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Modeling of nonlinear envelope solitons in strongly coupled dusty plasmas: Instability and collision

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Modeling of instability and collision of nonlinear dust-acoustic (NDA) envelope solitons in strongly coupled dusty plasmas (SCDPs) is theoretically investigated. The SCDPs consists of strongly correlated negatively variable-charged dust grains and weakly correlated Boltzmann electrons and ions. Using the derivative expansion perturbation technique, a nonlinear Schrödinger-type (NLST) equation for describing the propagation of NDA envelope solitons is derived. Moreover, the extended Poincaré–Lighthill–Kuo (EPLK) method is employed to deduce the analytical phase shifts and the trajectories after the collision of NDA envelope solitons. In detail, the results show that both modulation instability and phase shift after collision of NDA envelope solitons will modify with the increase in the effects of the viscosity, the relaxation time, and the dust charge fluctuation. Crucially, the modeling of dust-acoustic envelope solitons collision, as reported here, is helpful for understanding the propagation of NDA envelope solitons in strongly coupled dusty plasmas.

Keywords: dust acoustic wave envelopes, modulational instability, head-on collision, polarization effects

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1. Introduction

At the present time, there is a great deal of interest in perception the physics of dusty plasmas (containing besides electrons and ions also charged microparticles) because of their vital role in understanding the properties of different collective processes in space and laboratory dusty plasmas.[1–3] It is well known that nonlinear dust-acoustic NDA wave represents one of the important aspects of nonlinear waves in modern dusty plasma researches. Actually, one of the most interesting characteristics of a dusty plasma is the dust charge fluctuation which enters as an extra dynamical variable controlling the dust grain motion.[4] A great number of investigators[5–7] were studied the modification of the dusty plasma collective properties due to including the dust charge variation. For example, Xie *et al.*[5] derived dust-acoustic wave with varying dust charge and they showed that only rarefactive solitary waves exist when the Mach number lies within an appropriate regime, depending on the system parameters. On the other side, because of the large charges on the individual dust particles, the dust components in a plasma can be easily in the strongly coupled regime where the electrostatic energy of dust particle interactions greatly exceeds the dust kinetic energy.[8] Theoretically, Ikezi[9] first pointed out that the Coulomb crystallization of charged dust grains interacting via a repulsive Yukawa force in a plasma, when the Coulomb coupling parameter Γ [the ratio between the coulomb interaction energy density and the dust thermal energy T_d] exceeds 171. Clearly, large Γ at room temperature is achieved when dust grains acquire tens of thousands electrons on their surface. This theoretical prediction was verified experimentally with the observations of Thomas *et al.*[10] It is well known that, there are two regimes in the generalized hydrodynamic model: i) the hydrodynamic regime (i.e., $\omega\tau_m \ll 1$) and ii) kinetic regime (i.e., $\omega\tau_m \gg 1$), where τ_m is the memory (viscoelastic) relaxation time and ω^{-1} is the typical time scale of the wave under consideration.[11] On one hand, for the hydrodynamic regime, the viscoelastic relaxation is instantaneous, and one has the usual hydrodynamic equation. In this case dust grains support only the longitudinal dust acoustic wave (LDAW), which suffers only viscous dissipation. On the other hand, for the kinetic regime, the viscoelastic relaxation is not instantaneous, and the dusty plasma supports both LDAW as well as the transverse shear wave.[7,12]

In last few years, there have been a few theoretical investigations of dust-acoustic wave, double layer, and dust acoustic shock wave in strongly coupled dusty plasmas (SCDPs).[13–19] For instance, Mamun and Cairns[16] have studied low frequency electrostatic dust modes in SCDPs including dust charge fluctuations. They found that the dust-acoustic wave mode becomes unstable due to the effect of equilibrium dust grain charge inhomogeneity. In addition, they observed that the influence of strong correlations in the dust fluid signifi-

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cantly modify the dispersion properties of the existing dust-acoustic wave mode. Recently, Rahman and Mamun[18] have investigated a SCDP containing strongly correlated negatively charged dust grains and weakly correlated adiabatic effects. They demonstrated that both the dust-acoustic wave and the dust acoustic shock wave are found to exist with negative potential only[18] Furthermore, the effects of a modulational instability (MI) of nonlinear wave in SCDPs have not investigated enough.[20] Moreover, Veeresha *et al.*[8] reported that the dust-acoustic wave can suffer a MI at short wavelengths such that a slow parallel modulation of a finite amplitude monochromatic plane wave and in some limits lead to the formation of an envelope soliton pulse. Additionally, they[8] have demonstrated that the effective dust temperature and the effects of polarization force, respectively, are very important to illustrate various characteristics of the dust-acoustic wave propagation in many space and laboratory dusty plasma situations. Very recently, El-Labany *et al.*[21] examined the MI of the modulated dust acoustic envelope solitons in SCDPs including the polarization force effect only. They demonstrated that as the effect of polarization force increases, both the angular frequency and the group velocity of the dust acoustic envelope solitons decrease.

Presently, in the process of soliton propagation in SCDPs, wave–wave collision has attracted more and more attention in modern plasma researches. Actually, one of the striking properties of solitons is their asymptotic preservation of form when they undergo a collision.[22] The unique effects due to the collision are their phase shifts and trajectories. Many authors have investigated the collision of two solitary waves in various plasma models using the extended Poincare–Lighthill–Kuo (EPLK) method.[23–27] The EPLK method has been employed to study wave–wave collision in dusty plasmas.[28,31] For example, Xue[28,29] demonstrated that the dust charge fluctuation and the magnitude of the magnetic field have strong effects on the phase shifts. Later, Li *et al.*[30] illuminated that the value of phase shift increases as the angle between the propagation directions of the two dust acoustic solitary waves increases. Recently, El-Labany *et al.*[31] stated that the oblique collision strongly affects the trajectories of dust acoustic solitary waves after collision. In addition, very recently, El-Labany *et al.*[21] have firstly addressed the head on collision of two modulated dust acoustic envelope dark solitons in the hydrodynamic regime in SCDPs through the nonlinear Schrödinger equation framework. They[21] showed that the phase shifts for the dark solitons increase as the effect of polarization force decreases. Up to now, the collision of two NDA envelope solitons in the kinetic regime of SCDPs has not been performed. Furthermore, particular questions to be answered are: how do MI, angular frequency, group velocity, and the collision of the NDA envelope solitons are influenced by the incorporated new effects; the viscosity, the relaxation time and the dust charge fluctuation. Therefore, it is expected that the answers of these questions will lead to a significant improvement for understanding nonlinear wave collision observed in SCDPs experiments.

This paper is organized in the follows: In Section 2, the basic equations for describing NDA envelope solitons in SCDPs within the kinetic regime are introduced. In Section 3, the derivative expansion technique is used to derive a nonlinear Schrödinger-type (NLST) equation, then studying MI of NDA envelope solitons in various regions of the physical parameters involved. In Section 4 using the EPLK method, the analytical phase shifts and the trajectories after the collision are deduced, and the collision between two NDA envelope dark solitons is discussed. Based on physical parameters, Section 5 is devoted to show numerical illustrations and the discussion of the present findings. Section 6 is devoted to concluding remarks.

2. Govern equations

Let us consider SCDPs consisting of negatively variable-charged dust grains, electrons, and ions. The electrons and the ions are considered weakly coupled due to their higher temperatures and smaller electric charges, while the dust grains are assumed to be strongly coupled because of their low temperature and large electric charges. The charge neutrality condition reads $Z_{d0}n_{d0}+n_{e0}=n_{i0}$, where n_{e0}, n_{i0}, and n_{d0} are the unperturbed number densities of electrons, ions, and dust grains, respectively, and Z_{d0} is the unperturbed number of electrons residing on the dust grain surface. Owing to the low phase velocity of dust acoustic waves (in comparison with electron and ion thermal velocities), the electrons and the ions are assumed to behave as light fluids compared to the dust fluid and model them by Boltzmann distribution

$$n_e = n_{e0}\exp\left(\frac{e\varphi}{T_e}\right), \tag{1}$$

$$n_i = n_{i0}\exp\left(-\frac{e\varphi}{T_i}\right). \tag{2}$$

The set of equations describing the proposed SCDPs fluid in kinetic regime is governed by[17,19]

$$\frac{\partial n_d}{\partial t}+\frac{\partial(u_d n_d)}{\partial x}=0, \tag{3}$$

$$D_\tau\left[n_d m_d D_t u_d - e n_d Z_d\frac{\partial\varphi}{\partial x} + e n_d Z_d R\left(\frac{n_i}{n_{i0}}\right)^{1/2}\frac{\partial\varphi}{\partial x}+T_{ef}\frac{\partial n_d}{\partial x}\right]-\eta_l\frac{\partial^2 u_d}{\partial x^2}=0, \tag{4}$$

