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Overtaking phenomenon of nonlinear ultra-low frequency multi-shock structures in ultra-relativistic degenerate plasmas

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ABSTRACT

Overtaking collisions of nonlinear ultra-low frequency (NULF) shock waves are examined in a degenerate ultra-relativistic plasma, containing inertial mobile heavy-ion fluid and inertialess degenerate light ions and electrons. The well-known reductive perturbation analysis then is used to obtain a Burgers' equation (BE), which describes the propagation of ultra-low frequency shock waves in the plasma model. The Cole-Hopf transformation and the exponential function have been applied to get NULF multi-shock waves. Here, the findings illustrate that the excitation, amplitude, and steepness of overtaking collisions of NULF multi-shock waves and their electric fields are modified with the influence of heavy-ion kinematic viscosity. It is found that the amplitude and the steepness of NULF multi-shock waves increase with the decrease in the light-to-heavy ion density ratio. The numerical simulation gives rise to significant highlights on the dynamics of overtaking collisions of NULF multi-shock waves in astrophysical plasmas such as white dwarfs.

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Nonlinear ultra-low frequency multi-shock waves; overtaking collisions; ultra-relativistic degenerate plasmas

1. Introduction

Nonlinear shock waves in physics are defined as the propagation of disturbance characterized by an extremely rapid increase in physical quantities such as density, pressure, and temperature. The excitation of nonlinear shock waves in different media has been a topic of interest because of its wide range of applications, such as in solid explosives (*1*), ultra-small electronic devices (*2*), astrophysical objects (*3, 4*), and high-intensity laser-produced plasmas (*5*). In general, shock structures may play critical and effective roles in many astrophysical situations. For example, strong field generation and nonlinear dynamics are observed in the bowshock region and the solar wind, respectively (*6, 7*). In addition, the nonlinear shock waves are observed in the interaction of relativistic electrons/ions with plasma. Due to explosions that can be observed in the stars, the stars' luminosity is amplified as the total energy emitted from the galaxy is increased. Actually, various ways are applied to interpret the light emitted from the explosion (*8*). However, the formation of nonlinear shock waves is one of the most probable mechanisms because they propagate

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outward. Moreover, it is well-known that nonlinear shock wave propagation in degenerate relativistic plasma plays a critical role in understanding the formation and the collapse of compact objects. Consequently, the understanding of the nonlinear dynamics is coming into focus; there is a very interesting development of the shock structures in astrophysical compact objects, such as neutron stars and white dwarf stars. In this situation, lighter plasma particles are considered as degenerate; hence, the degenerate pressure dominates over the thermal pressure of these particles and the plasma can be generally treated as relativistic. Specifically, it is found that the central region of the white dwarf consists mostly of helium/carbon (*9*) and oxygen (*10*) as heavy elements, but the inner core contains degenerate relativistic electrons; while the outer envelope has nonrelativistic electrons (*11*). Clearly, the main components of white dwarfs are degenerate electrons and lighter ions, and non-degenerate heavier ions (*12*). The degenerate pressure of degenerate particles can be given by $P_s = \Gamma_s n^{\gamma_s}$ with $\gamma_s = 4/3$; $\Gamma_s = 3\hbar c(\pi/3)^{2/3}/4$ for the ultra-relativistic limit (*13*–*16*) and $\gamma_s = 5/3$; $\Gamma_s = 3\pi\hbar^2(\pi/3)^{1/3}/5m_s$ for the non-relativistic limit (*13*–*16*),where m_s is the mass of the degenerate particles $\hbar(= h/2\pi)$ is the reduced Planck's constant, c is the speed of light and the subscript s refers, respectively, to e and ℓ for the electron and for the light ion.

