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Automatic determination of refractive index profile, sectional area, and shape of fibers having regular and/or irregular transverse sections

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Abstract

A new method based on a mathematical model and a computer program is suggested to determine the refractive index profile of fibers having regular and/or irregular transverse sectional shape. Microinterferogram of both multiple-beam Fizeau fringes and the duplicated image from two-beam interference microscope are used for the determination of refractive index profile, cross-sectional area and shape of three different types of fibers. To confirm the suggested model, the calculated area and the shape of the transverse section of these fibers are compared with those results obtained using conventional methods.

Keywords: Irregular fiber; Refractive index profile; Interferometry

1. Introduction

It is widely recognized that the main characteristics of physical properties of fibers can be related to their refractive index profiles. In other words, the values of refractive indices of the fibers give useful information about the structural parameters of these fibers. Also, they provide information about the microscopic parameters such as oscillation energy, dispersion energy, and the molecular polarizability. So it is important to establish not only easy and simple but also efficient and accurate methods and tools for measuring the refractive index profiles. The interferometric methods represent easy and precise tools for investigating the fibers. Different interferometric methods have been applied successfully to measure the refractive indices of natural and synthetic fibers [1–13].

It is well known that there are many natural and synthetic fibers having different cross-sectional shapes and areas due to their nature and fabrication. These geometrical parameters are important and necessary for the determination of the refractive indices of fibers.

Many authors described different interferometric techniques to overcome the difficulty of irregular transverse sections of fibers [1–4,14,15]. Simmens [1] described a technique using the Babinet compensator to determine the optical properties of fibers of constant weight per unit length but irregular sectional shape. Hamza [2] introduced a method to measure the mean refractive indices and birefringence of fibers with irregular transverse sections by a complementary technique using double beam interference and scanning electron microscopy. Hamza et al. [3,4] applied multiple-beam Fizeau fringes to obtain mean refractive indices and birefringence of fibers with irregular transverse sections. Hamza et al. [14] applied variable wavelength microinterferometry to determine the dispersion curves of irregular fibers. Hamza et al. [15] suggested a method to determine the regular and/or irregular transverse sectional shape and area of homogeneous fibers. In this method, the transverse sectional shapes of the fibers are determined by measuring the thickness profiles and varying the angle of rotation for these fibers.

In this paper, we developed an accurate and simple method with the aid of automatic fringe analysis not only

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to measure the mean refractive indices but also to obtain the refractive index profile for natural, synthetic, and optical fibers with regular and/or irregular transverse sections. Also in this work, we are able to calculate the transverse sectional area of the fiber with any cross-sectional shape.

2. Theoretical considerations

A mathematical model will be derived to determine the refractive index profile of fibers in case of two- and multiple-beam interferometric methods. In these methods, the fiber under study is immersed in a suitable liquid with refractive index different from that of the fiber (Fig. 1(a)). Due to this difference the interference fringes will be deformed across the fiber as shown in Fig. 1(b). The shape of the deformed fringes depends on the refractive index and the thickness of the fiber. In case of multiple-beam interference fringes, the equation that governs the shape of the deformed fringes [16] can be written as follows:

$$2\Delta S = \Delta m\lambda \tag{1}$$

where ΔS is the change in the path difference due to the passing of light through the fiber and Δm is the change in the fringe order which is given by

$$\Delta m = \frac{z}{h} \tag{2}$$

where z is the fringe shift across the fiber and h the interfringe spacing.

Therefore,

$$\Delta S = \frac{z\lambda}{2h} \tag{3}$$

where ΔS is a function of the refractive index of the fiber and the optical path length through it. To obtain the refractive index profile according to this equation we must know the fringe shift across the fiber and this optical path length. The accuracy of measuring these two parameters determines the accuracy of the produced refractive index profile.

2.1. Measuring the fringe shift (Z)

In order to obtain precise values for the fringe shift z, the experimental microinterferogram for fringes along the fiber are processed using a prepared computer program. Using this program we obtain precise contour lines of the fringes, from which we can automatically measure the shift z with high accuracy.

2.2. Determination of the total path length of light through the fiber

When the fiber is immersed in two liquids of different refractive indices, two different values of the fringe shift are obtained. Suppose the shift z_a is obtained using the first liquid and z_b when using the second liquid. The two equations that describe the fringe shift in the two cases using multiple-beam interferometer [6] are

$$nl_a - n_0 l_0 = \frac{z_a\lambda}{2h_a} \tag{4}$$

where n is the fiber mean refractive index, l_a the optical path length into the fiber at position x, l_0 the optical path length at the same position x if the fiber is not existing, n_0 the liquid refractive index, z_a the fringe shift due to the fiber at position x (see Fig. 2a), λ the light wavelength, and h_a the interfringe spacing, and

$$nl_b - n_0^* l_0 = \frac{z_b\lambda}{2h_b} \tag{5}$$

where n_0^* is the refractive index of the second liquid, l_b the optical path length into the fiber at the same position x, z_b the fringe shift using the second liquid at position x (see Fig. 2b), and h_b the interfringe spacing.