$$\frac{\partial^2\varphi}{\partial x^2}=4\pi e\left[n_e+n_d Z_d-n_i\right], \tag{5}$$

where n_d is the dust number density, u_d is the dust fluid speed, φ is the electric potential. t and x are the time and the space variables, respectively. The third term on the left-hand side of Eq. (4) is due to the polarization force, which arises due to the interaction between thermal ions and highly negatively charged dust grains, one can defined the polarization force as $F_{\rm p} = -eZ_{\rm d}R(n_{\rm i}/n_{\rm i0})^{1/2}\nabla\varphi$, where $R\left(=Z_{\rm d}e^2/4\lambda_{\rm Di0}T_{\rm i}\right)$ is a parameter determining the effect of polarization force, and $\lambda_{\rm Di0} = (T_{\rm i}/4\pi n_{\rm i0}e^2)^{1/2}$. $T_{\rm i(e)}$ is the ion (electron) temperature in energy unit, $n_{\rm i(e)}$ is the ion (electron) number density, $Z_{\rm d}$ is the number of electrons residing on the dust grain surface, and e is the magnitude of the electron charge. The fourth term represents the pressure force, and the fifth term is the viscosity force, which is introduced by taking the kinematic viscosity among the plasma constituents into account. $m_{\rm d}$ is the dust grain mass, $T_{\rm ef}= (T_{\rm d}\mu_{\rm d} + T_*)$ is the effective dust temperature consisting of two parts: T_* arising from the electrostatic interactions among highly negatively variable-charged dust grains and $T_{\rm d}\mu_{\rm d}$ arising from thermal pressure and

$$D_\tau = 1+\tau_{\rm m}\frac{\partial}{\partial t},\quad D_t=\frac{\partial}{\partial t}+u_{\rm d}\frac{\partial}{\partial x},$$

where $\tau_{\rm m}$ is the viscoelastic relaxation time, $\mu_{\rm d}$ is the compressibility, and $\eta_{\rm l}$ is the longitudinal viscosity coefficient. The parameters T_*, $\tau_{\rm m}$, and $\mu_{\rm d}$ are given by[32]

$$T_* = \frac{N_{\rm nn}}{3}\frac{e^2Z_{\rm d}^2}{a_{\rm d}}(1+\kappa)\,{\rm e}^{-\kappa},$$
$$\tau_{\rm m} = \frac{\eta_{\rm l}}{n_{\rm d0}T_{\rm d}}\left[1-\mu_{\rm d}+\frac{4}{15u(\Gamma)}\right]^{-1},$$
$$\mu_{\rm d} = \frac{1}{T_{\rm d}}\frac{\partial p_{\rm d}}{\partial n_{\rm d}} = 1+\frac{1}{3}u(\Gamma)+\frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial\Gamma},$$

where $N_{\rm nn}$ is determined by the dust structure and corresponds to the number of the nearest neighbors, $\Gamma = \left(q_{\rm d}^2/a_{\rm d}T_{\rm d}\right)\exp(\kappa)$, $\kappa[=a_{\rm d}/\lambda_{\rm D}]$ is the screening parameter, and $u(\Gamma)$ is a measure of the excess internal energy, $q_{\rm d}$ is the dust grain charge, $a_{\rm d}$ is the intergrain distance, $T_{\rm d}$ is the dust temperature in the energy unit, and $\lambda_{\rm D}$ is the dusty plasma Debye radius. In fact, $u(\Gamma)$ can be written as: $u(\Gamma)=a(k)\Gamma+b(k)\Gamma^{1/3}+c(k)+d(k)\Gamma^{-1/3}$. For a Yukawa fluid, the coefficients up to order k^4 are given by

$$a(k) = k/2-0.899-0.103k^2+0.003k^4,$$
$$b(k) = 0.565-0.026k^2-0.003k^4,$$
$$c(k) = -0.207-0.086k^2+0.018k^4,$$
$$d(k) = -0.031+0.042k^2-0.008k^4.$$

It is well known that the dust grains are negatively charged due to plasma electron current

$$I_{\rm e} = -e\pi r_{\rm d}^2(8T_{\rm e}/\pi m_{\rm e})^{1/2}n_{\rm e}\exp(eq_{\rm d}/r_{\rm d}T_{\rm e}),$$

and ion current

$$I_{\rm i} = e\pi r_{\rm d}^2(8T_{\rm i}/\pi m_{\rm i})^{1/2}n_{\rm i}(1-eq_{\rm d}/r_{\rm d}T_{\rm i}),$$

and the charges on the dust grains varies continuously with time. The varying dust charge $q_{\rm d}$ is governed by the current balance equation[33]

$$\left(\frac{\partial}{\partial t}+u_{\rm d}\frac{\partial}{\partial x}\right)q_{\rm d} = I_{\rm e}+I_{\rm i}. \tag{6}$$

Now, let us write the normalized set of equations for SCDPs fluid as follows:

$$\frac{\partial n_{\rm d}}{\partial t}+\frac{\partial(n_{\rm d}u_{\rm d})}{\partial x} = 0, \tag{7}$$

$$\left(1+\tau_{\rm m}\frac{\partial}{\partial t}\right)\left[n_{\rm d}\frac{\partial u_{\rm d}}{\partial t}+n_{\rm d}u_{\rm d}\frac{\partial u_{\rm d}}{\partial x}-n_{\rm d}Z_{\rm d}\frac{\partial\varphi}{\partial x}\right.$$
$$\left.+Z_{\rm d}^2R\left(\frac{n_{\rm i}}{\mu_{\rm i}}\right)^{1/2}n_{\rm d}\frac{\partial\varphi}{\partial x}+\gamma\frac{\partial n_{\rm d}}{\partial x}\right]-\eta_{\rm l}\frac{\partial^2u_{\rm d}}{\partial x^2} = 0, \tag{8}$$

$$\frac{\partial^2\varphi}{\partial x^2} = Z_{\rm d}n_{\rm d}+\mu_{\rm e}\exp(\sigma\varphi)-\mu_{\rm i}\exp(-\varphi), \tag{9}$$

$$\left(\frac{\partial}{\partial t}+u_{\rm d}\frac{\partial}{\partial x}\right)Q_{\rm d} = I_{\rm e}+I_{\rm i}, \tag{10}$$

where the normalized electron current and normalized ion current are given by

$$I_{\rm e} = -\exp[\sigma(\varphi+\psi)],$$
$$I_{\rm i} = \alpha_1\delta\exp(-\varphi)(1-\psi). \tag{11}$$

Here, the variables $n_{\rm e}$, $n_{\rm i}$, and $n_{\rm d}$ are normalized by $Z_{\rm d0}n_{\rm d0}$, $Z_{\rm d0}n_{\rm d0}$, and $n_{\rm d0}$, respectively, $u_{\rm d}$ by $\lambda_{\rm D}\omega_{\rm pd}$, φ by $T_{\rm i}/e$, t by $\omega_{\rm pd}^{-1}$, x by $\lambda_{\rm D}$, $\tau_{\rm m}$ by $\omega_{\rm pd}^{-1}$, η by $m_{\rm d}n_{\rm d0}\omega_{\rm pd}\lambda_{\rm D}^2$, and $I_{\rm e}$ and $I_{\rm i}$ by $er_{\rm d}^2\sqrt{8\pi T_{\rm e}}/m_{\rm e}$, where $\psi\,(=e\Phi/T_{\rm i})$ denotes the normalized dust grain surface potential relative to the plasma potential φ. At equilibrium $I_{\rm e}+I_{\rm i}\approx 0$, with the aid of Eq. (11), one can write Eq. (6) as

$$\alpha_1\delta\exp(-\varphi)(1-\psi)-\exp[\sigma(\varphi+\psi)] = 0. \tag{12}$$

The dust charge is defined by $q_{\rm d} = -eZ_{\rm d} = r_{\rm d}\Phi = r_{\rm d}T_{\rm i}\psi/e$; accordingly the normalized dust charges $Z_{\rm d}=\psi/\psi_0$ where $\psi_0 = \psi$ at $\varphi = 0$) is the surface floating potential with respect to the unperturbed plasma potential at an infinite place, which can be calculated from $\alpha_1\delta(1-\psi_0)-\exp(\sigma\psi_0) = 0$. Let us introduce the following notations: The dust plasma frequency $\omega_{\rm pd} = \sqrt{4\pi(Z_{\rm d0}e)^2n_{\rm d0}/m_{\rm d}}$, the plasma Debye length $\lambda_{\rm D} = \sqrt{T_{\rm i}/4\pi Z_{\rm d0}n_{\rm d0}e^2}$, $\alpha_1 = \sqrt{\sigma m_{\rm e}/m_{\rm i}} = \sqrt{\sigma/\mu_{\rm ie}}$, $\mu_{\rm ie} = m_{\rm i}/m_{\rm e}$, $\delta = \mu_{\rm i}/\mu_{\rm e}$, $\sigma = T_{\rm i}/T_{\rm e}$, $\mu_{\rm e} = n_{\rm e0}/Z_{\rm d0}n_{\rm d0}$, $\mu_{\rm i} = n_{\rm i0}/Z_{\rm d0}n_{\rm d0}$, $\gamma = T_{\rm ef}/T_{\rm i}$, $c_{\rm d} = \sqrt{Z_{\rm d0}T_{\rm i}/m_{\rm d}}$.

