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The Global Attractivity of Difference Equations of Nonincreasing Nonlinearities with Applications

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Abstract—The global attractivity of the unique positive equilibrium of the equation $x_{n+1} = g(x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_m})$, $n = 0, 1, 2, \ldots$, is investigated where g is a nonincreasing continuous function in each of its arguments and the k are positive integers. We are able to use our results to extract some important global attractivity criteria of the positive equilibria of the discrete Clark model $x_{n+1} = \alpha x_n + f(x_{n-k})$ and the rational recursive sequence $x_{n+1} = (a + bx_n)/(A + x_{n-1})$ as well.

Keywords—Global attractivity, Clark model, Difference equation.

1. INTRODUCTION

Consider the following difference equation:

$$x_{n+1} = g(x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_m}), \qquad n = 0, 1, 2, \ldots, \tag{E}$$

where $G : I^m \to (0, \infty)$, $I = (0, \infty)$, is nonincreasing in each of its arguments, and $k_1, k_2, \ldots, k_m$ are nonnegative integers. We define $k = \max\{k_i : i = 1, 2, \ldots, m\}$ and associate for each solution, $\{x_n\}_{n>0}$, the initial values $x_{-i} > 0$, $i = 0, 1, 2, \ldots, k$. Let

$$G(y) = g(y, y, \ldots, y), \qquad \text{where } (y, y, \ldots, y) \in I^m.$$

Then $G : I \to (0, \infty)$ is a nonincreasing function. So, (E) has a unique equilibrium, say $\bar{x}$, given by the equation

$$G(\bar{x}) = \bar{x}. \tag{1.1}$$

Throughout this work, function G will be assumed to satisfy either one of the following assumptions on I:

$$G(G(x)) \begin{cases} < x, & \text{if } x > \bar{x}, \\ > x, & \text{if } x < \bar{x}, \end{cases} \tag{H$_1$}$$

$$G(G(x)) = x, \qquad \text{for all } x > 0. \tag{H$_2$}$$

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