Eur. Phys. J. Plus (2021) 136:455
https://doi.org/10.1140/epjp/s13360-021-01472-2

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Closed-form solutions to the new coupled Konno–Oono equation and the Kaup–Newell model equation in magnetic field with novel statistic application

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Received: 28 July 2020 / Accepted: 21 April 2021

Abstract This paper employs the new robust solver to retrieve exact solutions to a Kaup–Newell model equation and the nonlinear coupled Konno–Oono equation. This solver yields the closed formula for the solutions. Different types of travelling wave solutions, i.e., rational function, hyperbolic function and trigonometric function solutions with many capricious parameters are revealed. Subsequently, by utilizing Matlab 18 we plot 2D and 3D surfaces of obtained analytical solutions for suitable values of the free parameters. The depiction of the solver is direct, vital, sturdy and can be applied to other nonlinear partial differential equations. We also show that some proposed rational solutions can be rapprochement with some known probability distributions.

1 Introduction

The nonlinear partial differential equations (NPDEs.) have become most examined subject of all-embracing studies in several branches of applied science, such as fluid mechanics, optics, ecology, engineering, electromagnetic theory, chemical physics, plasma physics, solid state physics, [1–10]. Due to the complexity of the nonlinear wave equations, there is no unified technique to obtain all solutions of NPDEs. Namely, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as: tanh–sech method [11], first integral method [12], $(\frac{G'}{G})-$ expansion method [13], modified Kudryashov method [14], sub-equation method [15], variational iteration method [16], exponential function method [17], fractional sub-equation method [18], Riccati–Bernoulli sub-ODE method [19], homotopy perturbation [20].

Suppose the NPDEs,

$$\Upsilon(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, ...) = 0. \tag{1.1}$$

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Published online: 27 April 2021

Utilizing the wave transformation:

$$u(x,t) = U(\zeta), \qquad \zeta = x - wt, \tag{1.2}$$

Eq. (1.1) reduced to the following ODE:

$$\Pi\big(U, U', U'', U''', ...\big) = 0. \tag{1.3}$$

It is well known that there are many models of NPDEs (1.1) in applied science converted to the following ODE:

$$\alpha U'' + \beta U^3 + \gamma U = 0, \tag{1.4}$$

see [21–30] and so on. Eq. (1.4) symbolizes a Hamiltonian system with many vital applications [31]. As a result of the importance of Eq. (1.3), we introduced the sturdy solver for the widely used NPDEs [3], using RB sub-ODE method [19].

The dimensionless form of Kaup–Newell equation (KNE) is given by [32–34]

$$\chi_t + ia\chi_{xx} + b\big(\mid \chi \mid^2 \chi\big)_x = 0 \qquad i = \sqrt{-1}, \tag{1.5}$$

where $a, b \in \mathbb{R} - \{0\}$, represent the parameter of group velocity dispersion and the nonlinearity coefficient, respectively. Here, $\chi(x,t)$ denotes the magnetic field transverse component to lowest order [35]. This equation was defined as an alternative model to the nonlinear Schrödinger equation, see [36]. Biswas et al. [32] applied the modified simple equation method and trial equation approach to give dark, bright and singular solitons to Eq. (1.5). Biswas et al. [33] employed the extended trial function method to get sub-pico-second optical soliton solutions of Eq. (1.5). Souleymanou et al. [34] considered the extended direct algebraic method to give exact solutions to Eq. (1.5).

The nonlinear coupled Konno–Oono (CKO) equation was presented by Konno and Oono [37]:

$$\begin{aligned} &q_{xt} - 2Lq\, r_{xx} - 2Mq\, s_x + N(rs)_x = 0,\\ &r_{xt} - 2Lr\, r_x - 2M(2qq_x + r_x s) - 2N(q)_x r = 0,\\ &s_{xt} - 2Ms\, s_x - 2L(2qq_x + rs_x) - 2Ns(q)_x = 0, \end{aligned} \tag{1.6}$$

where L, M and N are constants. This system has been scrutinized as applications for current-filed string interacting with an external magnetic field [37–39], and the parallel transport of each point of the curve along the direction of time where the connection is magnetic-valued [40]. Koçak et al. [41] obtained trawling wave solution to CKO equations utilizing the modified exponential function method.

