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On the global attractivity and oscillations in a class of second-order difference equations from macroeconomics

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New global attractivity criteria are obtained for the second-order difference equation

$$x_{n+1} = cx_n + f(x_n - x_{n-1}), \quad n = 1, 2, \ldots$$

via a Lyapunov-like method. Some of these results are sharp and support recent related conjectures. Also, a necessary and sufficient condition for the oscillation of this equation is obtained using comparison with a second-order linear difference equation with positive coefficients.

Keywords: nonlinear difference equations; global attractivity; oscillation; macroeconomics models

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1. Introduction

Consider the second-order difference equation

$$x_{n+1} = cx_n + f(x_n - x_{n-1}), \quad n = 1, 2, \ldots \tag{1.1}$$

where $c \in [0, 1), f : \mathcal{R} \to \mathcal{R}$ is a continuous real function and the initial values x_0, x_1 are real numbers. Various particular cases of (1.1) have appeared in the mathematical models of macroeconomics. For prototype examples, the reader is referred to Samuelson [11], Hicks [7], and Puu [10]. Motivated by those examples, Sedaghat [12] proposed and investigated the general form (1.1). We mention here that for sigmoidal or tanh-like nonlinearities, equation (1.1) can also be regarded as the discrete analogue of the single delayed neuron model

$$x'(t) = -\alpha x(t) + f(x(t) - x(t - \tau)),$$

using (forward) Newton discretization scheme with step size equals τ. An account of the stability analysis and/or the oscillations of the above continuous *neuronic* equation and some related equations can be found in [2–5], while a higher-order discrete neuronic version was been investigated by Hamaya [6].

The global attractivity (stability), boundedness and/or oscillations of (1.1) were considered by [8,12–14]. Very recently, Li and Zhang [9] studied its bifurcation.

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