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Oscillation and asymptotic behaviour of a class of higher-order non-linear difference equations

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Abstract. The dynamical characteristics of scalar difference equations of the form

$$x_{n+1} = f_1(x_{n-\tau_1}) + f_2(x_{n-\tau_2}), \qquad n = 0, 1, 2, \dots,$$

are investigated. A necessary and sufficient condition is obtained for all positive solutions to be oscillatory about a unique positive equilibrium point and sufficient criteria for the global attractivity of the equilibrium are established. Also, the stability and periodicity of more general equations are studied via comparison with the corresponding properties of an associated first-order non-linear equation.

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Key words. global attractivity – periodicity – oscillation – difference equation

1. Introduction

The dynamical characteristics of the family of non-linear difference equations

$$x_{n+1} = h_\mu(x_n), \qquad n = 0, 1, 2, \dots, \tag{1}$$

where $\mu \in R$ is a parameter, $h_\mu \in C(I)$, $I \subseteq R$, have been investigated extensively by many authors. Some members of this family may look simple but their solutions display complicated behaviour (chaos) as the parameter μ increases beyond a certain critical value (see [5,16,17,21]). The famous result of Li and Yorke [12] confirms the occurrence of that chaotic behaviour if h_μ has period 3 point. On the other hand, Sharkovsky [20] (see also [12]) has pointed out that the lack of period 2 points of the map h_μ implies the absence of all periodic points of higher orders and hence (1) will not exhibit such chaotic behaviour in the sense of Li and Yorke [12]. Moreover, for some members of (1), the lack of period 2 points is also a sufficient condition for the unique equilibrium point to attract all solutions of (1). For instance, Cull [1–3] has studied a prototype of (1), namely

$$x_{n+1} = f(x_n), \qquad n = 0, 1, 2, \dots, \tag{2}$$

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