In general, in a one dimensional system, multi-nonlinear waves may interact with each other via two different methods (*i.e.* the overtaking collision and the head-on collision). In the overtaking collision, where multi-nonlinear waves move along the same direction, can be investigated analytically by the inverse scattering transformation technique (*17*). On the other side, the head-on collision (*18*, *19*), where the angle between two propagation directions of two nonlinear waves is π, can be examined analytically by the extended Poincare– Lighthill–Kuo technique (*20*, *21*). Besides the analytical methods, one can apply the numerical method, such as the FORTRAN software package, to study the propagation of steady-state solitons and steady-state shock waves in plasma physics (*22*–*26*). In this present work, we focus on the analytical study of the nonlinear phenomena in the ultra-relativistic case. Therefore, a number of investigators have recently made a number of theoretical studies on the propagation of nonlinear shock waves in degenerate relativistic plasma (*27*–*32*). For instance, Mamun and Zobaer (*28*) investigated the characteristics of shock waves and double layers propagating in a degenerate dense plasma, which contains inertial viscous ion fluid, non-relativistic and ultra-relativistic degenerate electron fluid as well as negatively charged stationary heavy element. Islam et al. (*31*) studied nonlinear ultra-low frequency (NULF) shock waves in a degenerate relativistic three-component plasma model through Burgers' nonlinear equation. Hafez et al. (*32*) discussed the ion acoustic shock waves in weakly and highly relativistic plasmas consisting of relativistic ion fluids, nonextensive electrons, and positrons by deriving Burgers' equation (BE). Recently, El-Shamy et al. (*33*) examined the effects of the chemical potentials, ultra-relativistic and degenerate of electrons and positrons on the physical nature of three-dimensional isothermal ion-acoustic shock waves. It is interesting to state here that all the earlier studies (*27*–*33*) are only limited to propagating shock waves in degenerate relativistic plasmas. Indeed, shock wave propagation may not be just an actual situation, but the collisions of multi-shock structures in degenerate relativistic plasmas are also a more realistic phenomenon. Furthermore, a few authors have studied overtaking collisions of multi-shock wave excitations (*34*, *35*). For example, Hafez et al. (*35*) discussed the effects of plasma parameters, such as the population of nonthermal electrons, α,and strength of non-extensive electrons, q, on overtaking collisions of ion-acoustic multi-shocks in an unmagnetized plasma with positive and negative ions, and (α, q)-distributed electrons. They found that the amplitude (thickness) of ion-acoustic shock wave excitations increases (decreases) with the increase in q and α. However, the overtaking collisions of ultra-low frequency multi-shock structures in degenerate relativistic plasmas have not yet been discussed. Therefore, the motivation for the present study is to investigate the influences of heavy-ion kinematic viscosity, the light-to-heavy ion density ratio and the relativistic effect on the overtaking phenomenon of NULF multi-shock waves.

2. Simulation model

Let us consider a three-component relativistic degenerate plasma containing nondegenerate inertial heavy-ion fluid, inertialess degenerate light ion and electron fluids. The set of normalized dynamic equations for the NULF multi-shock waves in relativistic degenerate plasmas can be written as follows (*12*, *28*):

$$\frac{\partial n_h}{\partial t}+\frac{\partial(n_h u_h)}{\partial x}=0, \tag{1}$$

$$\left(\frac{\partial u_h}{\partial t}+u_h\frac{\partial u_h}{\partial x}\right)=-\frac{\partial \phi}{\partial x}+\eta\frac{\partial^2 u_h}{\partial x^2}. \tag{2}$$

$$\mu_\ell\frac{\partial n_\ell^{\gamma_\ell}}{\partial x}+n_\ell\frac{\partial \phi}{\partial x}=0, \tag{3}$$

$$\mu_e\frac{\partial n_\ell^{\gamma_e}}{\partial x}-n_e\frac{\partial \phi}{\partial x}=0, \tag{4}$$

and Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2}\approx(\beta n_e-\alpha n_\ell-n_h), \tag{5}$$

where $\mu_\ell=\Gamma_\ell(n_\ell^{(0)})^{\gamma_\ell-1}/Z_\ell m_e c^2$,$\mu_e=\Gamma_e(n_e^{(0)})^{\gamma_e-1}/m_e c^2$, $\beta=n_e^{(0)}/Z_h n_h^{(0)}$, and $\alpha=Z_\ell n_\ell^{(0)}/Z_h n_h^{(0)}$. Here, Z_h and Z_ℓ are the charge states of heavy and light ions, respectively. $n_h^{(0)}$, $n_\ell^{(0)}$,and $n_e^{(0)}$ are, respectively, the unperturbed number density of heavy ions, light ions, and electrons. γ_ℓ and γ_e are the relativistic parameters of light ions and electrons, respectively. Here, the subscripts h, ℓ, and e refer, respectively, to the heavy ions, light ions, and electrons . The quasi-neutrality condition at equilibrium reads:$\beta=1+\alpha$. The physical quantities n_h, n_ℓ and n_e are the number densities of heavy ions, light ions and electrons scaled by $n_h^{(0)}$,$n_\ell^{(0)}$ and $n_e^{(0)}$,respectively. u_h is the velocity of inertial heavy ions normalized by the ion acoustic speed $C_{IAS}\left(=\sqrt{Z_h m_e c^2/m_h}\right)$, ϕ is the electrostatic potential scaled by $m_e c^2/e$. Space and time variables are scaled by the Debye length $\lambda_{DL}\left(=\sqrt{m_e c^2/4\pi Z_h e^2 n_h^{(0)}}\right)$ and the inverse of the heavy ion plasma frequency $\omega_{ph}^{-1}\left(=\sqrt{m_h/4\pi e^2 Z_h^2 n_h^{(0)}}\right)$, respectively. The heavy-ion kinematic viscosity is given by $\eta(=\eta_h C_{IAS}/m_h n_h^{(0)}\lambda_{DL}^2)$, where η_h is the heavy-ion dynamic viscosity.