Eqs. (4) and (5) contain four unknowns' l_a, l_b, l_0, and n, and so we cannot solve them by algebraic methods. To solve these equations we interpret some experimental assumptions. These assumptions (in case of two- and multiple-beam interference fringes) are based on the experimental fact that the small change in the immersion liquid refractive index causes a slight change in the optical path length through the fiber, while it causes a large change

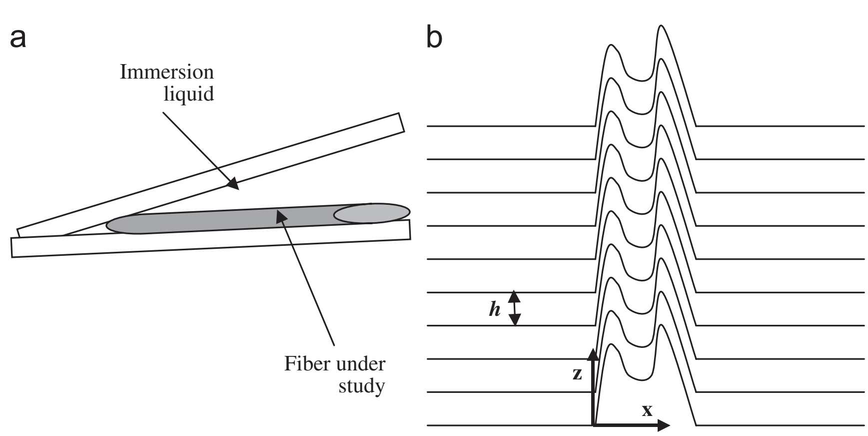

Fig. 1. (a) Liquid wedge interferometer containing a fiber with irregular transverse section. (b) The shape of the deformed Fizeau fringes across the fiber.

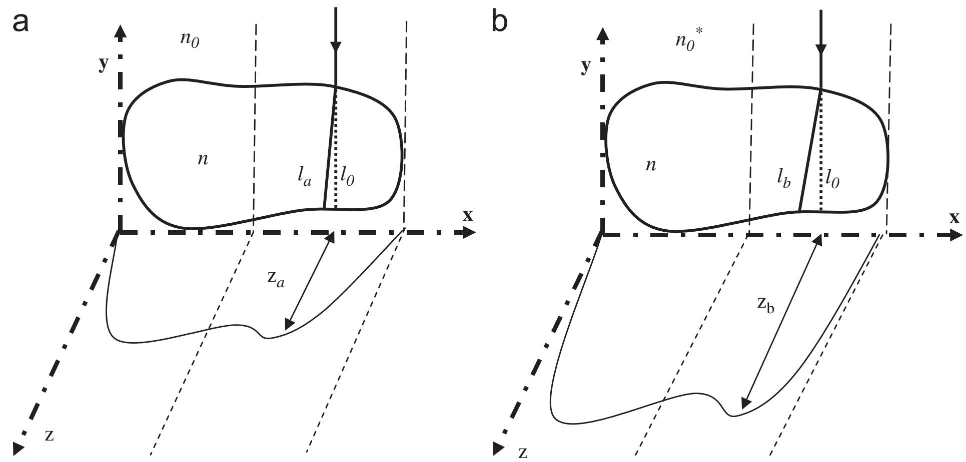

Fig. 2. The light ray path through the fiber when it is immersed in a liquid of refractive index: (a) n_0 and (b) n_0^*.

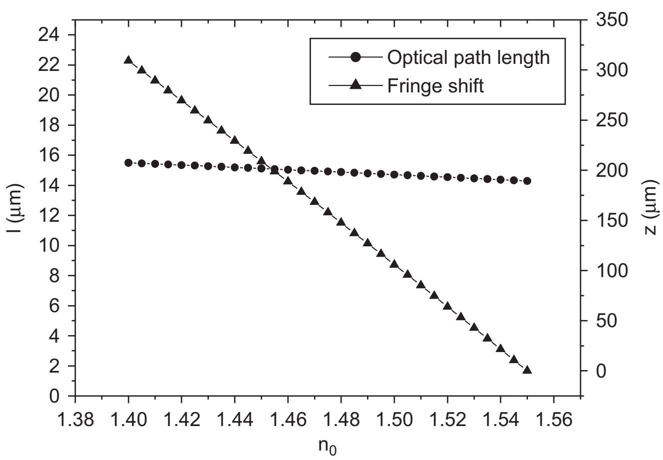

Fig. 3. Comparison between the values of the optical path length and the fringe shift when changing the immersion liquid refractive index.

in the fringe shift (see Fig. 2(a) and (b)). Fig. 3 shows the comparison of the changes in the values of optical path length at certain position and its corresponding fringe shift for imaginary circular fiber of refractive index 1.55 and radius 10 μm immersed in liquid of refractive index n_0 (1.4–1.55).