3. Derivation of the NLST equation

To examine the propagation of NDA envelope solitons in the proposed SCDPs, we analyze the outgoing solutions of Eqs. (7)–(12) by introducing the stretched coordinates[34]

$$\xi = \epsilon(x-\lambda t),\quad \tau = \epsilon^2 t,$$
$$\eta = \epsilon^{1/4}\eta,\quad \tau_{\rm m} = \epsilon^{1/4}\tau_{\rm m}, \tag{13}$$

where ϵ is a small (real) parameter and λ is the envelop group velocity to be determined later. The dependent variables are expanded as

$$G(x,t) = G_0 + \sum_{n=1}^{\infty} \epsilon^n \sum_{L=-\infty}^{\infty} G_L^n(\xi,\tau)\exp(\mathrm{i}L\Theta), \tag{14}$$

where $G_L^{(m)} = \left[n_{\mathrm{d}_L}^{(n)} u_{\mathrm{d}_L}^{(n)} \varphi_L^{(n)} Z_{\mathrm{d}_L}^{(n)}\right]$, $G_L^{(0)} = [1001]$, and $\Theta = kx - \omega t$. Here k and ω are real variables representing the fundamental (carrier) wave number and frequency, respectively. Since $G_L^{(m)}$ must be real, the coefficients in Eq. (14) have to satisfy the conditions $G_{-L}^{(m)} = G_L^{*(m)}$, where the asterisk indicates the complex conjugate. Substituting Eqs. (13) and (14) into Eqs. (7)–(12) and collecting terms of the same powers of ϵ. The first order ($n = 1$) equations read

$$-\omega L n_{\mathrm{d}}^{(1)} + kLu_{\mathrm{d}}^{(1)} = 0, \tag{15}$$

$$\begin{aligned}&(-\omega^2\tau_{\mathrm{m}} + k^2\eta_{\mathrm{l}})L^2u_{\mathrm{d}}^{(1)} + \omega k\tau_{\mathrm{m}}L^2(R-1)\varphi^{(1)}\\ &+\omega k\tau_{\mathrm{m}}L^2\mu_{\mathrm{d}}\gamma_* n_{\mathrm{d}}^{(1)} = 0,\end{aligned} \tag{16}$$

$$Z_{\mathrm{d}}^{(1)} = g_1\varphi^{(1)}, \tag{17}$$

$$-\left(\alpha + k^2\right)\varphi^{(1)} = n_{\mathrm{d}}^{(1)} + Z_{\mathrm{d}}^{(1)}. \tag{18}$$

The $L = 1$ (first harmonic) components lead to

$$\begin{aligned} n_{\mathrm{d}1}^{(1)} &= -\left(\alpha + k^2 + g_1\right)\varphi_1^{(1)},\\ Z_{\mathrm{d}1}^{(1)} &= g_1\varphi_1^{(1)},\\ u_{\mathrm{d}1}^{(1)} &= -\frac{\omega}{k}(\alpha + k^2 + g_1)\varphi_1^{(1)},\end{aligned} \tag{19}$$

where

$$\alpha = \mu_{\mathrm{e}}\sigma_{\mathrm{i}} + \mu_{\mathrm{i}}, \quad g_1 = -\frac{(1-\Psi_0)(1+\sigma)}{\Psi_0[1+\sigma(1-\Psi_0)]},$$

with the neutrality condition $\mu_{\mathrm{i}} = 1 + \mu_{\mathrm{e}}$. Moreover, in this order, we can derive the linear dispersion relation as

$$\omega^2 = k^2\left[\frac{\eta_{\mathrm{l}}}{\tau_{\mathrm{m}}} + \mu_{\mathrm{d}}\gamma_* + \frac{1-R}{\alpha_1 + k^2 + g_1}\right]. \tag{20}$$

This dispersion relation coincides exactly with that derived by El-Taibany and Kourakis,[6] upon omitting the nonthermality $a = 0$ (a is the nonthermal parameter introduced in their work)[6] and neglecting the viscosity η_{l} and the polarization effect. In addition, it agrees with the expression derived by Veeresha *et al.*[8] for the dust acoustic mode in SCDPs in short wavelength regime with R and η_{l} equal zero. On the other side, if we return back to the hydrodynamic regime (ignoring dust charge variation) $\omega\tau_{\mathrm{m}} \ll 1$, equation (20) agrees exactly with the form of Amin *et al.*[35] by setting ($R = \gamma_* = \eta_{\mathrm{l}} = \theta = 0$); ($\theta$ is introduced in the work of Amin *et al.*[35]). Ignoring the dust charging fluctuation, by setting $g_1 = 0$, and with $\eta_{\mathrm{l}} = 0$, this dispersion relation coincide with that derived in El-Labany *et al.*[21] Moreover, the group velocity λ is given by

$$\lambda = \frac{\partial\omega}{\partial k} = \frac{k}{\omega}\left[\frac{\eta_{\mathrm{l}}}{\tau_{\mathrm{m}}} + \mu_{\mathrm{d}}\gamma_* + \frac{\alpha(1-R)}{(\alpha_1 + k^2 + g_1)^2}\right]. \tag{21}$$

Furthermore, from the ($L = 1$) component of the third-order equations ($n = 3$), and after some algebraic manipulation, we obtain the following nonlinear Schrödinger-type (NLST) equation for the NDA envelope solitons as[36]

$$\mathrm{i}\frac{\partial\Psi}{\partial\tau} + P\frac{\partial^2\Psi}{\partial\xi^2} + Q|\Psi|^2\Psi + H\Psi = 0. \tag{22}$$

In Appendix A, we derive Eq. (22). For simplicity, we have denoted $\varphi_1^{(1)} \equiv \Psi$. Here, P, Q, and H are the dispersion coefficient, the nonlinear coefficient, and the damping term, respectively.

It is noticed here, as well-known, if $PQ > 0$, the modulated envelop is "unstable" for $k < \sqrt{2Q/P}\varphi_0$; i.e., for perturbation wavelengths larger than a critical value. Otherwise, if $PQ < 0$, the modulated envelope becomes "stable" against external perturbation. In other words, for "positive" PQ, the carrier wave is modulationally "unstable"; it may either "collapse", due to (possibly random) external perturbations, or lead to the formation of "bright" envelope modulated wave packets, i.e., localized envelope "pulses" confining the carrier wave. The instability usually saturates by the formation of a train of envelop pulses, the so-called bright solitons. The latter can be stationary in time, but the system also oscillates periodically back and forth between the soliton state and an almost homogeneous state, usually referred to as the Fermi–Pasta–Ulam. For $PQ < 0$, the carrier wave modullationally "stable" and may propagate in the form of a "dark" ("black") envelope wave packet, i.e., a propagating localized "hole" (a "void") amidst a uniform wave energy region. Here, the wave train is stable to the MI and the wave train will not fall apart into a train of solitons. Thus there exist dark solitons, which are local depletions of the amplitude, while the amplitude of the wave train remains stable on both sides of the solitons. Thus, in the following section, it is worthwhile to investigate the interaction of the two NDA envelope dark solitons.

4. The collision of NDA envelope dark solitons

It is well known that the most interesting feature of dark solitions is their threshold generations. Therefore, dark solitons can be created by an arbitrary small dip on a continuous wave background. In this section, we try to describe the interaction of two NDA envelope dark solitons in the kinetic regime of SCDPs using the EPLK perturbation method.[24–27] Firstly, we assume two NDA envelope dark solitons, one of which is traveling to the right, and the other one is going to the left.

