Assignment No (2)

Name:		Date	
Topic:		Lecture No:	

Answer all of the following questions:

Algorithms, Complexity

Question 1: (2-6)

Consider Algorithm 2.3, which finds the location LOC and the value MAX of the largest element in an array DATA with n elements. Consider the complexity function $C(n)$, which measures the number of times LOC and MAX are updated in Step 3. (The number of comparisons is independent of the order of the elements in DATA.)

(a) Describe and find $C(n)$ for the worst case.
(b) Describe and find $C(n)$ for the best case.
(c) Find $C(n)$ for the average case when $n = 3$, assuming all arrangements of the elements in DATA are equally likely.

Question 2: (2-7)

Suppose Module A requires M units of time to be executed, where M is a constant. Find the complexity $C(n)$ of each algorithm, where n is the size of the input data and b is a positive integer greater than 1.

(a) **Algorithm P2.7A:**

1. Repeat for I = 1 to N:

2. Repeat for J = 1 to N:
3. Repeat for K = I to N:
4. Module A.

[End of Step 3 loop.]
[End of Step 2 loop.]
[End of Step 1 loop.]
5. Exit.

(b) **Algorithm P2.7B:**

1. Set J := 1.
2. Repeat Steps 3 and 4 while J ≤ N:
3. Module A.
4. Set J := B × J.
[End of Step 2 loop.]
5. Exit.

Observe that the algorithms use N for n and B for b.)

Question 3: (2-8)

(a) Write a procedure FIND(DATA, N, LOC1, LOC2) which finds the location LOC1 of the largest element and the location LOC2 of the second largest element in an array DATA with $n > 1$ elements.

(b) Why not let FIND also find the values of the largest and second largest elements?

Question 4: (2-9)

An integer $n > 1$ is called a *prime* number if its only positive divisors are 1 and n; otherwise, n is called a *composite* number. For example, the following are the prime numbers less than 20:

2, 3, 5, 7, 11, 13, 17, 19

If $n > 1$ is not prime, i.e., if n is composite, then n must have a divisor $k \neq 1$ such that $k \leq \sqrt{n}$ or, in other words, $k^2 \leq n$.

Question 5: (2-10)

Suppose $P(n) = a_0 + a_1 n + a_2 n^2 + \cdots + a_m n^m$; that is, suppose degree $P(n) = m$. Prove that $P(n) = O(n^m)$.

Question 6: (additional)

Suppose that $T_1(n)$ and $T_2(n)$ are the time complexities of two program fragments P_1 and P_2 where $T_1(n) = O(f(n))$ and $T_2(n) = O(g(n))$, what is the time complexity of program fragment P_1 followed by P_2?

Supplementary

Question 7: (2-5)

Consider the complexity function $C(n)$ which measures the number of times LOC is updated in Step 3 of Algorithm 2.3. Find $C(n)$ for the average case when $n = 4$, assuming all arrangements of the given four elements are equally likely. (Compare with Solved Problem 2.6.)

Question 8: (2-6)

Consider Procedure P2.8, which finds the location LOC1 of the largest element and the location LOC2 of the second largest element in an array DATA with $n > 1$ elements. Let $C(n)$ denote the number of comparisons during the execution of the procedure.

(a) Find $C(n)$ for the best case.
(b) Find $C(n)$ for the worst case.
(c) Find $C(n)$ for the average case for $n = 4$, assuming all arrangements of the given elements in DATA are equally likely.

Question 9: (2-7)

Repeat Supplementary Problem 2.6, except now let $C(n)$ denote the number of times the values of FIRST and SECOND (or LOC1 and LOC2) must be updated.

Question 10: (2-8)

Suppose the running time of a Module A is a constant M. Find the order of magnitude of the complexity function $C(n)$ which measures the execution time of each of the following algorithms, where n is the size of the input data (denoted by N in the algorithms).

(a) **Procedure P2.8A:**

```
1. Repeat for I = 1 to N:
2.     Repeat for J = 1 to I:
3.         Repeat for K = 1 to J:
4.             Module A.
           [End of Step 3 loop.]
       [End of Step 2 loop.]
       [End of Step 1 loop.]
5. Exit.
```

(b) **Procedure P2.8B:**

```
1. Set J := N.
2. Repeat Steps 3 and 4 while J > 1.
3.     Module A.
4.     Set J := J/2.
   [End of Step 2 loop.]
5. Return.
```

Question 11: (Additional)

Find the order of complexity of the following program.

```
fun(n)
  {if(n<=2)return (1); else
    return ((fun(n-1)*fun(n-2));}
```

Variables, Data Types

Question 12: (additional)

Define Abstract Data Type. Explain it briefly.

