Optics Communications 282 (2009) 27–35

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Automatic determination of refractive index profile of fibers having regular and/or irregular transverse sections considering the refraction of light rays by the fiber

A.A. Hamza [b,c], T.Z.N. Sokkar [b], M.A. El-Morsy [a], M.A.E. Nawareg [a,*]

[a] Department of Physics, Faculty of Science, Mansoura University, New Damietta, Egypt
[b] Department of Physics, Faculty of Science, Mansoura University, Mansoura, Egypt
[c] The British University in Egypt, El Sherouk City, Cairo, Egypt

ARTICLE INFO

Article history:
Received 27 June 2008
Received in revised form 17 September 2008
Accepted 17 September 2008

Keywords:
Refractive index profiling
Irregular fiber
Refraction
Interferometry
Optical fiber

ABSTRACT

A new model, using non-destructive two- and/or multiple-beam interferometric techniques, is suggested for measuring the refractive index profile of fibers having regular and/or irregular cross-sectional shape taking into consideration the refraction of the light rays by the fiber. The proposed model is applied for three different fibers having different cross-sectional shapes and different refractive index profiles. These fibers are PPT, homogeneous fiber, with circular cross-section, graded index optical fiber of circular cross-sectional shape and Dralon fiber of irregular cross-section. To validate the proposed model it is used, firstly, to calculate the index profile for a standard PPT fiber. Secondly, the calculated results for the irregular Dralon fiber and GR-IN optical fiber are compared with that calculated using other conventional method. From this comparison, we recommend that the refraction must be taken into account to obtain accurate results especially for birefringent fibers and graded index optical fibers.

1. Introduction

The optical properties of any material can be characterized by measuring its refractive index profile in addition to its absorption index. In the following study, we confine our selves to overcome the problem of measuring the refractive index profile of fibers. Where, the majority of the physical properties of fibers can be related to their refractive index profiles. In other words, the values of refractive indices of the fibers give useful information about the structural parameters of these fibers. In addition, they provide information about the microscopic parameters such as oscillation energy, dispersion energy, and the molecular polarizability. To measure these physical properties of fibers we need to develop an easy and accurate model to determine its refractive index profile. More generally, this model must be able to detect the refractive index profile for irregular fibers without any previous knowledge about the fiber inner structure.

To develop this model we are dealing with two principle problems; the irregularity of the fibers transverse cross-sectional shapes and the unknown structure of the fiber. The term unknown, structure means that we do not know if the fiber is homogeneous, skin-core, multi-layer, or graded index fiber.

Many experimental techniques and theoretical models were developed in the past decades to fully characterize the different types of fibers. Instances for these techniques are X-ray structural analysis [1], infrared dichroism analysis [2], neutron scattering studies [3,4], etc. In this work, we used interferometric techniques. Transversal interferometry of fibers has been established in the last few decades as one of the most useful and accurate tools for refractive index profiling. Interferometry is considered as one of the best non-destructive techniques in the field of fiber research. Useful information, nearly at each point, about the fiber sample under study can be obtained using interferometric methods.

Authors have done many efforts to determine the refractive indices and birefringence of fibers with different types and shapes [5–14]. Hamza et al. [15,16] succeeded in developing an approximated zonal model to measure the refractive indices of skin-core and graded index fibers having circular cross-section shape. Tomographic imaging techniques [17–21] have been succeeded in measuring the refractive index profiles of non-azimuthally symmetric optical fibers. Hamza et al. [14,22,23] proposed a series of research to overcome the problem of irregularity of the fiber transverse cross-sections and they already succeeded in measuring the mean refractive indices of homogeneous [14] and skin-core [22] fibers but having irregular cross-sections. Hamza et al. [23] succeeded

---

* Corresponding author. Tel.: +20 010 7724365; fax: +20 057 3403868.
E-mail address: maen.fsh@gmail.com (M.A.E. Nawareg).

0030-4018/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2008.09.073

in developing a model for measuring the refractive index profile of irregular fibers without considering the refraction of light rays by the fiber under certain experimental conditions.

In this paper, we developed a new model with the aid of automatic fringe analysis to measure the refractive index profile of fibers having regular or irregular transverse cross-sectional shape with taking the refraction of light rays by the fiber into considerations. This represents an important development in this subject.

2. Theoretical considerations

Suppose we have a fiber with irregular shape as seen in Fig. 1. The aim of this work is to obtain the function, which describes the fiber refractive index profile taking into consideration the refraction of the incident light beam by the fiber.

If this fiber is immersed into a liquid in an interferometer, the resulted fringes will be deformed across the fiber. The basic two factors that determine the shape of the deformed fringes across the fiber are the geometrical shape of the fiber cross-section and the shape of the function, which represent the refractive index profile of this fiber. In the following, we will derive a model to measure the refractive index profile of the fiber using two parameters, the function describing the fringe shift and the cross-section shape of the fiber.

The equation gives the relation between the refractive index of the fiber and the fringe shift [24,25] is given by,

$$\int_L n\mathrm{d}S - n_0 L_0 = \frac{z\lambda}{2h} \tag{1}$$

where L represents the total ray path through the fiber, L_0 is the corresponding path if there is no fiber, n is the refractive index at position (x, y, z), $\mathrm{d}S = (\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)^{0.5}$, n_0 is the refractive index of the immersion liquid, z is the fringe shift, λ is the light wavelength, and h is the interfringe spacing.

To calculate the integration in Eq. (1) we divide the fiber into large number of irregular zones that have the same shape as the outer surface and each one has a constant refractive index, see Fig. 2. Therefore, if the light ray passes through m layers, we can write Eq. (1) as follows:

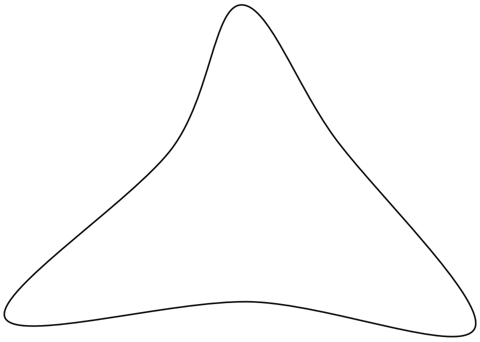

Fig. 1. A fiber with irregular cross-sectional shape.