It is interesting to mention here that we can apply the term NULF shock waves instead of ion-acoustic shock waves because the inertia is supplied by the heavier ions, while the restoring force is given by the degenerate pressure of the lighter ions and electrons. Therefore, NULF shock waves are propagated in comparison with ion-acoustic shock waves in normal plasmas (*i.e.* electron-ion plasmas). In this present case, applying the reductive perturbation method (*36*), we first introduce the independent variables:

$$\zeta = \varepsilon(x - \lambda t), \text{ and } \tau = \varepsilon^2 t, \tag{6}$$

where ε is a small($\ll 1$) and real parameter, and λ is the unknown propagation speed of shock waves normalized by the ion acoustic speed. In addition, the dependent variables are expanded as

$$\psi = \psi^{(0)} + \sum_{n=1}^{\infty} \varepsilon^n \psi^{(n)}, \tag{7}$$

where

$$\psi = [n_h, u_h, \phi] \text{ and } \psi^{(0)} = [1, 0, 0]. \tag{8}$$

Substituting (6) and (7) into the simulation model and collecting the lowest-order terms in ε, we can obtain the following relations**:**

$$n_h^{(1)} = \frac{1}{\lambda^2}\phi^{(1)}, \tag{9}$$

$$u_h^{(1)} = \frac{1}{\lambda}\phi^{(1)}, \tag{10}$$

$$n_\ell^{(1)} = -\frac{1}{\Gamma_\ell \gamma_\ell}\phi^{(1)}, \tag{11}$$

$$n_e^{(1)} = \frac{1}{\Gamma_e \gamma_e}\phi^{(1)}, \tag{12}$$

Let us now substitute the first order number densities $n_h^{(1)}, n_\ell^{(1)}$, and $n_e^{(1)}$ in Poisson equation, the propagation speed λ of the NULF shock waves is given by

$$\lambda = \sqrt{\frac{1}{(\beta/\Gamma_e \gamma_e) + (\alpha/\Gamma_\ell \gamma_\ell)}}, \tag{13}$$

In the same way, the next-higher order in ε is obtained, we can finally obtain the Burgers' equation (BE) (*31*)

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} = B \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}, \tag{14}$$

where $A \left(= \frac{3}{2\lambda} - \frac{\lambda^3}{2} \left(\beta \frac{(2-\gamma_e)}{(\Gamma_e \gamma_e)^2} - \alpha \frac{(2-\gamma_\ell)}{(\Gamma_\ell \gamma_\ell)^2} \right) \right)$ is the nonlinear coefficient, $B \left(= \frac{\eta}{2} \right)$ is the dissipative coefficient.

The NULF shock wave solution of the BE is derived by transforming the independent variables ζ and τ to $\xi = \zeta - v_0 \tau$, where v_0 is a constant velocity normalized to C_{IAS}, representing the speed of the shock wave in the moving reference frame. It is well known that the velocity of the moving frame is the sound speed λ, and the realistic velocity of the shock wave is $\lambda + v_0$. Further, we can write the NULF shock wave solution of (14) as (31).

$$\phi^{(1)} = \phi_0\{1 - \tanh(\xi/W)\}, \tag{15}$$

where the amplitude, ϕ_0, and the width, W, of a single NULF shock wave are given by v_0/A and $2B/v_0$, respectively. Furthermore, the electric field, E, is given by (31)

$$E = -d\phi^{(1)}/d\xi = (v_0^2/2AB)\sec h^2(\xi/W). \tag{16}$$

3. NULF multi-shock waves' collisions

In this section, we apply the Cole-Hopf transformation and the exponential function to obtain NULF multi-shock wave solutions (37). We can now transform Equation (14) into the standard BE equation by changing the variables $\tau \to T, \zeta \to -\sqrt{B}X$ and$\phi^{(1)} \to -\sqrt{B}U/A$ to write

$$\frac{\partial U}{\partial T} + U\frac{\partial U}{\partial X} = \frac{\partial^2 U}{\partial X^2}. \tag{17}$$