The proposed assumptions are as follows:

(i) The refractive index of the first liquid should be close to that of the second liquid:

$$n_{\text{liquid}}(1) \cong n_{\text{liquid}}(2) \tag{6}$$

(ii) The refractive index of the used liquids should be chosen near to that of the fiber itself:

$$n_{\text{liquid}} - n_{\text{fiber}} \cong \text{zero} \tag{7}$$

The first assumption means that the optical path length in the fiber at certain position is nearly the same when the fiber is immersed into these two liquids; so we can write

$$l_{\text{a}} = l_{\text{b}} \tag{8}$$

The second assumption means that the ray path into the fiber is nearly equal to the vertical distance into the fiber (l_0) at position x, so we have

$$l_{\text{a}} = l_{\text{b}} = l_0 = l \tag{9}$$

From Eqs. (4), (5), and (9), we can derive the following equation:

$$l = \frac{1}{(n_0 - n_0^*)}\left(\frac{z_{\text{b}}\lambda}{2h_{\text{b}}} - \frac{z_{\text{a}}\lambda}{2h_{\text{a}}}\right) \tag{10}$$

Similarly in case of two-beam interference fringes the following equation can be derived:

$$l = \frac{1}{(n_0 - n_0^*)}\left(\frac{z_{\text{b}}\lambda}{h_{\text{b}}} - \frac{z_{\text{a}}\lambda}{h_{\text{a}}}\right) \tag{11}$$

Eqs. (10) and (11) give the value of the total path length of light at position x in case of multiple- and two-beam interference fringes, respectively. They are valid for homogeneous, skin-core and multi-layer fiber under the assumptions given by Eqs. (6) and (7).

2.3. *Refractive index profile for fiber having irregular transverse sections*

In this section, a mathematical formula will be derived to determine the refractive index profiles of homogeneous and multi-layer fibers having irregular transverse cross-section shapes:

(i) Refractive index profile for homogeneous fiber having irregular cross-section.

Suppose we have a fiber with irregular cross-section and its refractive index is n_1. Under the assumption given by Eq. (7), the refractive index profile is described by the following equation:

$$n_1 = \frac{z_1\lambda}{2hl} + n_0 \quad (12)$$

The fringe shift z_1 will be measured accurately using the automatic method. This method is based on obtaining the contour lines for the fringe shift. Using Eq. (10), we obtain the values of l and then Eq. (12) is used to determine the value of the refractive index n_1 of the fiber at any position x along the fiber axis.

(ii) Refractive index profile for multi-layer fiber having irregular cross-section.

Firstly, we consider a fiber having skin-core structure. The equation that describes the shift in the skin is the same as Eq. (12). So if we know the fringe shift due to the light path through the skin we can easily determine the refractive index of the skin n_1. But when the light passes through the skin and core of the fiber (see Fig. 4) the equation that describes the shift is given by

$$n_1 l_1 + n_2 l_2 - n_0(l_1 + l_2) = \frac{z_2\lambda}{2h} \quad (13)$$

This equation contains three unknowns l_1, l_2, and n_2. So we cannot solve this problem by algebraic methods. This problem becomes more difficult if the number of layers is more than two.

To overcome the above difficulty, we will interpret a solution based on experimental and numerical methods. As discussed previously we can measure the total path length l_T into the fiber at a certain position with a good accuracy using Eq. (10). So we can write

$$n_1 l_1 + n_2(l_T - l_1) - n_0 l_T = \frac{z_2\lambda}{2h} \quad (14)$$

where $l_T = l_1 + l_2$.