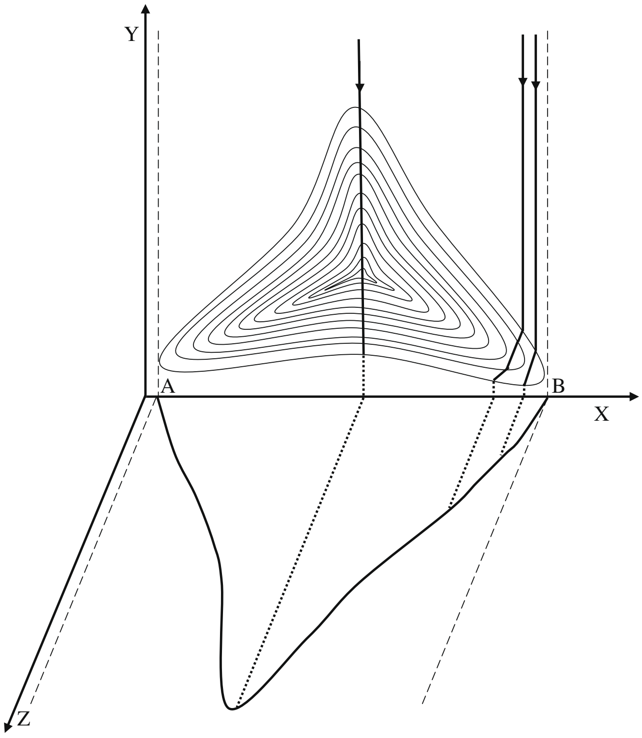

Fig. 2. Light rays passes through the different layers of the fiber and their corresponding fringe shift.

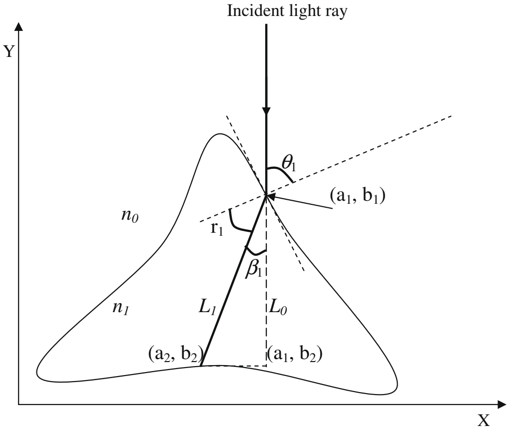

Fig. 3. A light ray passes through irregular fiber having refractive index n_1 immersed in a liquid of refractive index n_0.

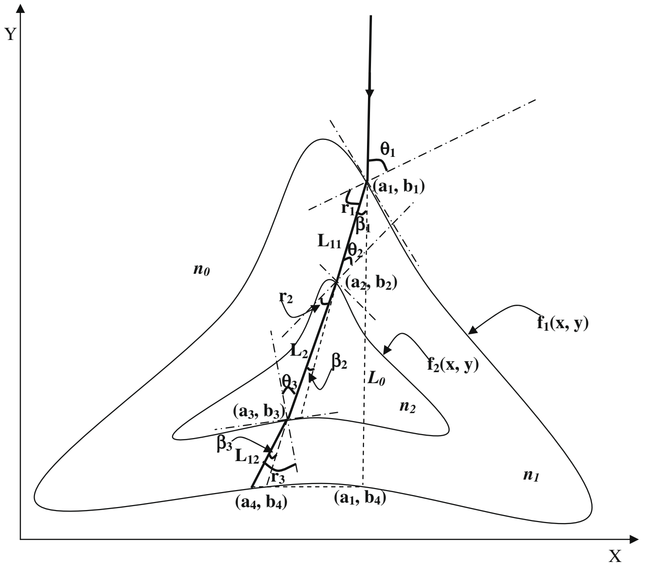

Fig. 4. A light ray passes through irregular fiber having two layers of refractive index n_1 and n_2, respectively, and immersed in a liquid of refractive index n_0.

$$\sum_{i=1}^{m} n_i L_i - n_0 L_0 = \frac{z\lambda}{2h} \tag{2}$$

where m is the number of the zones, n_i is the refractive index of the layer number i, and L_i is the ray path through the ith layer. Then we calculate the refractive index of each layer from the values of the corresponding fringe shift (see Fig. 2). In these calculations, we depend on deducing the ray equation through each layer separately.

Fig. 2 shows a vertical parallel beam of light incident on the fiber. Some rays of this beam will pass only through one layer of the fiber, some will pass through two layers of the fiber and others will pass through k layers. We will discuss each case as follows.

2.1. Light ray passes through one layer

In this case, Eq. (2) can be written as

$$n_1 = \frac{z\lambda}{2hL_1} + \frac{n_0 L_0}{L_1} \tag{3}$$

To calculate n_1, one needs to determine L_1 and L_0. This can be done if we know the ray equation outside and inside the fiber and the equation which describes the fiber cross-section shape $f_1(x,y) = 0$. The equation describes the incident light ray at certain position (see Fig. 3) is given by

$$x = a_1 \tag{4}$$

The equation that describes the ray path through the fiber is deduced and it is found to be

