILLATION CRITERIA FOR HALF-LINEAR SECOND ORDER DIFFERENCE EQUATIONS

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ABSTRACT. New oscillation and nonoscillation criteria are established for the second order half-linear difference equation

$$\Delta(r_n\Phi(\Delta x_n)) + q_n\Phi(x_{n+1}) = 0, \quad \Phi(x) = |x|^{p-2}x, \quad p > 1,$$

via the Riccati technique. Some known results are also improved including the discrete version of the Hille-Wintner comparison theorem.

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1. INTRODUCTION

The aim of this paper is to study the oscillatory character of the half-linear difference equation

$$\Delta(r_n\Phi(\Delta x_n)) + q_n\Phi(x_{n+1}) = 0, \tag{1.1}$$

in which $\Phi(x) = |x|^{p-2}x$, $p > 1$, and $\{r_n\}$, $\{q_n\}$ are real sequences.

An interval $(m, m+1]$ is said to contain a generalized zero of a solution $\{x_n\}$ if $x_m \neq 0$ and $r_m x_m x_{m+1} \leq 0$. A solution is called oscillatory if it has infinitely many generalized zeros in the set of positive integers. Otherwise, the solution is called nonoscillatory. Equation (1.1) is called oscillatory if all its solutions are oscillatory. Equation (1.1) is said to be nonoscillatory if it has at least one nonoscillatory solution. If $r_n > 0$ for all $n \geq n_0$ and some positive integer n_0, then the nonoscillation of $\{x_n\}$ is equivalent to saying that x_n is either eventually positive or eventually negative.

Recently, there have been an extensive investigation on the various qualitative properties of equation (1.1) (e.g., oscillation, nonoscillation, conjugacy). Among the papers dealing with the oscillation and/or the nonoscillation of (1.1) and some related equations we refer to [3, 7, 8, 14, 18], [20]-[25] and to [1] for further results on the oscillation as well as the general theory of the difference equations. The study of (1.1)

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