We can reduce (17) to linear heat equation by the Cole-Hopf transformation (37) namely

$$U = -2(\log \Psi)_X. \tag{18}$$

Then, one can obtain

$$\Psi_T - \Psi_{XX} = 0. \tag{19}$$

It should be mentioned here that the derivatives are denoted by lower-case subscript X and T. The simple solution of the heat equation (*i.e.* Equation (19)) reads

$$\Psi_i = \alpha_i e^{(k_i X - \omega_i T)}. \tag{20}$$

where i $= 1,2,3,\dots N, k_i$ is the wavenumber, and $\omega_i$is the frequency. In general, the following linear superposition considers a general solution of the heat equation:

$$\Psi = \alpha_0 + \sum_{i=1}^{N} \alpha_i e^{(k_i X - \omega_i T)}, \tag{21}$$

where α_0 and α_i are arbitrary constants. It is clear that one can directly apply the rational exponential function to satisfy the single as well as multi-shock wave solutions of (14) (34). First, the single NULF shock wave solution of (17) is given by (34)

$$U = \frac{\alpha_1 k_1 e^{(k_1 X - \omega_1 T)}}{\alpha_0 + \alpha_1 e^{(k_1 X - \omega_1 T)}}. \tag{22}$$

Therefore, after a bit of manipulation, one can easily obtain the linear dispersion relation of (17) as $(\omega_1 = -k_1^2)$. Then the analytical single NULF shock wave solution of (14) is written as (34)

$$\phi^{(1)} = \frac{2\sqrt{B}}{A}\frac{\alpha_1 k_1 e^{\left((-k_1/\sqrt{B})\zeta + k_1^2\tau\right)}}{\alpha_0 + \alpha_1 e^{\left((-k_1/\sqrt{B})\zeta + k_1^2\tau\right)}}. \tag{23}$$

For two-overtaking NULF shock wave structures, the following rational exponential function is considered (34):

$$\phi^{(1)} = \frac{2\sqrt{B}}{A}\left[\frac{\sum_{i=1}^{2}\left(\alpha_i k_i e^{\left((-k_i/\sqrt{B})\zeta-\omega_i\tau\right)}\right)+\alpha_{12}(k_1+k_2)\alpha_1\alpha_2 e^{\left(\sum_{i=1}^{2}\left((-k_i/\sqrt{B})\zeta\right)-\sum_{i=1}^{2}(\omega_i\tau)\right)}}{\alpha_0+\sum_{i=1}^{2}\left(\alpha_i e^{\left((-k_i/\sqrt{B})\zeta-\omega_i\tau\right)}\right)+\alpha_{12}\alpha_1\alpha_2 e^{\left(\sum_{i=1}^{2}\left((-k_i/\sqrt{B})\zeta\right)-\sum_{i=1}^{2}(\omega_i\tau)\right)}}\right]. \tag{24}$$

Putting (24) into (14) and equating the coefficients of different power of exponential functions equal to zero. One can notice that there are three cases: Case I: $\omega_1 = -k_1^2; \omega_2 = -k_2^2; \alpha_{12} = 0$, the two-overtaking NULF shock wave analytical solution of BE is represented by (*34*)

$$\phi^{(1)} = \frac{2\sqrt{B}}{A}\left[\frac{\sum_{i=1}^{2}\left(\alpha_i k_i e^{\left((-k_i/)\sqrt{B})\zeta+k_i^2\tau\right)}\right)}{\alpha_0+\sum_{i=1}^{2}\left(\alpha_i e^{\left((-k_i/)\sqrt{B})\zeta+k_i^2\tau\right)}\right)}\right]. \tag{25}$$

From the in-depth analyses of Equation (25), it is clear that for all the ranges of two arbitrary parameters k_1 and k_2 the two NULF shock waves were temporarily (at $\tau = 0$) merged into a single shock wave, as time goes on, the two NULF shock waves continue to move away from each other. Here, we expect that the collision will be quasielastic. Case II:$\alpha_0 = 0; \omega_2 = \omega_1 + k_1^2 - k_2^2; \alpha_{12} = 0$. Then the two-overtaking NULF shock wave analytical solution of BE is given by (*24*)

$$\phi^{(1)} = \frac{2\sqrt{B}}{A}\left[\frac{\sum_{i=1}^{2}\left(\alpha_i k_i e^{\left((-k_i/)\sqrt{B})\zeta-\omega_i\tau\right)}\right)}{\sum_{i=1}^{2}\left(\alpha_i e^{\left((-k_i/)\sqrt{B})\zeta-\omega_i\tau\right)}\right)}\right]. \tag{26}$$