$$y = s_1 x + t_1 \tag{5}$$

where; $s_1 = \tan(\pi/2 - \gamma_1)$, $t_1 = b_1 - s_1 a_1$
and

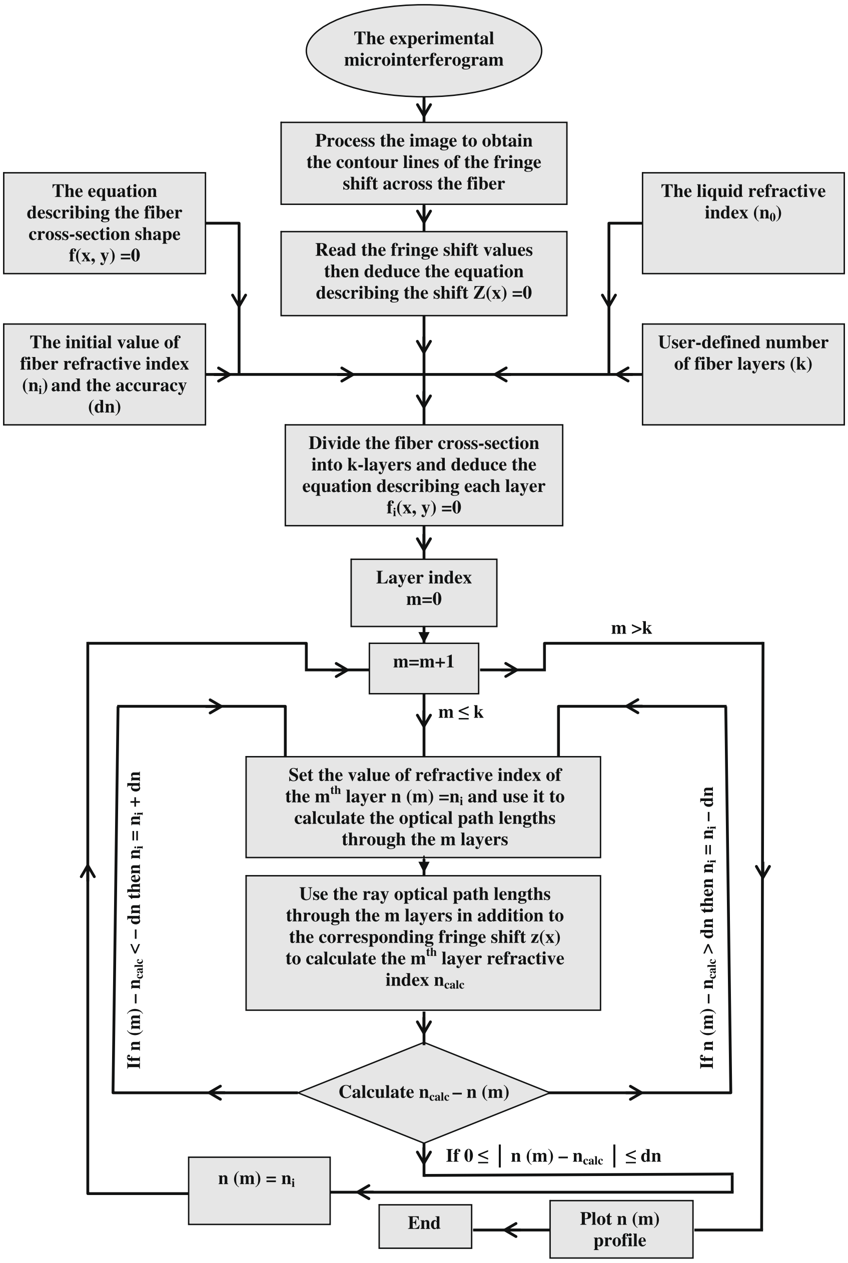

Fig. 5. The flowchart which illustrates the calculation of the refractive index profiles of fibers using the suggested model.

$$\gamma_1 = \beta_1 = \theta_1 - r_1 \tag{6}$$

$$r_1 = \sin^{-1}\left[\frac{n_0}{n_1}\sin(\theta_1)\right] \tag{7}$$

$$\theta_1 = \cos^{-1}\left[\frac{c_1}{\sqrt{c_1^2+1}}\right] \tag{8}$$

where

$$c_1 = -\frac{f_{1y}}{f_{1x}} \tag{9}$$

where f_{1y} and f_{1x} are the partial differentiations of $f_1(x,y)$ with respect to y and x, respectively, at the point (a_1,b_1) (see Fig. 3).

When solving, simultaneously, the ray Eq. (5) with the equation of the cross-section shape $f_1(x,y) = 0$ we can obtain the intersection points (a_1,b_1) and (a_2,b_2) (see Fig. 3). Therefore, we obtain L_0 and L_1 as follows:

$$L_0 = |b_2 - b_1| \tag{10}$$

$$L_1 = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2} \tag{11}$$

An iteration method is then used to predict the exact value of n_1.

2.2. Light ray passes through k layers

To deduce the general equations by which we can measure the refractive index of the kth layer we consider the case of two-layer fiber (see Fig. 4). Using Eq. (2) we can write the equation which describes the relation between the refractive indices and the fringe shift in the following form:

$$n_2 = \frac{z\lambda}{2hL_2} - \frac{1}{L_2}[n_1L_1 - n_0L_0] \tag{12}$$

The incident ray is parallel to y-axis and reaches the fiber at the point (a_1,b_1). The incidence and refraction angles r_1 and θ_1, respectively, are given by the same Eqs. (7) and (8). By the same method discussed in Section 1 we can obtain the ray equation after refraction at the point (a_1,b_1) which is given by Eq. (5) (see Fig. 4). If the general equation of the layer number k is given by

$$f_k(x,y) = 0 \tag{13}$$

The general equation of the ray through the fiber is proved to be expressed as

$$y = s_kx + t_k \quad ,k = 1,2,3,\ \ldots\ ,2m-1. \tag{14}$$

where

$$s_k = \tan(\pi/2 - \gamma_k) \tag{15}$$

$$t_k = b_k - sa_k \tag{16}$$

and

$$\gamma_k = \sum_{j=1}^{k}\beta_j \tag{17}$$

$$\beta_j = \theta_j - r_i \tag{18}$$

where

$$\theta_j = \cos^{-1}\left[\frac{s_{j-1}c_j + 1}{\sqrt{c_j^2+1}\sqrt{s_{j-1}^2+1}}\right] \tag{19}$$

$$c_j = -\frac{f_{jy}}{f_{jx}} \tag{20}$$

$$r_j = \sin^{-1}\left[\frac{n_{j-1}}{n_j}\sin(\theta_j)\right] \tag{21}$$

The general equation gives the refractive index of the kth layer is written as

$$n_k = \frac{z\lambda}{2hL_k} - \frac{1}{L_k}\left[\sum_{i=1}^{k-1} n_iL_i - n_0L_0\right] \tag{22}$$

where

$$L_k = \sqrt{(a_{k+1} - a_k)^2 + (b_{k+1} - b_k)^2} \tag{23}$$

$$L_i = \sqrt{(a_{i+1} - a_i)^2 + (b_{i+1} - b_i)^2} + \sqrt{(a_{2k-i+1} - a_{2k-i})^2 + (b_{2k-i+1} - b_{2k-i})^2} \tag{24}$$

where $i = 1,2,\ldots,k-1$.