In this paper, we consider a special case of system (1.6), which is a new Konno–Oono equation system [42–45]:

$$\begin{aligned} &\phi_{xt} - 2\phi\,\psi = 0,\\ &\psi_t + 2\phi\phi_x = 0. \end{aligned} \tag{1.7}$$

In recent years, this system has gained a significant attention and has been the subject of various studies, like tanh-function method & extended tanh-function method [42], the sine-Gordon expansion method [43], the extended exp function method [44] and the external trial equation method [45].

The application in the field of probability theory has very prominence in the recently few years, attaching some of the obtained solutions with some known probability density functions such as t-distribution gives us more information about the properties and behavior of the solution. We show that the rational solutions form some probability function distributions such as Beta distribution, Gamma distribution, t-distribution,... etc., with fixed parameters. In this work, we can see that the solution (4.7) associated with t-distribution with parameter $k = 3$.

This article is ordered as follows. Section 2, gives the robust unified solver for the equation $\alpha U'' + \beta U^3 + \gamma U = 0$. Section 3 introduces the solutions to the KNE equation. Section 4 presents the solutions to the new CKO equation. Section 5 compares the new CKO rational function solution with an associated statistical distribution. Section 6 presents the physical explanation for the obtained results. Conclusions will appear in Sect. 7.

2 Unified solver

Here, we present the unified solver concerning the following equation

$$\alpha\Theta'' + \beta\Theta^3 + \gamma\Theta = 0. \tag{2.1}$$

According to the introduced solver in [3], the solutions to Eq. (2.1) are

Rational function solutions: (at $\gamma = 0$)

$$U_{1,2}(x,t) = \left(\mp\sqrt{\frac{-\beta}{2\alpha}}\,(\zeta + \mu)\right)^{-1}. \tag{2.2}$$

Trigonometric function solutions: (at $\frac{\gamma}{\alpha} < 0$)

$$U_{3,4}(x,t) = \pm\sqrt{\frac{\gamma}{\beta}}\tan\left(\sqrt{\frac{-\gamma}{2\alpha}}\,(\zeta + \mu)\right) \tag{2.3}$$

and

$$U_{5,6}(x,t) = \pm\sqrt{\frac{\gamma}{\beta}}\cot\left(\sqrt{\frac{-\gamma}{2\alpha}}\,(\zeta + \mu)\right). \tag{2.4}$$

Hyperbolic function solutions: (at $\frac{\gamma}{\alpha} > 0$)

$$U_{7,8}(x,t) = \pm\sqrt{\frac{-\gamma}{\beta}}\tanh\left(\sqrt{\frac{\gamma}{2\alpha}}\,(\zeta + \mu)\right) \tag{2.5}$$

and

$$U_{9,10}(x,t) = \pm\sqrt{\frac{-\gamma}{\beta}}\coth\left(\sqrt{\frac{\gamma}{2\alpha}}\,(\zeta + \mu)\right), \tag{2.6}$$

where μ is arbitrary constant. In the next sections, we employ the unified solver in order to solve the KNE equation and the new Konno–Oono equation system.

3 Solutions of the KNE equation

Using the transformation

$$\chi(x,t) = \mathrm{e}^{i(-kx+wt+\varsigma)} V(\zeta), \qquad \zeta = x - \nu t, \tag{3.1}$$

where k, w and ς denote, respectively, soliton frequency, soliton wave number and soliton phase. Setting (3.1) into (1.5), the real and imaginary components give rise to

$$\nu = 2ak + 3bV^2 \tag{3.2}$$

and

$$aU'' - bkV^3 + (w - ak^2)V = 0, \tag{3.3}$$

respectively. Comparing Eq. (3.3) with Eq. (2.1) yields $\alpha = a$, $\beta = -bk$ and $\gamma = w - ak^2$. Thus, the solutions of Eq. (1.5) are:

The rational solutions of Eq. (3.3) are

$$V_{1,2}(x,t) = \left(\mp\sqrt{\frac{bk}{2a}}\,(x - \nu t + \mu)\right)^{-1}. \tag{3.4}$$

As a result, the solutions of Eq. (1.5), using Eq. (3.1), are

$$\chi_{1,2}(x,t) = \mathrm{e}^{i(-kx+wt+\varsigma)}\left(\mp\sqrt{\frac{bk}{2a}}\,(x - \nu t + \mu)\right)^{-1}. \tag{3.5}$$

The trigonometric solutions of Eq. (3.3) are

$$V_{3,4}(x,t) = \pm\sqrt{\frac{ak^2 - w}{bk}}\,\tan\left(\sqrt{\frac{ak^2 - w}{2a}}\,(x - \nu t + \mu)\right) \tag{3.6}$$

and

$$V_{5,6}(x,t) = \pm\sqrt{\frac{ak^2 - w}{bk}}\,\cot\left(\sqrt{\frac{ak^2 - w}{2a}}\,(x - \nu t + \mu)\right). \tag{3.7}$$

As a result, the solutions of Eq. (1.5), using Eq. (3.1), are

$$\chi_{3,4}(x,t) = \pm\,\mathrm{e}^{i(-kx+wt+\varsigma)}\sqrt{\frac{ak^2 - w}{bk}}\,\tan\left(\sqrt{\frac{ak^2 - w}{2a}}\,(x - \nu t + \mu)\right) \tag{3.8}$$

and

$$\chi_{5,6}(x,t) = \pm\,\mathrm{e}^{i(-kx+wt+\varsigma)}\sqrt{\frac{ak^2 - w}{bk}}\,\cot\left(\sqrt{\frac{ak^2 - w}{2a}}\,(x - \nu t + \mu)\right). \tag{3.9}$$

The hyperbolic solutions of Eq. (3.3) are

$$V_{7,8}(x,t) = \pm\sqrt{\frac{w - ak^2}{bk}}\,\tanh\left(\sqrt{\frac{w - ak^2}{2a}}\,(x - \nu t + \mu)\right) \tag{3.10}$$

and

$$V_{9,10}(x,t) = \pm\sqrt{\frac{w-ak^2}{bk}}\coth\left(\sqrt{\frac{w-ak^2}{2a}}\,(x-\nu\,t+\mu)\right). \tag{3.11}$$

As a result, the solutions of Eq. (1.5), using Eq. (3.1), are

$$\chi_{7,8}(x,t) = \pm\,\mathrm{e}^{i(-kx+wt+\varsigma)}\sqrt{\frac{w-ak^2}{bk}}\tanh\left(\sqrt{\frac{w-ak^2}{2a}}\,(x-\nu\,t+\mu)\right) \tag{3.12}$$

and

$$\chi_{9,10}(x,t) = \pm\,\mathrm{e}^{i(-kx+wt+\varsigma)}\sqrt{\frac{w-ak^2}{bk}}\coth\left(\sqrt{\frac{w-ak^2}{2a}}\,(x-\nu\,t+\mu)\right). \tag{3.13}$$

4 Solutions of the new CKO equation

Now, to solve Eq. (1.7), using the transformation

$$\begin{aligned}\phi(x,t) &= \Phi(\zeta), \quad \zeta = c(x-\lambda\,t)\\ \psi(x,t) &= \Psi(\zeta), \quad \zeta = c(x-\lambda\,t),\end{aligned} \tag{4.1}$$

where c is the wave number and λ is the wave velocity. Plugging (3.1) into (1.5) yields

$$-\lambda\,c^2\Phi'' - 2\Phi\Psi = 0, \tag{4.2}$$

$$-\lambda\,c\Phi' + 2c\Phi\Phi' = 0, \tag{4.3}$$

Integrating Eq. (4.3) with respect to ζ gives

$$\Psi = \frac{1}{\lambda}(\Phi^2+\eta), \tag{4.4}$$

where η is an integral constant. Setting Eq. (4.4) into Eq. (4.3) gives

$$\lambda^2\,c^2\Phi'' + 2\Phi^3 + 2\eta\Phi = 0. \tag{4.5}$$

Comparing Eq. (4.5) with Eq. (2.1) yields $\alpha = \lambda^2\,c^2$, $\beta = 2$ and $\gamma = 2\eta$. Thus, the solutions of Eq. (1.7) are:

