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Periodic points and stability in Clark's delayed recruitment model

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Abstract

We provide further insight on the dynamics of Clark's delayed recruitment equation, depending on the relevant parameters involved in the model. We pay special attention to the stability and bifurcations from the positive equilibrium, and to the existence and attraction properties of nontrivial cycles. A detailed analysis is worked out for a three-dimensional example.

Keywords: Clark's model; Delayed discrete population model; Stability; Attracting cycle; Ricker oscillation; Bifurcation analysis

1. Introduction

Clark's equation is a simple discrete-time mathematical model to represent the evolution of a population in which the number of adults each year is calculated as the sum of the survival adults in the previous year and the recruitment, which is in general a nonlinear function of the size of population of adults a number of k years before. See, e.g., [4,5]. In general, it is written as

$$x_{n+1} = \alpha x_n + f(x_{n-k}), \tag{1}$$

where $\alpha \in [0, 1)$ is a survival rate, and $f : (0, \infty) \to [0, \infty)$ is the recruitment function. Here, we will use a slightly different form of Clark's equation, already employed by Fisher [12], and suggested by Botsford [4] to explain an apparent contradiction between two data tables. Namely, we will consider equation

$$x_{n+1} = \alpha x_n + (1 - \alpha)h(x_{n-k}). \tag{2}$$

Notice that, for a fixed $\alpha \in [0, 1)$, Eqs. (1) and (2) are equivalent, taking $h = (1 - \alpha)^{-1} f$. We believe that Eq. (2) is more convenient to investigate the variation in the dynamics of the solutions as the parameter α varies on [0, 1). We will try to justify this assertion. One of the key points in Clark's model is how the stock-recruitment relationship should be. Form (2) represents well two of the postulates indicated in [4], namely: (a) high fecundity at low stock sizes (hence, the recruitment should be larger for decreasing α), and (b) increasing pre-recruitment mortality at high stock

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