In this case, the collision will be completely elastic of the two NULF shock waves for all the ranges of two arbitrary parameters k_1 and k_2. Case III:$\alpha_0 = 0; \omega_1 = -k_1^2 - 2k_1k_2; \omega_2 = -k_2^2 - 2k_1k_2$ and the two-overtaking NULF shock wave reads (*34*)

$$\phi^{(1)} = \frac{2\sqrt{B}}{A}\left[\frac{\sum_{i=1}^{2}\left(\alpha_i k_i e^{\left((-k_i/)\sqrt{B})\zeta-\omega_i\tau\right)}\right)+\alpha_{12}(k_1+k_2)\alpha_1\alpha_2 e^{\left(\sum_{i=1}^{2}\left((-k_i/)\sqrt{B})\zeta\right)-\sum_{i=1}^{2}(\omega_i\tau)\right)}}{\sum_{i=1}^{2}\left(\alpha_i e^{\left((-k_i/)\sqrt{B})\zeta-\omega_i\tau\right)}\right)+\alpha_{12}\alpha_1\alpha_2 e^{\left(\sum_{i=1}^{2}\left((-k_i/)\sqrt{B})\zeta\right)-\sum_{i=1}^{2}(\omega_i\tau)\right)}}\right]. \tag{27}$$

From careful investigations of Equation (27), it is obvious that the collision will be elastic of the two NULF shock waves for all the ranges of two arbitrary parameters k_1 and k_2. In this study, we focus on the two-overtaking NULF shock wave analytical solution of BE in the case I and ignore the cases II and III for simplicity. It should be mentioned here that the rational exponential function (24) is also checked by considering $e^{(k_i X - \omega_i T)}$; and substituting the Burgers equations, which yields the linear dispersion relation $\omega_i = -k_i^2$, as mentioned in the first case. Interestingly, (25) illustrates that the Burgers Equation (14) is only supported by moving NULF multi-shock waves.

Following the same strategy, the three-overtaking NULF shock wave analytical solution of BE is given by (34)

$$\phi^{(1)} = \frac{2\sqrt{B}}{A}\left[\frac{\sum_{i=1}^{3}\left(\alpha_i k_i e^{\left((-k_i/)\sqrt{B})\zeta+k_i^2\tau\right)}\right)}{\alpha_0+\sum_{i=1}^{3}\left(\alpha_i e^{\left((-k_i/)\sqrt{B})\zeta+k_i^2\tau\right)}\right)}\right]. \tag{28}$$

In general, the overtaking phenomenon between NULF multi-shock waves is written as (24)

$$\phi^{(1)} = \frac{2\sqrt{B}}{A}\left[\frac{\sum_{i=1}^{N}\left(\alpha_i k_i e^{\left((-k_i/)\sqrt{B})\zeta+k_i^2\tau\right)}\right)}{\alpha_0+\sum_{i=1}^{N}\left(\alpha_i e^{\left((-k_i/)\sqrt{B})\zeta+k_i^2\tau\right)}\right)}\right], \tag{29}$$