After some time they interact, collide, and then depart. Secondly, the analytical solution of the NLST Eq. (22) of equal small amplitudes can be written as[37,38]

$$\varphi_1^{(1)}(\tau,\xi) = \varphi_0\left[1+a(\tau,\xi)\right] \times \exp\left[2\mathrm{i}\varphi_0^2 t+\mathrm{i}\phi(\tau,\xi)\right], \tag{23}$$

where φ_0 an arbitrary constant represents the amplitude of the background far from the dark soliton of amplitude $a(\tau,\xi) \ll \varphi_0$,[39] and $\phi(\tau,\xi)$ is the phase function contributed from the excitation which is assumed to be a function of τ and ξ. Substituting Eq. (23) into Eq. (22), we obtain the following two equations:

$$a_\tau+2Pa_\xi\phi_\xi+P(1+a)\phi_{\xi\xi}=0, \tag{24}$$

$$-(1+a)\phi_\tau+P\left(a_{\xi\xi}-(1+a)\phi_\xi^2\right) +Q\varphi_0^2\left(3a^2+a^3\right)+Q\varphi_0^2(1+3a) -2\varphi_0^2(1+a)+H(1+a)=0. \tag{25}$$

The following asymptotic expansions are used to obtain the NDA envelope dark solitons solutions of Eqs. (24) and (25)

$$\phi(\tau,\xi)=\varepsilon\phi^{(0)}(\tau,\xi)+\varepsilon^3\phi^{(1)}(\tau,\xi)+\cdots, \tag{26}$$

$$a(\tau,\xi)=\varepsilon^2 a^{(0)}(\tau,\xi)+\varepsilon^4 a^{(1)}(\tau,\xi)+\cdots, \tag{27}$$

where ε is a small parameter characterizing the strength of nonlinearity. We assume that $a^{(j)}$ and $\phi^{(j)}$ $(j=1,2,\ldots)$ are functions of the multiple-scale variables, which are given by[38]

$$\zeta=\varepsilon(\xi-c_{\mathrm{R}}\tau)+\varepsilon^2P^{(0)}(\eta)+\varepsilon^4P^{(1)}(\zeta,\eta)+\cdots, \tag{28}$$

$$\eta=\varepsilon(\xi+c_{\mathrm{L}}\tau)+\varepsilon^2Q^{(0)}(\zeta)+\varepsilon^4Q^{(1)}(\zeta,\eta)+\cdots, \tag{29}$$

referring in ζ and η to a right- and to a left-propagating dark soliton, S_{R} and S_{L}, respectively. The wave speed c_{R} and c_{L} are to be related to the amplitudes of the waves. The functions $P^{(j)}$ and $Q^{(j)}$ $(j=0,1,2,\ldots)$ are to be determined in the process of our solution of Eqs. (24) and (25). The aim of introducing these functions is to make a uniformly valid asymptotic expansion (i.e., to eliminate secular terms) and at the same time obtain the change of the trajectories (i.e., phase shifts) of the NDA envelope dark solitons after collision. Now, let us introduce the asymptotic expansion

$$c_{\mathrm{R}}=c+\varepsilon^2R^{(1)}+\varepsilon^4R^{(2)}+\cdots, \tag{30}$$

$$c_{\mathrm{L}}=c+\varepsilon^2L^{(1)}+\varepsilon^4L^{(2)}+\cdots. \tag{31}$$

Putting Eqs. (26)–(29) into Eqs. (24) and (25), we obtain

$$a^{(0)}=f_{\mathrm{R}}(\zeta)+f_{\mathrm{L}}(\eta), \tag{32}$$

$$\phi^{(0)}=\sqrt{\frac{2\varphi_0^2-3Q\varphi_0^2-H}{P}}\left(\int f_{\mathrm{R}}(\zeta)-\int f_{\mathrm{L}}(\eta)\right). \tag{33}$$

We can calculate the local wave speed as $c=\sqrt{P(2\varphi_0^2-3Q\varphi_0^2-H)}$. Clearly, we have two waves, one of them, $f_{\mathrm{R}}(\xi)$, is traveling to the right, and the other one, $f_{\mathrm{L}}(\eta)$, is going to the left.

To the next higher-order, the solvability conditions for $a^{(1)}$and $\varphi^{(1)}$ give

$$h_1R^{(1)}\frac{\partial f_{\mathrm{R}}(\zeta)}{\partial\zeta}-h_2f_{\mathrm{R}}(\zeta)\frac{\partial f_{\mathrm{R}}(\zeta)}{\partial\zeta}-h_3\frac{\partial^3f_{\mathrm{R}}(\zeta)}{\partial\zeta^3}=0, \tag{34}$$

$$-h_1L^{(1)}\frac{\partial f_{\mathrm{L}}(\eta)}{\partial\eta}+h_2f_{\mathrm{L}}(\eta)\frac{\partial f_{\mathrm{L}}(\eta)}{\partial\eta}+h_3\frac{\partial^3f_{\mathrm{L}}(\eta)}{\partial\eta^3}=0, \tag{35}$$

$$\frac{\partial P^{(0)}}{\partial\eta}=\frac{h_5}{h_4}f_{\mathrm{L}}(\eta), \tag{36}$$

$$\frac{\partial Q^{(0)}}{\partial\xi}=\frac{h_5}{h_4}f_{\mathrm{R}}(\zeta), \tag{37}$$

where

$$\begin{aligned}
R^{(1)} &= \frac{h_2}{3h_1}\varphi_{\mathrm{R}},\quad L^{(1)}=\frac{h_2}{3h_1}\varphi_{\mathrm{L}},\\
h_1 &= 2\left(2\varphi_0^2-3Q\varphi_0^2-H\right),\\
h_2 &= [P(2\varphi_0^2-3Q\varphi_0^2-H)^3]^{1/2}\\
&\quad -6Q\varphi_0^2\sqrt{P\left(2\varphi_0^2-3Q\varphi_0^2-H\right)},\\
h_3 &= -P^{3/2}\sqrt{\left(2\varphi_0^2-3Q\varphi_0^2-H\right)},\\
h_4 &= 4[P(2\varphi_0^2-3Q\varphi_0^2-H)^3]^{1/2},\\
h_5 &= 3[P(2\varphi_0^2-3Q\varphi_0^2-H)^3]^{1/2}\\
&\quad +6Q\varphi_0^2\sqrt{P(2\varphi_0^2-3Q\varphi_0^2-H)}.
\end{aligned}$$

Equations (34) and (35) are the two-side traveling wave Korteweg–de Vries (KdV) equations in the reference frames of ξ and η, the one-soliton solutions of Eqs. (34) and (35), are given by[40]

$$f_{\mathrm{R}}(\zeta)=\varphi_{\mathrm{R}}\mathrm{sech}^2\left(\sqrt{\varphi_{\mathrm{R}}h_2\zeta/12h_3}\right), \tag{38}$$

$$f_{\mathrm{L}}(\eta)=\varphi_{\mathrm{L}}\mathrm{sech}^2\left(\sqrt{\varphi_{\mathrm{L}}h_2\eta/12h_3}\right), \tag{39}$$

where φ_{R} and φ_{L} are the initial amplitudes of the two NDA envelope dark solitons S_{R} and S_{L} in their initial positions. Moreover, after some algebraic steps, one can obtain the corresponding phase shifts.[40,41] Details are given in Appendix B, that is

$$\Delta P_0=-2\varepsilon^2\frac{h_5}{h_4}\left(\frac{12h_3\varphi_{\mathrm{L}}}{h_2}\right)^{1/2}, \tag{40}$$

$$\Delta Q_0=2\varepsilon^2\frac{h_5}{h_4}\left(\frac{12h_3\varphi_{\mathrm{R}}}{h_2}\right)^{1/2}. \tag{41}$$

5. Discussion

Before going to the discussion, it should be mentioned here that, the numerical values of the physical parameters are selected based on actual experimental data as follows:[17,42]

$$n_{\mathrm{e}0}=4\times10^7\ \mathrm{cm}^{-3},\quad n_{\mathrm{i}0}=7\times10^7\ \mathrm{cm}^{-3},\quad T_{\mathrm{i}}=0.1\ \mathrm{eV},$$

$$T_e = 1\ \text{eV},\ T_d = 0.03\ \text{eV},\ R = 0\text{–}1,\ Z_{d0} = 3\times 10^3.$$

Furthermore, under the condition $\omega\tau_m \gg 1$ (i.e., the kinetic regime), η_l is a finite quantity, which makes the transition from the fluid to the solid state. This means, physically, that correlation energies of dust particles change due to a change in the order of the arrangement of the dust particles with increased correlation effect. Accordingly, the viscosity force represented by $\eta_l(\partial^2 u_d/\partial x^2)$ plays in this case the role of a restoring force rather than a dissipative one.[21]

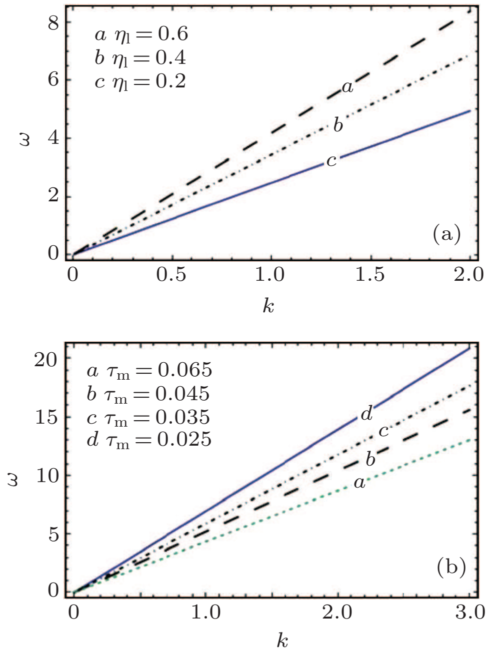

Fig. 1. (color online) Variation of ω against the wave number k for different values of viscosity coefficient η_l with $R = 0.2$ (a) and the relaxation time τ_m with $R = 0.4$ (b).