From the ray and cross-sectional shape equations, we can obtain the values of the intersection points (a_j,b_j) between the light ray and the different layers of the fiber. Moreover, from these values we can calculate the path length L_i and L_k of the light ray at any position inside the fiber.

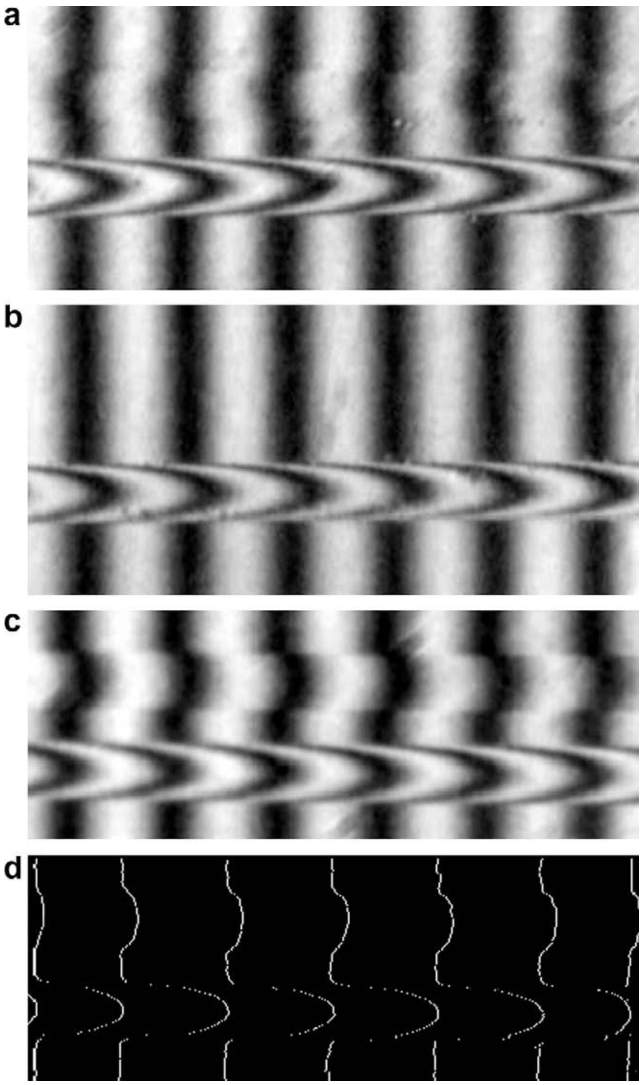

Fig. 6. (a) Microinterferogram of totally duplicated image of PPT fiber in which the fringe shift due to the parallel component of light is very close to match; (b) the parallel component is exactly in the matching case using immersion liquid of refractive index $n_0 = 1.5205$ and (c and d) Microinterferogram of totally duplicated image of PPT fiber and its contour lines using light with wavelength $\lambda = 546.1$ nm and immersion liquid of refractive index $n_0 = 1.5158$ at temperature 29.5 °C.

An iteration method is used to predict the exact value of n_k. This model is used for calculating the refractive index profile of fibers having irregular and/or regular cross-section shape.

3. Experimental results and discussion

Using the suggested model the refractive index profile is calculated for three fibers. Firstly, we apply the model for measuring the refractive index profile of a homogeneous PPT fiber and comparing the results with that measured using conventional matching method. Secondly, the model is applied to measure the refractive index profile for GR-IN multimode optical fiber, which has circular cross-section shape. Then the refractive index profile is calculated for Dralon fiber, which has irregular transverse cross-sectional shape. To confirm the importance of taking the refraction of light rays by the fiber into consideration, we compare the refractive index profile calculated using this model, with that calculated using other model [23] which neglects the refraction of light rays through the fibers. Fig. 5 shows the flowchart for calculating the refractive index profile of fibers using the suggested model.

3.1. PPT (polypropylene terephthalate) fiber (regular fiber)

This sample is a homogeneous, i.e. it has a constant refractive index, circular fiber with radius 10.6 ± 0.109 μm. The Pluta polarizing interference microscope [12,13] is used to determine the refractive index profile for this fiber. The fiber is fixed at two ends on a microscope slide, and a drop of suitable liquid is placed on it. It is used as a standard fiber to validate the suggested model. We use the matching method for the determination of the refractive index for both components of light, which vibrates parallel and perpendicular to the fiber axis. In this method we use an immersion liquid with refractive index exactly equals to that of the fiber, which means that the fringe shift is equal to zero. Fig. 6a shows the microinterferogram of totally duplicated image of PPT fiber in which the parallel component is very close to match. In Fig. 6b the parallel component is exactly in the matching case using