Question 13: (2-13)

Describe briefly the difference between local variables, parameters and global variables.

Question 14: (2-14)

Suppose NUM denotes the number of records in a file. Describe the advantages in defining NUM to be a global variable. Describe the disadvantages in using global variables in general.

Question 15: (2-15)

Suppose a 32 bit memory location AAA contains the following sequence of bits:

0100 1101 1100 0001 1110 1001 0101 1101

Determine the data stored in AAA.

Question 16 : (2-16)

Mathematically speaking, integers may also be viewed as real numbers. Give some reasons for having two different data types.

Programming Problems

2.1 Write a function subprogram DIV(J, K), where J and K are positive integers such that DIV(J, K) = 1 if J divides K but otherwise DIV(J, K) = 0. (For example, DIV(3, 15) = 1 but DIV(3, 16) = 0.)

2.2 Write a program using DIV(J, K) which reads a positive integer N > 10 and determines whether or not N is a prime number. (*Hint:* N is prime if (i) DIV(2, N) = 0 (i.e., N is odd) and (ii) DIV(K, N) = 0 for all odd integers K where $1 < K^2 \leq N$.)

2.3 Translate Procedure P2.8 into a C program; i.e., write a program which finds the location LOC1 of the largest element and the location LOC2 of the second largest element in an array DATA with N > 1 elements. Test the program using 70, 30, 25, 80, 60, 50, 30, 75, 25, and 60.

2.4 Translate the sieve method for finding prime numbers, described in Solved Problem 2.9, into a C program to find the prime numbers less than N. Test the program using (a) N = 1000 and (b) N = 10 000.

2.5 Let C denote the number of times LOC is updated using Algorithm 2.3 to find the largest element in an array A with N elements.

(a) Write a subprogram COUNT(A, N, C) which finds C.

(b) Write a Procedure P2.27 which (i) reads N random numbers between 0 and 1 into an array A and (ii) uses COUNT(A, N, C) to find the value of C.

(c) Write a program which repeats Procedure P2.27 1000 times and finds the average of the 1000 C's.

(i) Test the program for N = 3 and compare the result with the value obtained in Solved Problem 2.6.

(ii) Test the program for N = 4 and compare the result with the value in Supplementary Problem 2.5.

2.6 Write a pseudocode for an algorithm that receives an integer, prints the number of digits and the sum of digits in the integer.

Multiple Choice Questions

2.1 _____ of a set of n elements is an arrangement of the elements in a given order.
(a) Combination (b) Permutation
(c) Exponent (d) Logarithm

2.2 There are _____ permutations of a set of n elements.
(a) $n!$ (b) n
(c) $n2$ (d) $n+1$

2.3 Logarithms to the base 10 are called _____ logarithms.
(a) Natural (b) Simple
(c) Common (d) Binary

2.4 The first part of an algorithm tells the _____ of the algorithm.
(a) Logic (b) Process
(c) Purpose (d) Steps

2.5 Each step of an algorithm may contain its _____ in brackets.
(a) Purpose (b) Functions
(c) Steps (d) Comments

2.6 The term _____ will be used for an independent algorithmic module which solves a particular problem.
(a) Program (b) Logic
(c) Procedure (d) Name

2.7 _____ logic employs a number of conditions which lead to a selection of one out of several alternative modules.
(a) Selection (b) Sequential
(c) Iteration (d) Procedural

2.8 A structure is of the form:

```
If condition, then:
        [Module A]
  Else:
        [Module B]
[End of if structures]
```

What is this structure?
(a) Multiple Alternative
(b) Double Alternative
(c) Single Alternative
(d) None of the above

2.9 _____ loop uses a condition to control the loop.

(a) Repeat-for (b) Repeat

(c) Continue (d) Repeat-while

2.10 In complexity theory, _____ case refers to the expected value of $f(n)$.

(a) Average (b) Best

(c) Worst (d) Good

2.11 $O(n^2)$ is the complexity of which searching and sorting algorithm?

(a) Binary search (b) Linear search

(c) Merge sort (d) Bubble sort

2.12 The _____ notation is used when the function $g(n)$ defines a lower bound for the function $f(n)$.

(a) Omega (b) Big O

(c) Theta (d) Little Oh

2.13 Each program module contains its own list of variables called _____

(a) Global (b) Local

(c) Search (d) Binary

2.14 _____ function of C is used to allocate a block of memory.

(a) malloc() (b) calloc()

(c) free (d) realloc()

2.15 Variables that can be accessed by some program modules are called _____ variables.

(a) Global (b) Local

(c) Search (d) Nonlocal