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On the Oscillation and Asymptotic Behaviour of Solutions of a Neuronic Equation

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1. Introduction

One of the models proposed in [3] for the study of the dynamics of a single isolated neuron is the first order nonlinear delay differential equation

$$(E) \qquad \frac{dx(t)}{dt} = -x(t) + a\tanh(x(t) - bx(t-\tau)), \qquad t \geq 0$$

in which $a, b \in R$, $\tau > 0$. The stability characteristics of (E) have been studied by [3], but nothing is known about the oscillation of (E). It is the purpose of this paper to investigate the oscillation of (E). Other results regarding the estimation of the distance between adjacent zeros and the asymptotic behaviour of all solutions of (E) will be obtained.

By a solution of (E) we mean a real valued continuous function x that satisfies (E) for $t \geq 0$ and

$$x(s) = \phi(s), \qquad s \in [-\tau, 0]$$

where ϕ is assumed to be a continuous real valued function on $[-\tau, 0]$. As usual a nontrivial solution of (E) is called oscillatory if it has an infinite number of zeros on $[T, \infty)$ for all $T \geq 0$, otherwise the solution is called nonoscillatory. Thus any nonoscillatory solution is eventually of one sign. Equation (E) is called oscillatory if all its solutions are oscillatory. Equation (E) is said to be nonoscillatory if at least one solution of (E) is nonoscillatory.

The oscillatory behaviour of delay differential equations of various types has been the subject of interest of many authors. During the past few years hundreds of papers have been published as well as a number of books (e.g. [1, 2, 4]). One of the main tools of studying the oscillation and nonoscillation of nonlinear delay differential equations is the linearization technique by which the oscillatory behaviour of the nonlinear equation may be, completely, characterized from the oscillation of a related linear one. Unfortunately, for the best of the authors knowledge, none of the known linearization results can be applied to (E).

The following result, [4, Theorem 2.3.1], will be needed.