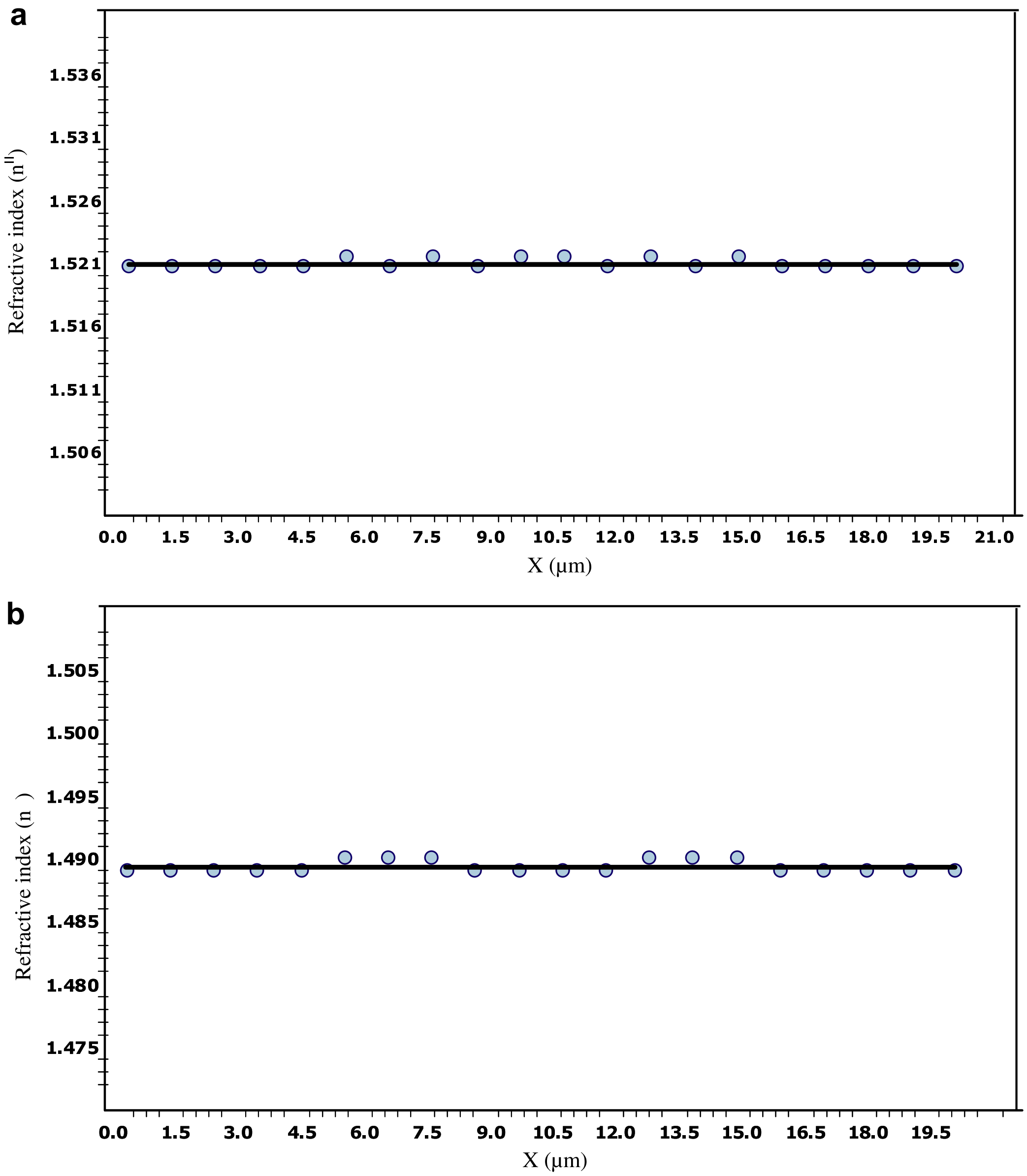

Fig. 7. (a and b) the refractive index profiles of PPT fiber for light vibrating parallel and perpendicular to the fiber axis, respectively, calculated using the suggested model.

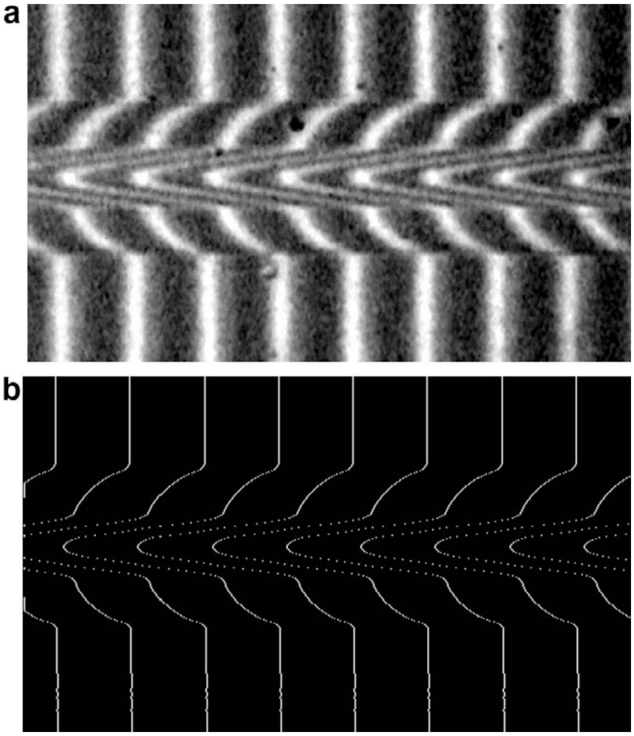

Fig. 8. (a and b) Microinterferogram of Fizeau fringes crossing LDF fiber and its contour lines, respectively, light with wavelength $\lambda = 535.1$ nm and immersion liquid of refractive index $n_0 = 1.4613$ at temperature 25.2 °C.

immersion liquid of refractive index $n_0 = 1.5205$. The liquid which makes the perpendicular component in the matching case is found to be $n_0 = 1.4893$. The immersion liquids refractive indices are measured using Abbe's refractometer with an error $\pm 5 \times 10^{-4}$. Due to the uncertainty in obtaining the matching case, it has been found that the accuracy in measuring the refractive indices cannot be better than 1×10^{-3}. Fig. 6c and d shows the microinterferogram of totally duplicated image of PPT fiber in the mismatching case for both components, parallel and perpendicular, and its contour lines using light with wavelength $\lambda = 546.1$ nm and immersion liquid of refractive index $n_0 = 1.5158$ at temperature 29.5 °C. This mismatching microinterferogram is used to measure the refractive index profiles of the PPT fiber for both components of light using the suggested model. Where the equation describes the fiber cross-section in addition to that describes the fringe shift is fed to the prepared program (Fig. 5) which calculates the refractive index profile of the fiber. The calculated profiles for both parallel and perpendicular components of light are given in Fig. 7a and b, respectively. The error sources in the calculations using our model are the error in calculating the fiber radius, noise in the fringe shift, error due to the interferometer setup, measuring the refractive index of the immersion liquid, and error due to the theoretical model itself. The accuracy [12] in measuring the optical path difference, with a fringe field (normal Wollaston prism), is about 0.05λ, and there for the error in the assessment of the values of refractive indices using Pluta microscope cannot be better than 0.003–0.001 [12]. Using automatic fringe analysis in addition to averaging the fringe shift values over a number of fringes reduces the error in measuring the fringe shift. This enables us to reach the maximum possible accuracy of the used interferometer. Where, the calculated error in measuring the fringe shift due to the noise is about ±1.38%. The measured refractive indices for both component of light, vibrates parallel and perpendicular to the fiber axis, using the suggested model are found to be 1.520 and 1.489, respectively, with an error $\pm 1.015 \times 10^{-3}$. These results compared with that calculated using the matching method strongly support the suggested model.