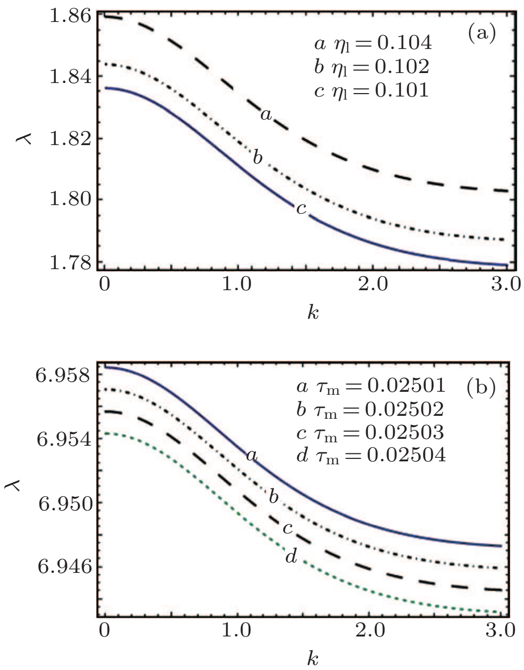

Fig. 2. (color online) Variation of the group velocity λ against the wave number k for different values of η_l with $R = 0.2$ (a) and τ_m with $R = 0.4$ (b).

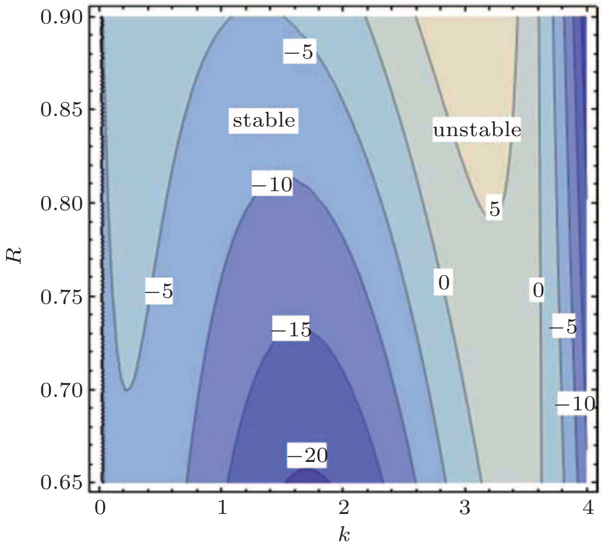

Fig. 3. (color online) The contour plot of the product PQ defining stable/unstable regime with $\tau_m = 0.025$. The contours with the value "0" are separators between stable and unstable regimes.

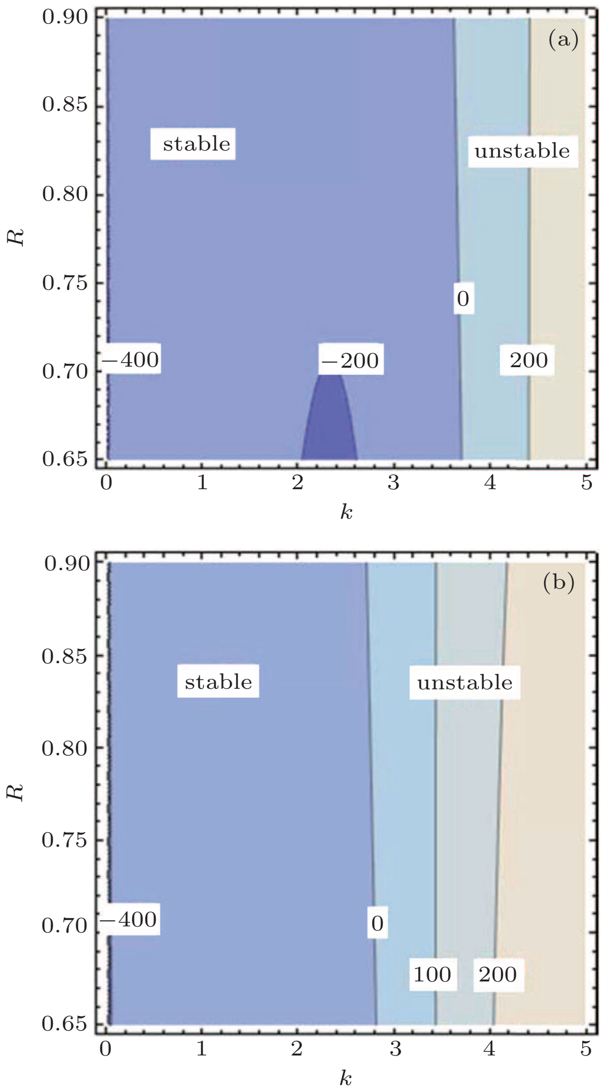

Fig. 4. (color online) The contour plot of the product PQ with $\eta_l = 0$ by including dust charge variation effect (a) and with constant dust charge (b).

We can summarize the effects of these new physical parameters on the MI and collision of NDA envelope solitons as follows. Figure 1 shows that ω increases as η_{l} or k increases though it decreases as τ_{m} increases. On the contrary, we have found that λ (decreases) increases as (either k or τ_{m}) η_{l} increases, (cf., Fig. 2). Figure 3 demonstrates the contour plot of the product PQ in the k–R domain. It illustrates two specific stable regimes for the NDA envelope solitons; in between an unstable regime is created. The middle (unstable regime is located insides the two contours denoted by "0". Figures 4–6 illustrate the contour plots of the product PQ against including dust charge variation, η_{l}, and τ_{m}, respectively. These illustrations reveal that including the dust charging fluctuations leads to an expansion of the stable regime of the NDA envelope solitons by shifting the critical wavenumber boundary to a higher value. However, increasing the longitudinal viscosity (relaxation time), η_{l} (τ_{m}) leads to a reduction (an expansion) of the stable domain for NDA envelope dark solitons, respectively. In all cases, for large wavelength limit seems to predict stability, as physically expected. Let us now interpret the dependence of the NDA envelope dark solitons phase shifts and trajectories, released after collision occurred, on the system physical parameter variations. Cleary, the magnitude of the phase shift depends on the initial amplitudes of NDA envelope dark solitons with large amplitudes causing large phase shifts (see. Eqs. (40) and (41)). Furthermore, dark soliton S_{R} is traveling to the right and dark soliton S_{L} is going to the left, we see from Eqs. (40) and (41) that due to collision, each dark soliton has a negative phase shift in its traveling direction. Physical interpretation for a negative phase shift phenomenon is that during the colliding process, two NDA envelope dark solitons decrease their velocities, which lead to change in their trajectories after collision stage.

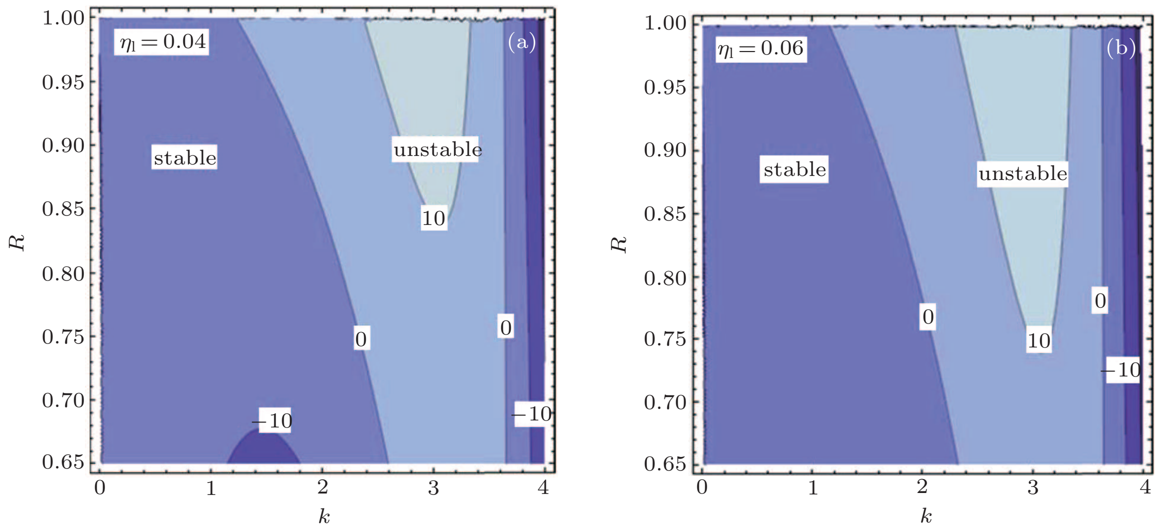

Fig. 5. (color online) The contour plots of the product "PQ" in the k–R domain are presented for $R = 0.2$.