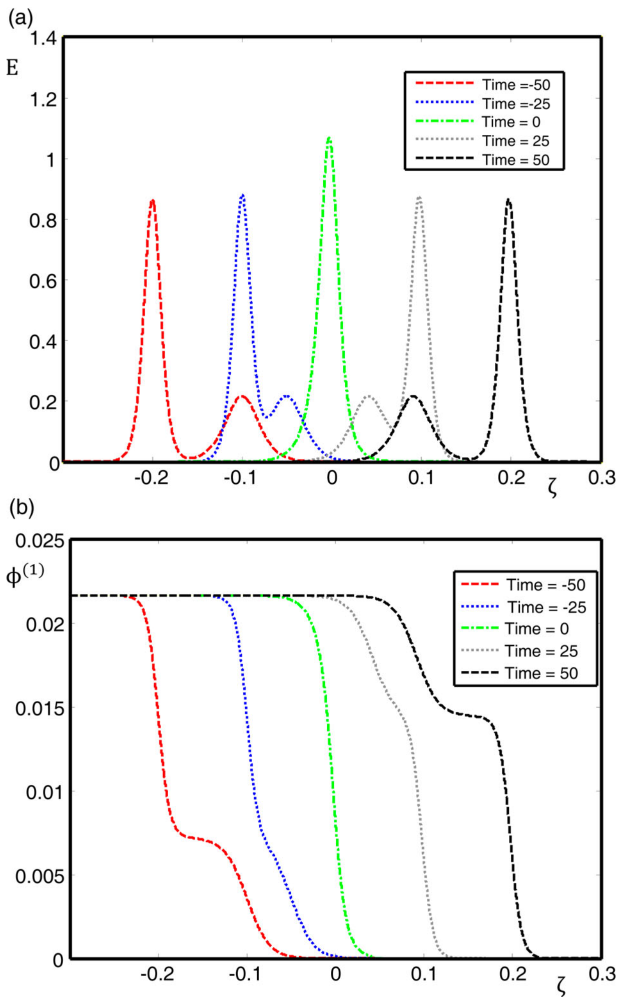

Figure 1. The overtaking collision profiles between (a) two single-E and (b) two single-ULFSW at the different values of time for $\alpha = 0.66$, $\eta = 0.01$, $\gamma_e = \gamma_\ell = 4/3$, $k_1 = 0.4$, and $k_2 = 0.8$.

where N is a positive integer number, α_0 and α_i are free parameters, which determine the phase shifts of the respective NULF multi-shock waves after overtaking collisions. However, based on the above discussion, NULF multi-shock waves are merely superposition of each other for all time, it follows that there may not be a phase shift, since each NULF multi-shock waves is consequently unaffected by a collision. Therefore, one can use the free parametric values $\alpha_0 = \alpha_i = 1, k_i = i$, where $i = 1, 2, 3, \ldots N$. Now, let us rewrite the two-overtaking NULF shock wave analytical solution of BE in the following mathematical expression:

$$\phi^{(1)} = \frac{2\sqrt{B}}{A}\left\{\frac{\alpha_1 k_1 e^{\theta_1} + \alpha_2 k_2 e^{\theta_2}}{(\alpha_0 + (\alpha_1 e^{\theta_1} + \alpha_2 e^{\theta_2}))}\right\}, \tag{30}$$

where $\theta_{1,2}\left(= \frac{-k_{1,2}}{\sqrt{B}}\zeta + k_{1,2}^2\tau\right)$ is the notation that contains the linear dispersion law, $\omega_{1,2} = -k_{1,2}^3$, describing the BE. For the condition; $\tau >> 1$, the two-overtaking NULF shock wave

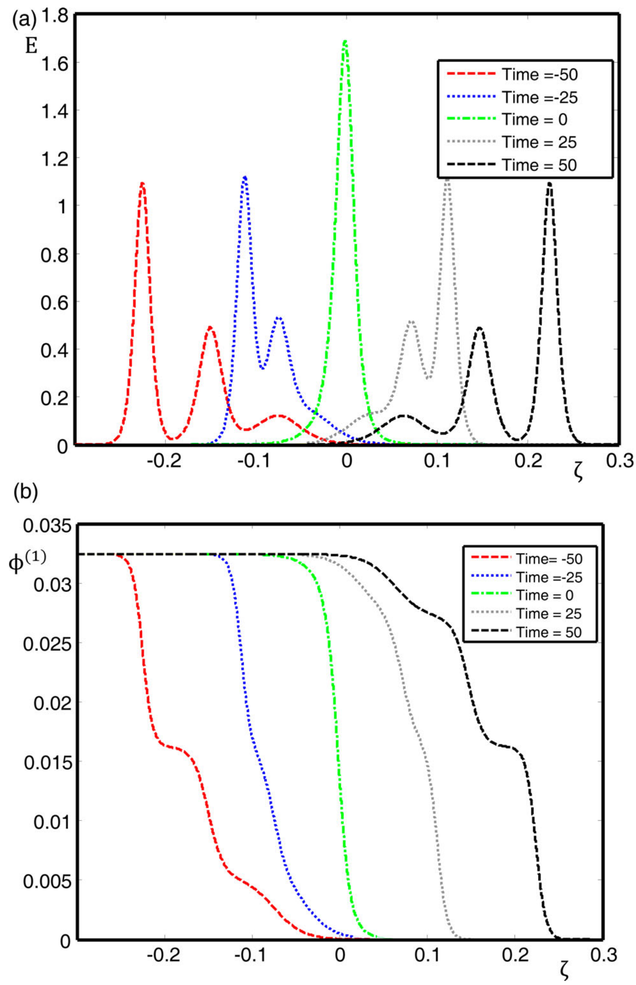

Figure 2. The overtaking collision profiles between (a) three single-E and (b) three single-ULFSW at the different values of time for $\alpha = 0.66$, $\eta = 0.01$, $\gamma_e = \gamma_\ell = 4/3$, $k_1 = 0.3$, $k_2 = 0.6$ and $k_3 = 0.9$.