3.2. LDF optical fiber (regular fiber)

This sample is a GR-IN optical fiber with circular cross-sectional shape. It has a diameter 125 ± 1.2 μm so it cannot be tested using Pluta polarizing interference microscope due to this large diameter. Where this large diameter will causes an overlapping of the resulted fringe pattern. GR-IN optical fibers have relatively large difference between the cladding and core refractive indices. This means that the refraction of light rays by the fiber will have significant values. We choose such a fiber to confirm the importance of considering the refraction of light beam by the fiber. The multiple-beam Fizeau interferometer in transmission attached to a CCD camera, which is connected to a PC [26], is used to determine the refractive index profile for this fiber. In this interferometer, the fiber under study is immersed in a liquid of suitable refractive index in a silvered wedge. We used plane-polarized light of wavelength $\lambda = 535.1$ nm at temperature 25.2 °C. Fig. 8a and b show the microinterferogram of Fizeau fringes crossing the multimode optical fiber and its contour lines using an immersion liquid of refractive index $n_0 = 1.4613$. Fig. 9 shows a comparison between the refractive index profiles calculated using the suggested model and that model [23] which does not take the refraction of light rays by the fiber into consideration. It is clear that there is a considerable difference between the value of the refractive index calculated with and without considering the refraction. The uncertainty in measuring the fringe shift using Fizeau interferometer is equal to ±0.97%. The accuracy of the measured refractive index profile using the suggested model is about $\pm 3.7 \times 10^{-4}$ while the accuracy due to using the model [23] which neglects the refraction is $\pm 5.47 \times 10^{-4}$.

3.3. Dralon fiber (irregular fiber)

The Pluta polarizing interference microscope is used to determine the refractive index profile for this fiber. The fiber is fixed at two ends on a microscope slide (with the ability for rotation around its axis), and a drop of suitable liquid is placed on it. Fig. 10a and b show the microinterferogram of totally duplicated image of Dralon fiber and its contour lines using an immersion liquid of refractive index $n_0 = 1.5189$ at temperature 29.5 °C. In case of irregular fibers, the optical path difference at a given point of the object cannot readily be determined. The purpose of the suggested model is to overcome this difficulty. Firstly, we deduced the equation that describes the cross-sectional shape of the fiber. The following steps can do this:

(1) A cross-section image for a bundle of the fiber is captured using an optical microscope (see Fig. 11a).
(2) Taking into consideration the captured cross-section image we rotate the fiber about its axis into the interferometer to have, in our case, the maximum possible width AB (see Fig. 2). Where AB is measured using a ruler program. Then we capture the interferogram using a CCD camera.

Notice:

The fiber width AB measured during the rotation can be chosen to have maximum, minimum or any characterizing width. Its importance is to define the orientation of the fiber only.

(3) We automatically analyze the cross-section image to obtain the shape with its real dimensions (see Fig. 11b).

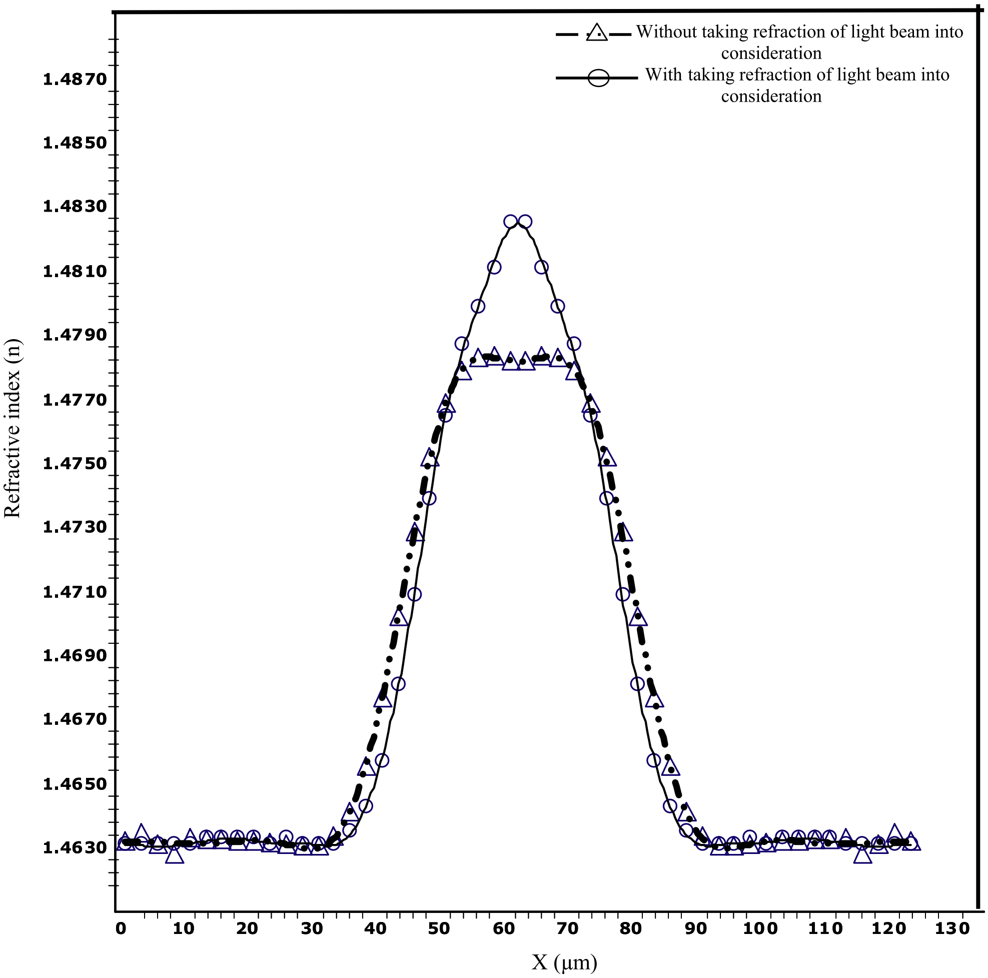

Fig. 9. Refractive index profile for LDF fiber considering the refraction (using the suggested model) and when neglecting it.