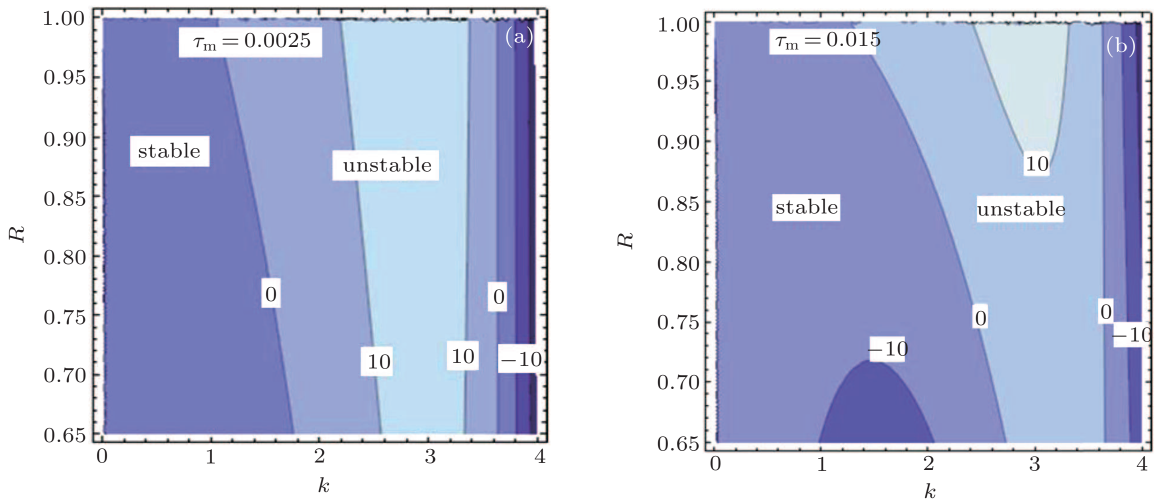

Fig. 6. (color online) The contour plots of the product "PQ" in the k–R domain are presented for $R = 0.2$.

Figure 7 demonstrates the numerical simulation result of the amplitude $a(\tau,\xi)$ of the two NDA envelope dark solitons against the space coordinate ξ and the time variable τ is represented in. It shows that after the collision, the two NDA envelope dark solitons propagate along the trajectories deviated from the initial trajectories, which is the phase shift. Figures 8 and 9, respectively, show how η_l and τ_m affect the profile of the two NDA envelope dark solitons against the space coordinate ξ and the time variable τ. It is obvious that the phase shift increases with increasing either η_l or τ_m. Therefore, the phase shifts for NDA envelope dark solitons depend directly on η_l and τ_m. Figure 10 represents the variation of the phase shift ΔQ_0 with k for various values of R in two cases: with and without dust charge fluctuation. The figure points out that the phase shift ΔQ_0 increases as R decreases. Owing to the inclusion of the charge fluctuation in our model, the magnitude of phases shift ΔQ_0 increases. The point to be stressed here is that, the dust charge fluctuation has a strong effect on the phase shift.

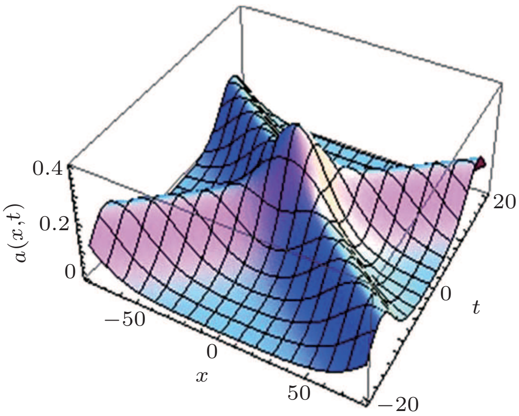

Fig. 7. (color online) The colliding process of two DA waves. The amplitude, $a(\tau,\xi)$, of two dark solitons is illustrated.

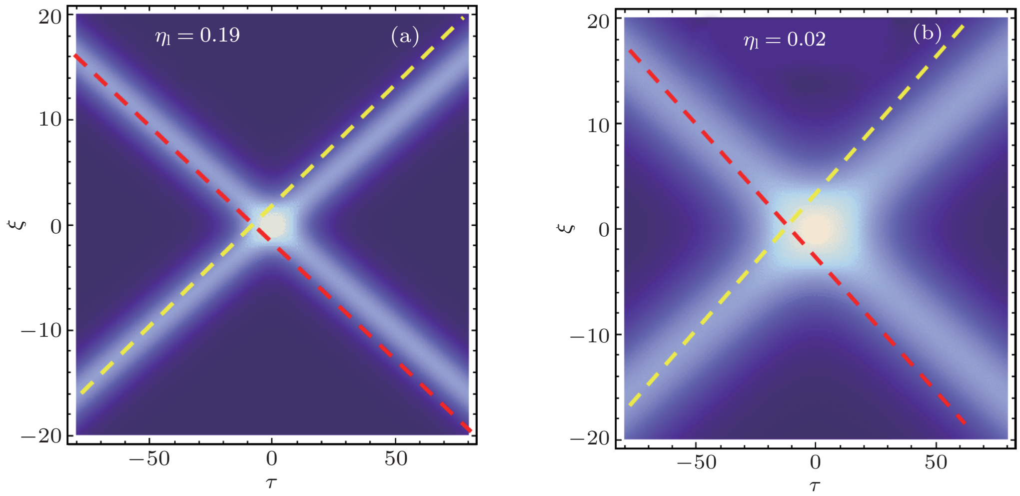

Fig. 8. (color online) Space-time plot is presented for different values of η_l with $k = 0.3$, $\varepsilon = 0.01$, $R = 0.2$, and $\varphi_a = \varphi_b = 0.2$.

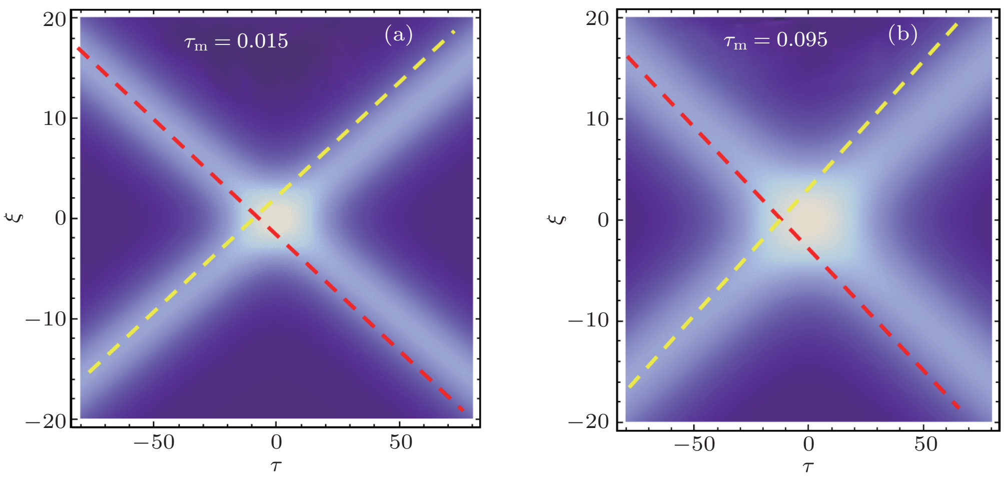

Fig. 9. (color online) Space-time plot is presented for different values of τ_m with $k = 0.3$, $\varepsilon = 0.01$, $R = 0.2$, $\eta_l = 0.02$, and $\varphi_a = \varphi_b = 0.2$.

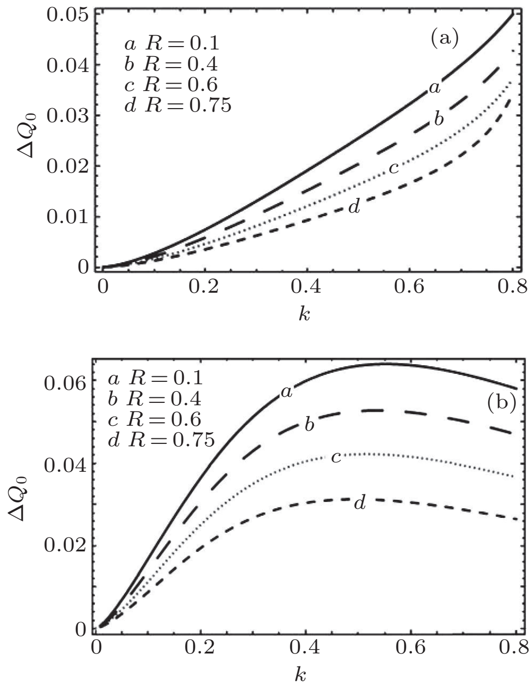

Fig. 10. The variation of the phase shift ΔQ_0 against k for different values of R without dust charge variation and viscosity effect η_l (a) and including dust charge variation while the viscosity effect is omitted (b).