solution (*i.e.* Equation (30)) asymptotically transforms into a superposition of two single-NULF shock wave solution

$$\phi^{(1)} \approx \frac{2\sqrt{B}}{A}\left(\frac{k_1}{(1+e^{-\theta_1})}+\frac{k_2}{(1+e^{-\theta_2})}\right). \tag{31}$$

By using the identity $1/(1+e^{-x}) = \frac{1}{2}\left(1+\tanh\left(\frac{x}{2}\right)\right)$, and after some algebraic manipulations, one can obtain the asymptotic solution of (14) to write

$$\phi^{(1)} \approx \sum_{i=1}^{2}\left\{\phi_i^{(0)}\left[1-\tanh\left(\frac{k_i}{2B}(\zeta - Bk_i\tau)\right)\right]\right\}. \tag{32}$$

where $\phi_i^{(0)}\left(=3k_i\sqrt{B}/2A\right)$ are the amplitudes of two single-NULF shock waves due to overtaking collisions. Moreover, (32) indicates that we are dealing with the superposition of two single-NULF shock waves moving in the same direction. In addition, the expression of electric field, E, can be written as

$$E = \sum_{i=1}^{2}\left\{\left(3k_i^2/4A\sqrt{B}\right)\sec h^2\left(\frac{k_i}{2B}(\zeta - Bk_i\tau)\right)\right\}, \tag{33}$$

In fact, we can use the same strategy to study the overtaking phenomenon between the NULF multi-shock wave solutions. In general, the structure of NULF multi-shock wave

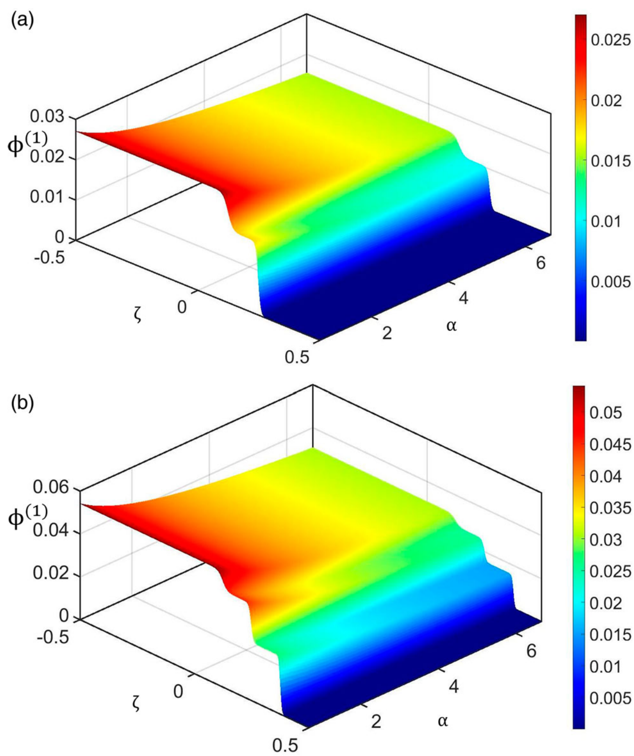

Figure 3. The overtaking collision profile between (a) two single-ULFSW and (b) three single-ULFSW for different values of α at $\tau = 40$, $\eta = 0.01$, $\gamma_e = \gamma_\ell = 4/3$, $k_1 = 0.5$, $k_2 = 1$ and $k_3 = 1$.

solutions and E are, respectively, given by:

$$\phi^{(1)} \approx \sum_{i=1}^{N} \left\{ \phi_i^{(0)} \left[1 - \tanh\left(\frac{k_i}{2B}(\zeta - Bk_i\tau) \right) \right] \right\}, \tag{34}$$

$$E = \sum_{i=1}^{N} \left\{ \left(3k_i^2/4A\sqrt{B} \right) \sec h^2 \left(\frac{k_i}{2B}(\zeta - Bk_i\tau) \right) \right\}. \tag{35}$$