(4) Using a suitable fitting method, we obtain the equation describing the fiber cross-section. For some irregularly shaped fibers, one equation may not be enough to describe the fiber cross-section so we can obtain more than one equation. In these cases, the program is ready to receive one or more equations for the fiber cross-section. The deduced equation for Dralon fiber cross-section is given as

$$Y^2 = \left(a_1 - a_2^* x + a_3^* x^2 + a_4^* x^3 + a_5^* x^4 - a_6^* x^5 - a_7^* x^6 + a_8^* x^7\right)^2 \qquad (25)$$

where $2 < x < 42$ and $-7.736 < Y < 7.74$ and a_1, a_2, a_3, a_4, a_5, a_6 and a_7 are constants.

Eq. (25) in addition to the equation describes the fringe shift across the fiber are fed to the prepared program which calculates the refractive index profile of the Dralon fiber taking the refraction of light rays by the fiber into consideration. The error in measuring the fiber cross-section dimensions using Eq. (25) is ±0.145 μm. The accuracy of the calculated refractive index profile is about $\pm 1.24 \times 10^{-3}$. Fig. 12 shows a comparison between the refractive index profiles calculated using the suggested model and that model [23] which does not take the refraction of light rays by the fiber into account for the parallel component of light. It is clear that there is a difference between the results calculated using the two models. To use the model [23] certain conditions are needed to obtain accurate results. In some cases these conditions can't be achieved as we see in case of the GR-IN optical fiber so that there were large errors in the results calculated using this model. However, in case of Dralon fiber these conditions are fulfilled so that more accurate results are obtained. Where the values of refractive index are nearly the same except at the fiber ends where the refraction becomes valuable due to the cross-section geometry. The uncertainty in measuring the refractive index profile using that model [23] which doesn't consider the refraction is $\pm 1 \times 10^{-3}$. Because another technique is used to measure the cross-sectional shape, it is clear that the uncertainty in measuring the refractive index profile using the suggested model is slightly greater than that when using the other model. Therefore, we can say that, although the suggested model can be used for measuring the refractive index profile for irregular fiber with considering the refraction of light rays by the fiber its importance appears only when the refraction has a significant value.

4. Conclusion

This work offers a model based on interferometry for measuring the refractive index profile of fibers having regular and/or irregular cross-sectional shape with considering the refraction of the light rays by the fiber. We applied the suggested method for three different fibers of different types and shapes. The calculated results using the suggested model were compared with that calculated using conventional methods. This comparison validates the model and confirms that its accuracy is limited by that of the used experimental techniques. The refractive index profile of an irregular fiber (Dralon fiber) is calculated using the suggested model with accu-

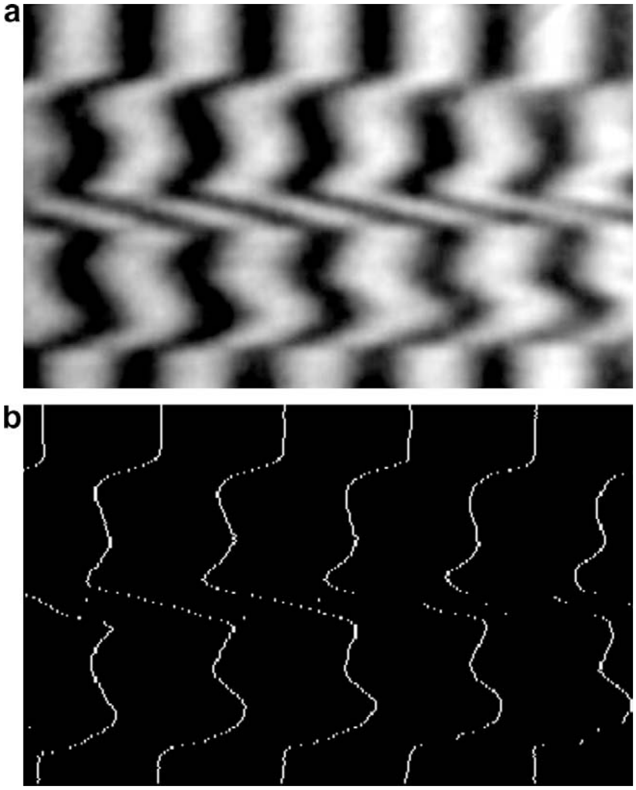

Fig. 10. (a and b) Microinterferogram of totally duplicated image of Dralon fiber and its contour lines using light with wavelength $\lambda = 546.1$ nm and immersion liquid of refractive index $n_0 = 1.5189$ at temperature 29.5 °C.

racy $\pm 1.24 \times 10^{-3}$ using two-beam interference Pluta microscope. Whereas, conventional methods could only calculate the mean refractive index profile of the irregularly shaped fibers. Comparison between the calculated results using the suggested model and that when neglecting the refraction of light beam by the fiber, using previously published model, is presented for Dralon and GR-IN optical fiber. The refractive index profile for the GR-IN optical fiber using the suggested model has an accuracy $\pm 3.7 \times 10^{-4}$ using mul-

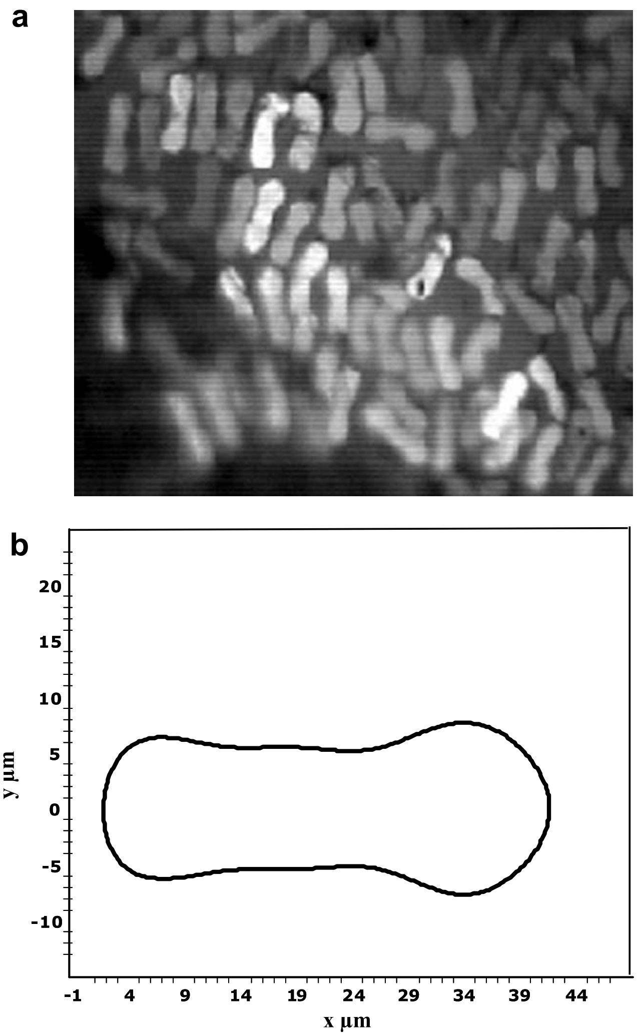

Fig. 11. (a) the shape of the cross section of Dralon fibers using the optical microscope and (b) the Dralon cross-section shape obtained from image processing of (a).