The rational solutions of Eq. (1.7) are

$$\phi_{1,2}(x,t) = \left(\mp\frac{i}{\lambda\,c}\,(c(x-\lambda\,t)+\mu)\right)^{-1}. \tag{4.6}$$

Utilizing Eq. (4.4) gives

$$\psi_{1,2}(x,t) = \frac{1}{\lambda}\left(\left(\left(\mp\frac{i}{\lambda\,c}\,(c(x-\lambda\,t)+\mu)\right)^{-1}\right)^2+\eta\right). \tag{4.7}$$

The trigonometric solutions of Eq. (1.7) are

$$\phi_{3,4}(x,t) = \pm\sqrt{\eta}\tan\left(\frac{\sqrt{-\eta}}{\lambda\,c}\,(c(x-\lambda\,t)+\mu)\right) \tag{4.8}$$

and

$$\phi_{5,6}(x,t) = \pm\sqrt{\eta}\cot\left(\frac{\sqrt{-\eta}}{\lambda c}\left(c(x-\lambda t)+\mu\right)\right). \tag{4.9}$$

Utilizing Eq. (4.4) gives

$$\psi_{3,4}(x,t) = \frac{1}{\lambda}\left(\left(\sqrt{\eta}\tan\left(\frac{\sqrt{-\eta}}{\lambda c}\left(c(x-\lambda t)+\mu\right)\right)\right)^2+\eta\right) \tag{4.10}$$

and

$$\psi_{5,6}(x,t) = \frac{1}{\lambda}\left(\left(\sqrt{\eta}\cot\left(\frac{\sqrt{-\eta}}{\lambda c}\left(c(x-\lambda t)+\mu\right)\right)\right)^2+\eta\right). \tag{4.11}$$

The hyperbolic solutions of Eq. (1.7) are

$$\phi_{7,8}(x,t) = \pm\sqrt{-\eta}\tanh\left(\frac{\sqrt{\eta}}{\lambda c}\left(c(x-\lambda t)+\mu\right)\right) \tag{4.12}$$

and

$$\phi_{9,10}(x,t) = \pm\sqrt{-\eta}\coth\left(\frac{\sqrt{\eta}}{\lambda c}\left(c(x-\lambda t)+\mu\right)\right). \tag{4.13}$$

Utilizing Eq. (4.4) gives

$$\psi_{7,8}(x,t) = \frac{1}{\lambda}\left(\left(\sqrt{-\eta}\tanh\left(\frac{\sqrt{\eta}}{\lambda c}\left(c(x-\lambda t)+\mu\right)\right)\right)^2+\eta\right) \tag{4.14}$$

and

$$\psi_{9,10}(x,t) = \frac{1}{\lambda}\left(\left(\sqrt{-\eta}\coth\left(\frac{\sqrt{\eta}}{\lambda c}\left(c(x-\lambda t)+\mu\right)\right)\right)^2+\eta\right). \tag{4.15}$$