4. Numerical simulation and discussion

Depending on the mathematical analysis discussed above, our interest is the investigation of the overtaking phenomenon between two/three single- NULF shock waves. Now, let us study the physical nature of NULF multi-shock waves and the electric field, E, during an overtaking collision by simulating the dynamic process of an overtaking collision of two/three single-NULF multi-shock waves/E propagating in the same directions and having different amplitudes**.** For numerical simulation (*31*), the range of considered typical parameters for white dwarf stars is $n_s^{(0)} \cong 10^{29} - 10^{31}\,\mathrm{cm}^{-3}$, which must satisfy the quasi-neutrality condition. Figure 1 (2) presents position/time graph for two (three) single-E/NULF shock waves. Furthermore, the overtaking collisions between two and three single-E/NULF shock waves are shown in Figures 1a,b and 2a,b, respectively. The positions are recorded for the discrete

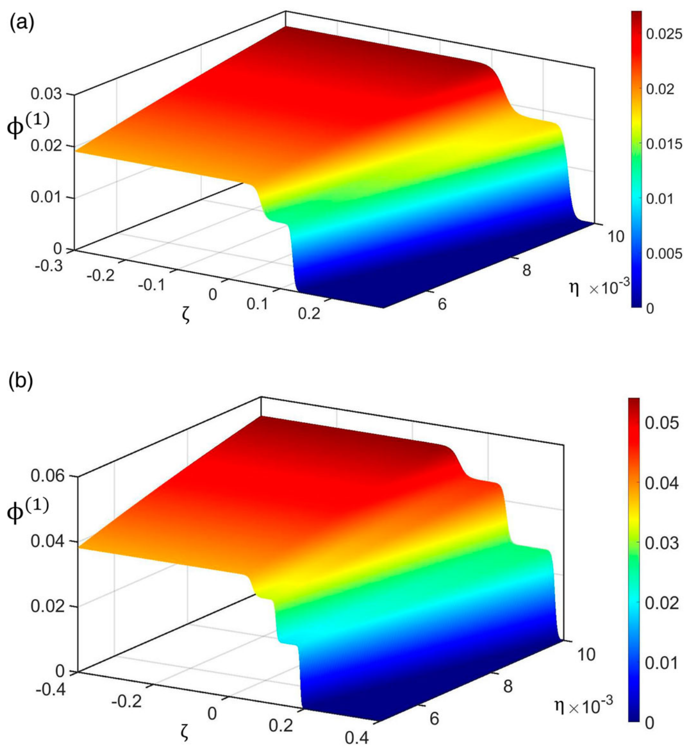

Figure 4. The overtaking collision profile between (a) two single-ULFSW and (b) three single-ULFSW for different values of η at $\tau = 40$, $\alpha = 0.66$, $\gamma_e = \gamma_\ell = 4/3$, $k_1 = 0.5$, $k_2 = 1$ and $k_3 = 1$.

various times $\tau = -50, -25, 0, 25$, and 50. Generally, the patterns illustrate the evolution in the profile of the progressive two and three single-E/NULF shock waves before ($\tau < 0$), during ($\tau = 0$), and after ($\tau > 0$) the overtaking collision. Let us now describe the features of NULF multi-shock waves due to the overtaking collisions. Now, we investigate the effects of the light-to-heavy ion density ratio, α, and the heavy-ion kinematic viscosity, η, on the amplitude and the steepness of NULF multi-shock waves. Figure 3a,b reflect the interesting effect of α on the profiles of the two and three single-NULF shock waves, respectively. It is found that the amplitude and the steepness of two/three single-NULF shock waves increase with the decrease in the light-to-heavy ion density ratio. Figure 4a,b illustrate how the physical nature of the two and three single- NULF shock waves varies, respectively, with the variation of the heavy-ion kinematic viscosity, η. One can observe in Figure 4a,b that, as η decreases, the strength (width) of the NULF multi-shock waves decreases (increases). Now, comparing our results with those of Islam et al. (*31*) would be interesting. The results for Islam et al. (*31*) are in agreement with the present results for the single-NULF shock wave, except that the shock wave amplitude is independent of η.

In this study, we investigated the excitation and the overtaking collision of NULF multi-shock waves in ultra-relativistic degenerate plasmas. By applying Cole-Hopf transformation and the exponential function, we have discussed the two/three single- NULF shock waves dynamics together with the physical parameters of the system, such as the light-to-heavy ion density ratio and the heavy-ion kinematic viscosity. In addition, the numerical simulation demonstrated that the light-to-heavy ion density ratio and the heavy-ion kinematic viscosity have substantial influences on the features of NULF multi-shock waves before, during, and after overtaking collisions. Finally, we believe that the present study may be helpful for an in-depth understanding of the fundamental properties of the nonlinear propagation of NULF multi-shock waves in ultra-relativistic degenerate plasmas that may occur in many astrophysical compact objects, like white dwarfs.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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