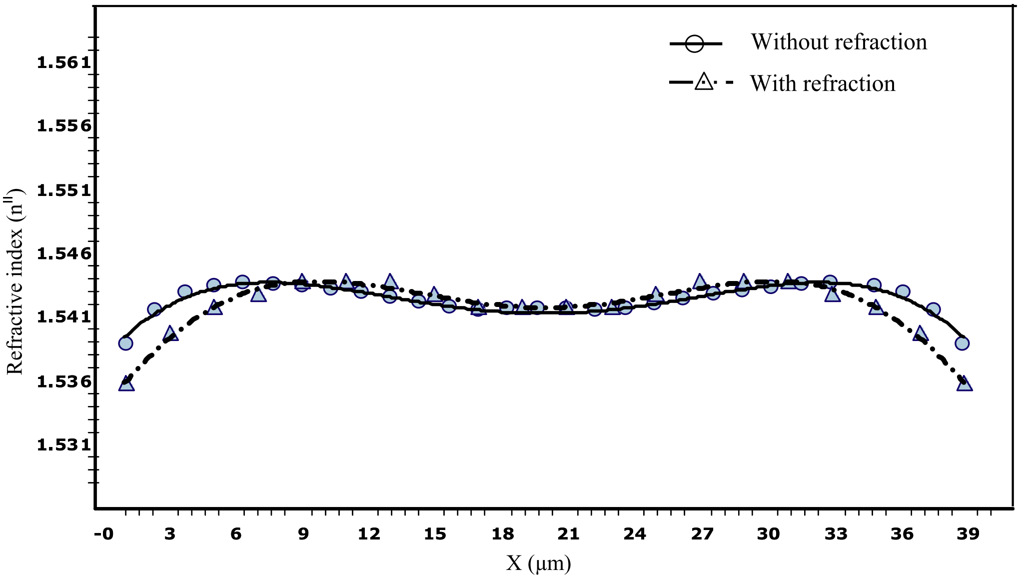

Fig. 12. Refractive index profiles for Dralon fiber with considering the refraction (using the suggested model) and when neglecting it.

tiple-beam interferometer. From this comparison, we conclude that when the refraction has a significant effect such a model is recommended to be used to obtain results that are more accurate. The most important advantage is that the developed method can be used for refractive index profiling of irregular fibers with taking into consideration the refraction of light rays by the fiber.

References

[1] D.C. Oda, G.C. Rotledge, Polymer 40 (1999) 4635.
[2] M. Yazdanian, I.M. Ward, Polymer 26 (1985) 1779.
[3] D.M. Sadler, P.J. Barham, Polymer 31 (1990) 43.
[4] D.M. Sadler, P.J. Barham, Polymer 31 (1990) 46.
[5] R.C. Faust, Physical methods of investigating textiles, in: R. Meredith, J.W.S. Hearle (Eds.), Textile Book Publishers, Inc., New York, 1959, p. 320.
[6] S.C. Simmens, Nature 181 (1958) 1260.
[7] N. Barakat, A.M. Hindeleh, Text. Res. J. 34 (1964) 357.
[8] N. Barakat, A.M. Hindeleh, Text. Res. J. 34 (1964) 581.
[9] A. Thetford, S.C. Simmens, J. Microsc. 89 (1969) 143.
[10] S.C. Simmens, J. Microsc. 89 (1969) 291.
[11] N. Barakat, Text. Res. J. 41 (1971) 167.
[12] M. Pluta, Opt. Acta 18 (1971) 661.
[13] M. Pluta, J. Microsc. 96 (1972) 309.
[14] A.A. Hamza, Text. Res. J. 50 (1980) 731.
[15] A.A. Hamza, A.M. Ghander, T.Z.N. Sokkar, M.A. Mabrouk, W.A. Ramadan, Pure Appl. Opt. 3 (1994) 943.
[16] A.A. Hamza, T.Z.N. Sokkar, A.M. Ghander, M.A. Mabrouk, W.A. Ramadan, Pure Appl. Opt. 4 (1995) 161.
[17] P.L. Chu, T. Whitbread, Appl. Opt. 18 (7) (1979) 1117.
[18] C.M. Vest, Appl. Opt. 24 (23) (1985) 4089.
[19] B.L. Bachim, T.K. Gaylord, S.C. Mettler, Opt. Lett. 30 (10) (2002) 1126.
[20] B.L. Bachim, T.K. Gaylord, Appl. Opt. 44 (3) (2005) 316.
[21] W. Gorski, W. Osten, Opt. Lett. 32 (14) (2007) 1977.
[22] A.A. Hamza, T.Z.N. Sokkar, M.A. Kabeel, J. Phys. D: Appl. Phys. 18 (1985) 2321.
[23] A.A. Hamza, T.Z.N. Sokkar, M.A. El-Morsy, M.A.E. Nawareg, Opt. Laser Technol. 40 (2008) 1082.
[24] E. Fazio, W.A. Ramadan, M. Bertolotti, G.C. Righini, Opt. Lett. 21 (16) (1996).
[25] A.W. Synder, J.D. Love, Optical Waveguide Theory, Chapman and Hall, London, 1995. p. 668.
[26] M.A. El-Morsy, T. Yatagai, A.A. Hamza, M.A. Mabrouk, T.Z.N. Sokkar, Opt. Laser. Eng. 38 (2002) 509.