5 Statistical evidence

In applied statistics, the primary application of t-distribution for determining confidence intervals and hypothesis testing. The density function of t-distribution is

$$f_k(y) = \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{k\pi}\,\Gamma\left(\frac{k}{2}\right)}\left(1+\frac{y^2}{k}\right)^{-\left(\frac{k+1}{2}\right)}, \tag{5.1}$$

where Γ is the gamma function defined by:

$$\Gamma(t) = (t-1)! = \int_{-\infty}^{+\infty} x^{t-1}\mathrm{e}^{-x}\mathrm{d}x,\ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

and k is non-negative parameter. We can see that the solution (4.7), since $\eta = 0$, can be written as following

$$\psi_{1,2}(x,t) = -\lambda c^2(c(x-\lambda t)+\mu)^{-2}. \tag{5.2}$$

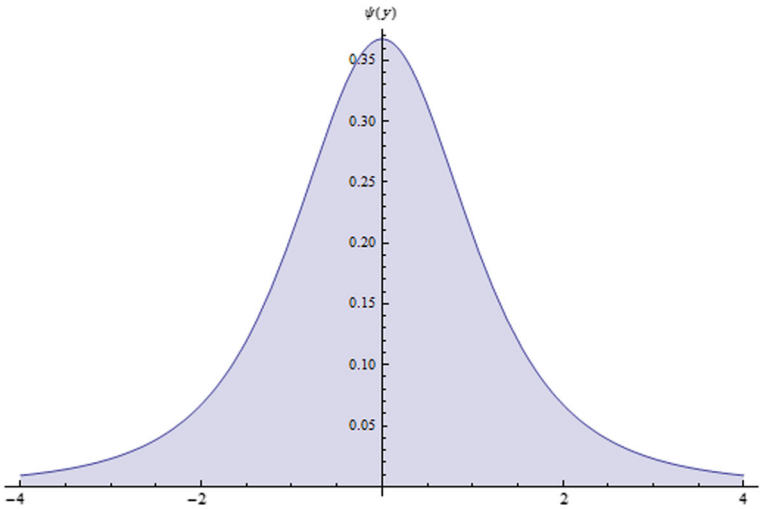

Fig. 1 T-distribution with parameter $k = 3$

This result adjusted to t-distribution with parameter $k = 3$, if we assume the random variable $y(x, t)$ satisfies the following $y^2(x, t) = 3(c(x-\lambda t)+\mu)-3$, we can also take $\frac{2}{\pi\sqrt{3}} = -\lambda c^2$, then we can write Eq. (5.2) as

$$\psi_{1,2}(y(x,t)) = \frac{\Gamma\left(\frac{3+1}{2}\right)}{\sqrt{3\pi}\,\Gamma\left(\frac{3}{2}\right)}\left(1+\frac{y^2}{3}\right)^{-\left(\frac{3+1}{2}\right)}. \tag{5.3}$$

Figure 1 represents the curve of $\Psi(y)$ as a probability density function of t-distribution.

6 Physical interpretation

Here, we illustrate the applications of the results constructed above. With the aid of the symbolic Matlab software, we applied the unified solver to construct solutions for a KNE and the CKO equation. These exact solutions of the proposed equations were given in the explicit form. Namely, we presented rational function, hyperbolic function and trigonometric function solutions. These solutions give some wave pictures in applied sciences and describe complex phenomena, namely in magnetic filed as shown in Figs. 2 and 3 . Indeed, the hyperbolic function solutions (4.12) and (4.13) represent the ranges and altitudes of seismic sea waves as shown in Fig. 4. The waves would be more dangerous to the entire world if the amplitudes of the wave are high. To decrease the disastrous power of such massive natural disasters or to convert them to useful energy sources, we should consider the mathematical structures of such natural problems. To understand such possible catastrophes, the best way is to solve these problems by making use of different approaches and then take necessary precautions.

