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Convergence to equilibria in discrete population models

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For a family of difference equations $x_{n+1} = \alpha x_n + f(x_{n-k})$, $n = 0, 1, \ldots$, where $\alpha \in (0, 1)$, $k \in \{1, 2, \ldots\}$, and $f : [0, \infty) \to (0, \infty)$ is continuous and decreasing, we find sufficient conditions for the convergence of all solutions to the unique positive equilibrium. In particular, we improve, up to our knowledge, all previous results on the global asymptotic stability of the equilibrium in the particular cases of the discrete Mackey–Glass and Lasota–Wazewska models in blood-cells production.

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Mathematics Subject Classificattion (2000): 39A10; 39A11; 92D25

1. Introduction

It is well-known that many discrete models in biology can be described by a difference equation

$$x_{n+1} = \alpha x_n + f(x_{n-k}), \quad n = 0, 1, \ldots, \tag{1.1}$$

where $\alpha \in (0, 1)$, $k \in \{1, 2, \ldots\}$, and $f : [0, \infty) \to (0, \infty)$ is a continuous function. For some examples, see sections 4.5–4.7 in the monograph [11]. Equation (1.1) is not only interesting by itself, but also as the discretization of an extensively studied family of delay differential equations, namely

$$x'(t) = -\delta x(t) + f(x(t - h)), \quad \delta > 0, \quad h > 0. \tag{1.2}$$

Relations between equations (1.1) and (1.2) can be found, for example, in refs. [1,10]. For processes modeled by equation (1.2), the reader can find many examples in the interesting list in [8, p. 78], including models in neurophysiology, metabolic regulation and agricultural commodity markets.

We will focus our attention on the case of a decreasing nonlinearity f. In fact, we will assume that f is differentiable and $f'(x) < 0$ for all $x > 0$. Under this hypothesis, it is clear that equation (1.1) has a unique constant solution $(\bar{x})$, where $\bar{x}$ solves the scalar equation

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