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Global attractors for difference equations dominated by one-dimensional maps

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The global attractivity character of nonlinear higher order difference equations of the form

$$x_{n+1} = g(x_n, x_{n-1}, \ldots, x_{n-k}), \quad n \geq 0$$

is investigated when g is dominated by an interval scalar map. Some basic properties of the interval map are obtained and used to prove new global attractivity criteria for the above equation with no monotonicity restrictions on g. Our results are applied to many models from mathematical biology and economy. The derived global attractivity criteria of these models are either new or improve substantially known ones.

Keywords: difference equations; global attractor; discrete population models; Schwarzian derivative

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1. Introduction

In this paper, we consider higher order difference equations of the form

$$x_{n+1} = g(x_n, x_{n-1}, \ldots, x_{n-k}), \quad n \geq 0 \tag{1}$$

where $g : I^{k+1} \to \mathrm{Cl}\, I$ is continuous, k is a nonnegative integer and I denotes a (closed, open or semiopen) subinterval of $\mathbb{R}$ (with $\mathrm{Cl}\, I$ being its closure as a subset of $\mathbb{R}$). We denote by a and b, respectively, the left and right endpoint of I ($a = \infty$ and/or $b = \infty$ is allowed).

If for an initial sequence $(x_n)_{n=-k}^{0} \subset I$ the sequence $(x_n)_{n=-k}^{\infty}$ is well defined by Equation (1), then we call it a *full orbit* of (1).

The global attractivity of the unique equilibrium of some particular cases of (1) can be characterized from the global attractivity of the fixed point of an underlying interval map. This is implemented in several proofs in Refs. [1,7,8,13,16,17,28] and appears explicitly in Refs. [9,10,15] where different prototypes of Clark's model

$$x_{n+1} = \alpha x_n + (1 - \alpha) h(x_{n-k}) \tag{2}$$

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