6. Conclusion

In summary, the MI and the collision of two NDA envelope solitons in a SCDP system in the kinetic regime have been investigated. Using the derivative expansion perturbation technique, the NLST equation is derived. The instability domain for the NDA wave is estimated. The EPLK method has been employed to obtain the analytical phase shifts after the collision of NDA envelope dark solitons. It is clear that the wave form of the colliding NDA envelope dark solitons remain unchanged. Analytically and numerically, the effects of the viscosity η_l the relaxation time τ_m, and the dust charge fluctuation are investigated. It is found that the phase shift and the stable domain increase with increasing the longitudinal viscosity. Moreover, increasing the relaxation time leads to an increase of the phase shift and the stable domain. This means, physically, that the increase of η_l (τ_m) leads to an increase (a decrease) of the group velocity λ, which in turn leads to an increase (a decrease) in the restoring force, which makes the NDA envelope dark soliton taller (shorter). Accordingly, the obtained result indicates that the stable domain and the phase shift for NDA envelope dark solitons decrease (increase) with the increase of η_l (τ_m). Finally, the dust charge fluctuation affects strongly on the nature, the stability domain and the phase shift of NDA envelope solitons. Evidently, including the dust charge fluctuation extends the stable domain and decreases the magnitude of phase shift. Furthermore at $\tau_m = 0$ (i.e., relations time is instantaneous), it is worth noticing that the usual hydrodynamic equation is considered. However, in our investigation we have considered the dust charge fluctuation in the kinetic regime, where the relaxation time τ_m is much larger than the dust fluid dynamic time (NDA envelope solitons), which gives rise to different physical behavior as interpreted. In addition, it is useful to compare our results with the results of El-Labany *et al.*[21] Properly, this work agrees with the results of El-Labany *et al.*[21] by neglecting the viscosity, the relaxation time, and the dust charge fluctuation. Hence our model, in the kinetic regime, is more general and can describe the propagation and the collision of two NDA envelope dark solitons by the derivative expansion perturbation technique and the EPLK method, respectively.

Appendix A

The second harmonic modes ($n = L = 2$) arising from nonlinear self-interactions of the carrier waves are obtained in terms of $(\varphi_1^{(1)})^2$ as

$$\varphi_2^{(2)} = \frac{g_2}{g_3}\varphi_1^{(1)2}, \quad Z_{d2}^{(2)} = g_4\varphi_1^{(1)2},$$
$$n_{d2}^{(2)} = g_5\varphi_1^{(1)2}, \quad u_{d2}^{(2)} = g_6\varphi_1^{(1)2}.$$

Proceeding to the first harmonics of order ε^2, the corresponding amplitudes are given by

$$\varphi_1^{(2)} = \mathrm{i}\frac{g_7}{g_8}\frac{\partial\varphi_1^{(1)}}{\partial\xi}, \quad Z_{d1}^{(2)} = \mathrm{i}g_9\frac{\partial\varphi_1^{(1)}}{\partial\xi},$$
$$n_{d1}^{(2)} = \mathrm{i}g_{10}\frac{\partial\varphi_1^{(1)}}{\partial\xi}, \quad u_{d1}^{(2)} = \mathrm{i}g_{11}\frac{\partial\varphi_1^{(1)}}{\partial\xi},$$

$$\begin{aligned}
g_2 = {} & \frac{g_1}{1+\sigma(1-\Psi_0)} \\
& - g_1\left(\alpha+k^2+g_1\right) + \frac{1}{2}\left(\mu_e\sigma_i^2-\mu_i\right) \\
& + \frac{1}{\frac{\omega}{k}\left(-4\omega^2\tau_m+4k^2\eta_l\right)+4k\omega\tau_m\mu_d} \\
& \times\left\{\frac{\omega}{k}\left(-4\omega^2\tau_m+4k^2\eta_l\right)\left(\alpha+k^2+g_1\right)^2\right. \\
& \left.+2k\omega\tau_m\left[-\left(\alpha+k^2\right)-Rg_1+\frac{R}{2}\right]\right\}, \\
g_3 = {} & -\left(\alpha+4k^2+g_1\right) \\
& + \frac{4k\omega\tau_m(R-1)}{\frac{\omega}{k}\left(-4\omega^2\tau_m+4k^2\eta_l\right)+4k\omega\gamma\tau_m\mu_d}, \\
g_4 = {} & \frac{g_1g_2}{g_3}+\frac{g_1}{1+\sigma(1-\Psi_0)}, \\
g_5 = {} & \frac{-(\alpha+4k^2)g_2}{g_3}-g_4 \\
& + g_1\left(\alpha+k^2+g_1\right)-\frac{1}{2}\left(\mu_e\sigma_i^2-\mu_i\right), \\
g_6 = {} & \frac{\omega\left(g_5-\left(\alpha+k^2+g_1\right)^2\right)}{k}, \\
g_7 = {} & \frac{\left(\alpha+k^2+g_1\right)}{\omega_1}\left\{\left(\frac{-\omega}{k^2}+\frac{\lambda}{k}\right)\frac{1}{\left(-\omega^2\tau_m+k^2\eta_l\right)}\right.
\end{aligned}$$

$$-\left[\left(\frac{2\lambda\tau_{\mathrm{m}}\omega^2}{k}-\omega\tau_{\mathrm{m}}\mu_{\mathrm{d}}\gamma-\mu_{\mathrm{d}}\gamma k\tau_{\mathrm{m}}-2\omega\eta_{\mathrm{l}}\right)\right.$$
$$\left.\left.-(1-R)\left[\omega\tau_{\mathrm{m}}+k\lambda\tau_{\mathrm{m}}\right]\right]\right\}-2k,$$
$$g_8=-\left(\alpha+k^2+g_1\right)+\frac{(1-R)k\omega\tau_{\mathrm{m}}}{\omega_1\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)},$$
$$g_9=\frac{g_1g_7}{g_8},\quad g_{10}=-g_9+2k-\frac{(\alpha+k^2)g_7}{g_8},$$
$$g_{11}=-\left(\alpha+k^2+g_1\right)\left(\frac{\omega}{k^2}-\frac{\lambda}{k}\right)+\frac{\omega g_{10}}{k}.$$

The nonlinear self-interaction of the carrier wave also results in the creation of a zeroth-harmonic, in this order; its strength is analytically determined by taking into account the $L=0$ components. We can express all of these quantities in terms of $\varphi_1^{(1)}\varphi_1^{*(1)}=|\varphi_1^{(1)}|^2$ as follows:

$$\varphi_0^{(2)}=\frac{g_{12}}{g_{13}}\left|\varphi_1^{(1)}\right|^2+c_4,$$
$$Z_{\mathrm{d}0}^{(2)}=g_{14}\left|\varphi_1^{(1)}\right|^2+g_1c_4,$$
$$n_{\mathrm{d}0}^{(2)}=g_{15}\left|\varphi_1^{(1)}\right|^2+c_5,$$
$$u_{\mathrm{d}0}^{(2)}=g_{16}\left|\varphi_1^{(1)}\right|^2+c_6,$$

where c_1, c_2, and c_3 are arbitrary constants and c_{i} $(i=4,5,6)$ are given by

$$c_4=c_2+\frac{\lambda c_1+c_3}{\lambda^2-\mu_{\mathrm{d}}\gamma},\quad c_5=c_2-(\alpha+g_1)c_4,\quad c_6=c_1+\lambda c_5,$$
$$g_{12}=2g_1\left(\alpha+k^2+g_1\right)+\left(-\mu_{\mathrm{e}}\sigma_{\mathrm{i}}^2+\mu_{\mathrm{i}}\right)-\frac{2g_1}{1+\sigma(1-\Psi_0)}+\frac{\left(\alpha+k^2+g_1\right)\left\{(1-R)+\frac{\omega}{k}\left(\alpha+k^2+g_1\right)\left[\frac{\omega}{k}-\lambda+2\right]-\frac{R}{2}\right\}}{\mu_{\mathrm{d}}\gamma-\lambda^2},$$
$$g_{13}=\alpha+g_1+\frac{1-R}{\lambda^2-\mu_{\mathrm{d}}\gamma_*},\quad g_{14}=g_1\left[\frac{g_{12}}{g_{13}}+\frac{2}{1+\sigma(1-\Psi_0)}\right],$$
$$g_{15}=-g_{14}+2g_1\left(\alpha+k^2+g_1\right)-\alpha\frac{g_{12}}{g_{13}}-\left(\mu_{\mathrm{e}}\sigma_{\mathrm{i}}^2-\mu_{\mathrm{i}}\right),\quad g_{16}=\lambda g_{15}-\frac{2\omega\left(\alpha+k^2+g_1\right)^2}{k}.$$

