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# New oscillation criteria for second order linear difference equations with positive and negative coefficients

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ABSTRACT

The oscillation of second order neutral difference equations with positive and negative coefficients of the form

$$\Delta^2(x_n+\lambda a_n x_{n-\tau})+p_n x_{n-\delta}-q_n x_{n-\sigma}=0,\quad \lambda=\pm 1$$

is investigated. We obtain many new results using the comparison between both first order and second order difference equations. An example is given to show the strength of the obtained results.



## 1. Introduction

The oscillatory properties of difference equations of many types have been the subject of extensive investigations reflecting the importance of this topic in applications (see [1–3]). Among the equations which did not undergo the deserved investigation are the equations with positive and negative coefficients. This, probably, is due to the difficulty associated with the presence of negative and positive coefficients in one equation. Some authors studied the oscillation and/or nonoscillation of first order difference equations of the form

$$\Delta(x_n-a_n x_{n-\tau})+p_n x_{n-\delta}-q_n x_{n-\sigma}=0; \tag{1}$$

see [1,3–6] and the references cited therein. For second order difference equations of this type, namely,

$$\Delta^2(x_n+\lambda a_n x_{n-\tau})+p_n x_{n-\delta}-q_n x_{n-\sigma}=0,\quad \lambda=\pm 1 \tag{2}$$

it seems that the nonoscillation theory is more developed than the oscillation theory (e.g., [7–9]). At least we know from [8] that (2) is nonoscillatory provided that

$$\sum_{i=n_0}^{\infty} n|p_n|<\infty \quad \text{and} \quad \sum_{i=n_0}^{\infty} n|q_n|<\infty, \tag{3}$$

when either $0\le a_n\le a<1$ or $-1<a<a_n\le 0$ for $n\ge n_0$.

In fact, as far as this author knows, the only known work on the oscillation of (2) is [10].

Our main objective here is to obtain new oscillation criteria for (2) when $\delta,\sigma$ and $\tau$ are nonnegative integers. We use the comparison with both first order delay and second order ordinary difference equations. In particular, we benefit from the theory developed in [11] for the oscillation and nonoscillation of second order functional difference equations and the advances of the oscillation theory of the second order ordinary difference equation

$$\Delta^2 u_n+h_n u_{n+1}=0.$$

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