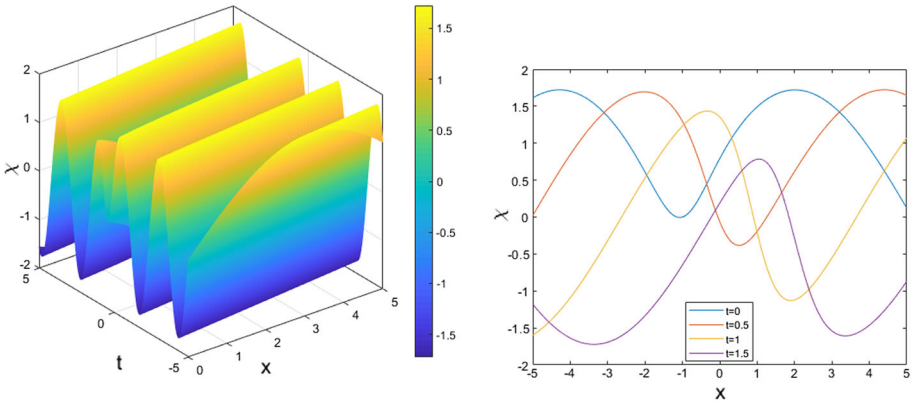

Fig. 2 Shapes of real part of $\chi = \chi_7$ with $k = 0.5$, $w = 2.4$, $\varsigma = 1$, $a = 0.7$, $b = 1.5$, $\nu = 1.9$, $\mu = 1$

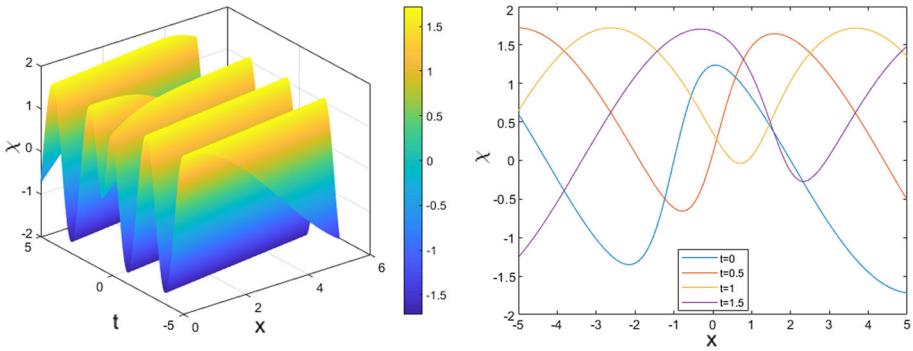

Fig. 3 Shape of imaginary part of $\chi = \chi_7$ with $k = 0.5$, $w = 2.4$, $\varsigma = 1$, $a = 0.7$, $b = 1.5$, $\nu = 1.9$, $\mu = 1$

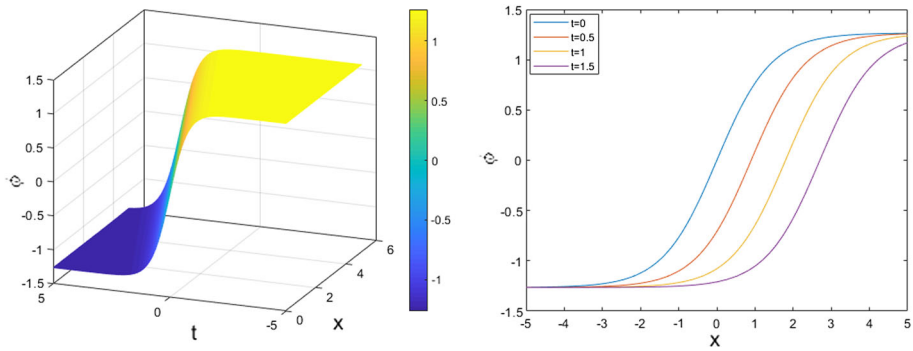

Fig. 4 Graphs of 3D and 2D of $\phi = \phi_7$ with $\eta = 1.6$; $\lambda = 1.8$; $c = 2.5$; $\mu = 0$

7 Conclusions

In this article, the sturdy unified solver was implemented for the KNE and the CKO equation to derive the solutions in various forms of traveling waves. This solver presents the closed form of the solutions, namely, rational function, hyperbolic function and trigonometric function solutions. We have clarified that the rational function solutions can be rapprochement with some known probability distributions, which is so interesting application in applied statistic. Our study shows that the presented solver is a powerful tool for obtaining analytical solutions. Indeed, this technique can also be applied to other nonlinear partial differential equations.

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