From the $(L=1)$ component of the third-order equations $(n=3)$, we obtain the following NLST equation for the NDA wave as

$$\mathrm{i}\frac{\partial\Psi}{\partial\tau}+P\frac{\partial^2\Psi}{\partial\xi^2}+Q|\Psi|^2\Psi+H\Psi=0,$$
$$P=\frac{Q_1}{Q_0},\quad Q=\frac{Q_2}{Q_0},\quad H=\frac{Q_3}{Q_0},$$
$$Q_0=-\frac{\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)\left(\alpha+k^2+g_1\right)-kD_4}{\omega\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)+k^2\omega\tau_{\mathrm{m}}\mu_{\mathrm{d}}\gamma},$$
$$Q_1=1+\frac{-D_3+\frac{\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)\left(\lambda g_{10}-g_{11}\right)}{k}}{\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)\frac{\omega}{k}+k\omega\tau_{\mathrm{m}}\mu_{\mathrm{d}}\gamma}-\frac{2kg_7}{g_8},$$
$$\omega_1=\frac{k\omega\tau_{\mathrm{m}}\mu_{\mathrm{d}}\gamma}{\left(\omega^2\tau_{\mathrm{m}}-k^2\eta_{\mathrm{l}}\right)}-\frac{\omega}{k},$$
$$Q_2=\frac{-D_8}{1+\sigma(1-\Psi_0)}+\frac{-D_1+\frac{\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)D_7}{k}}{\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)\frac{\omega}{k}+k\omega\tau_{\mathrm{m}}\mu_{\mathrm{d}}\gamma}-D_5,$$
$$Q_3=-D_6+\frac{-4g_1c_4}{1+\sigma(1-\Psi_0)}$$
$$+\frac{-D_2+2\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)\left(\alpha+k^2+g_1\right)\left[c_6+c_5\frac{\omega}{k}\right]}{\left(-\omega^2\tau_{\mathrm{m}}+k^2\eta_{\mathrm{l}}\right)\frac{\omega}{k}+k\omega\tau_{\mathrm{m}}\mu_{\mathrm{d}}\gamma},$$

where

$$D_1=\omega^2\tau_{\mathrm{m}}(\alpha+k^2+g_1)\Big(-2g_{16}(1-2\omega)+g_6$$
$$+\frac{\omega}{k}\left(-2g_{15}-g_5+3\left(\alpha+k^2+g_1\right)^2\right)\Big)$$
$$+\left(\alpha+k^2+g_1\right)\Big(-2\omega k\tau_{\mathrm{m}}\frac{g_{12}}{g_{13}}$$
$$-\omega k\tau_{\mathrm{m}}\frac{g_2}{g_3}+2\omega Rk\tau_{\mathrm{m}}\frac{g_{12}}{g_{13}}$$
$$+\omega kR\tau_{\mathrm{m}}\frac{g_2}{g_3}-3\omega k\tau_{\mathrm{m}}g_1-6\omega g_1kR\tau_{\mathrm{m}}\Big)$$
$$+\omega k\tau_{\mathrm{m}}\Big(2g_{15}(1-R)+g_5(1-R)$$
$$+2\frac{g_{12}}{g_{13}}(R+g_1+2Rg_1)+R\frac{g_2}{g_3}-\frac{R}{8}+\frac{g_1g_2}{g_3}+2g_{14}$$
$$+g_4-6g_1R+\frac{2g_1g_2R}{g_3}+4Rg_{14}+2Rg_4+3Rg_1^2\Big),$$
$$D_2=\left(\alpha+k^2+g_1\right)\Big(-2\omega^2\tau_{\mathrm{m}}c_6-\frac{2\omega^3\tau_{\mathrm{m}}}{k}c_5$$
$$+4\omega^2\tau_{\mathrm{m}}c_6-2\omega c_4k\tau_{\mathrm{m}}$$
$$+2\omega kR\tau_{\mathrm{m}}c_4\Big)+2c_5\omega k\tau_{\mathrm{m}}(1-R)$$
$$+2\omega k\tau_{\mathrm{m}}c_4[R(1+4g_1)+2g_1],$$
$$D_3=2\omega\lambda\tau_{\mathrm{m}}g_{11}+\left(\alpha+k^2+g_1\right)$$
$$\times\left(\frac{\lambda^2\tau_{\mathrm{m}}\omega}{k}-\tau_{\mathrm{m}}\mu_{\mathrm{d}}\gamma_*\lambda-\frac{\eta_{\mathrm{l}}\omega}{k}\right)$$
$$+[1-R]\tau_{\mathrm{m}}\left(\frac{\omega g_7}{g_8}+k\lambda\frac{g_7}{g_8}\right)$$

$$
\begin{aligned}
&+R\lambda\tau_{\rm m}-\tau_{\rm m}\mu_{\rm d}\gamma_{*}g_{10}(\omega+\lambda k-2k\eta_{\rm l}g_{11}),\\
D_4 &= k\tau_{\rm m}[1-R]+\left(\alpha+k^2+g_1\right)\left(-\frac{\omega^2\tau_{\rm m}}{k}+k\tau_{\rm m}\mu_{\rm d}\gamma_{*}\right),\\
D_5 &= -\left(\alpha+k^2+g_1\right)[2g_{14}+g_4]+g_1[2g_{15}+g_5]\\
&\quad+\left(\mu_{\rm e}\sigma_{\rm i}^2-\mu_{\rm i}\right)\left(\frac{2g_{12}}{g_{13}}+\frac{g_2}{g_3}\right)+\frac{\left(\mu_{\rm e}\sigma_{\rm i}^3+\mu_{\rm i}\right)}{2},\\
D_6 &= 2[-\left(\alpha+k^2+g_1\right)g_1c_4+g_1c_5+c_4\left(\mu_{\rm e}\sigma_{\rm i}^2-\mu_{\rm i}\right)],\\
D_7 &= \left(\alpha+k^2+g_1\right)[k(2g_{16}+g_6)+\omega(2g_{15}+g_5)],\\
D_8 &= 2g_{14}+g_4+\frac{2g_1g_{12}}{g_{13}}+\frac{g_1g_2}{g_3}.
\end{aligned}
$$

Appendix B

The leading phase changes due to the collision can be calculated

$$
P^{(0)}(\eta)=\frac{h_5}{h_4}\left(\frac{12h_3\varphi_{\rm L}}{h_2}\right)^{1/2}\left[\tanh\left(\left(\frac{h\varphi_{\rm L}}{12h_3}\right)^{1/2}\eta\right)+1\right],
$$

$$
Q^{(0)}(\xi)=\frac{h_5}{h_4}\left(\frac{12h_3\varphi_{\rm R}}{h_2}\right)^{1/2}\left[\tanh\left(\left(\frac{h_2\varphi_{\rm R}}{12h_3}\right)^{1/2}\zeta\right)-1\right].
$$

Therefore, the solution up to $O(\varepsilon^2)$ order can be obtained as following:

$$
\begin{aligned}
a(\tau,\xi) &= \varepsilon^2\left[\varphi_{\rm R}\,{\rm sech}^2\left(\sqrt{\frac{\varphi_{\rm R}h_2}{12h_3}}\zeta\right)\right.\\
&\quad\left.+\varphi_{\rm L}\,{\rm sech}^2\left(\sqrt{\frac{\varphi_{\rm L}h_2}{12h_3}}\eta\right)\right]+O(\varepsilon^4),\\
\phi(\tau,\xi) &= \varepsilon\sqrt{\frac{(2\varphi_0^2-3Q\varphi_0^2-H)}{P}}\\
&\quad\times\left\{\left(\frac{12h_3\varphi_{\rm L}}{h_2}\right)^{1/2}\left[\tanh\left(\left(\frac{h_2\varphi_{\rm L}}{12h_3}\right)^{1/2}\eta\right)+1\right]\right.\\
&\quad\left.-\left(\frac{12h_3\varphi_{\rm R}}{h_2}\right)^{1/2}\left[\tanh\left(\left(\frac{h_2\varphi_{\rm R}}{12h_3}\right)^{1/2}\zeta\right)-1\right]\right\}\\
&\quad+O(\varepsilon^3).
\end{aligned}
$$

Therefore, one can obtain the corresponding phase shifts

$$
\Delta P_0=-2\varepsilon^2\frac{h_5}{h_4}\left(\frac{12h_3\varphi_{\rm L}}{h_2}\right)^{1/2},
$$

$$
\Delta Q_0=2\varepsilon^2\frac{h_5}{h_4}\left(\frac{12h_3\varphi_{\rm R}}{h_2}\right)^{1/2},
$$

which are Eqs. (40) and (41), respectively.

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