To obtain n_2 we must know l_1 or l_2 which represents a very difficult task since we have irregular fiber with unknown dimensions. To solve this problem, we divide the fiber into a very large number of imaginary layers which satisfy the condition that all layers have the same thickness in the vertical direction at a certain position x as seen in Fig. 5. Each of these layers has constant refractive index. Then the refractive index of the first imaginary layer will be given by the same Eq. (12), where z_1 is the fringe shift due to that light which passes only through this layer. In case of the light passing through two layers the refractive index of the second layer is given by the following equation:

$$n_2 = \frac{z_2\lambda}{2hl} + 2n_0 - n_1 \quad (15)$$

where z_2 is the fringe shift due to that light which passes only through these two layers, and

$$l_1 = l_2 = l = \frac{l_T}{2} \quad (16)$$

Eq. (16) is valid since the imaginary layers have the same thickness in the vertical direction at position x, and the light path length is nearly equal to the vertical distance into the fiber.

The relation that gives the refractive index of the third layer when the light passes through three layers is given by

$$n_3 = \frac{z_3\lambda}{2hl} + 3n_0 - n_2 - n_1 \quad (17)$$

where

$$l = \frac{l_T}{3} \quad (18)$$

Eqs. (12), (15), and (17) can be generalized to give the refractive index for the layer number m as follows:

$$n_m = \frac{z_m\lambda}{2hl} + \sum_{i=1}^{m} n_0 - \sum_{j=1}^{m-1} n_j \quad (19)$$

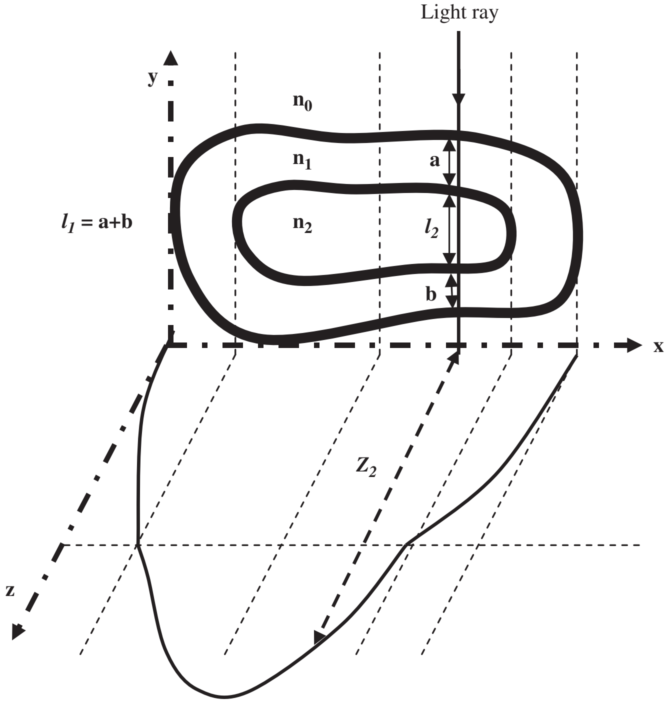

Fig. 4. Light ray passing through the skin-core fiber.

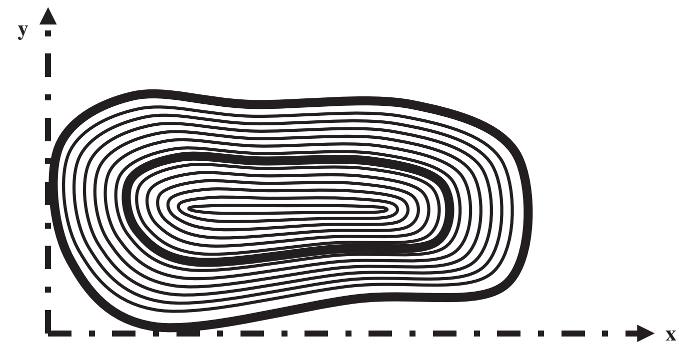

Fig. 5. Skin-core fiber divided into a very large number of imaginary layers.

where

$$l = \frac{l_T}{m} \tag{20}$$

and z_m is the fringe shift due to the light path through the m layers.

A prepared computer program based on Eqs. (10), (19), and (20) is used to calculate the refractive index profile of the multi-layer fiber, where the program read the values of the fringe shift as a function of the position. Then these values are used to calculate the refractive index of each layer of the fiber.

In case of two-beam interference fringes, Eq. (19) can be given in the following form:

$$n_m = \frac{z_m \lambda}{hl} + \sum_{i=1}^{m} n_0 - \sum_{j=1}^{m-1} n_j \tag{21}$$

2.4. Determining the area and drawing the shape of the fiber cross-section

The methods used for determining the cross-sectional area of fibers with an irregular cross-section [17,18] are difficult and yield only a statistical estimate of the area. Also, the method [15] is applicable only for homogeneous fibers. We suggest a method to measure the area of the irregular cross-section of the fiber. This method bases on interferometry and gives high accuracy. In this method, we calculate the summation

$$A = \sum_{i=1}^{m-1} x_i \left(\frac{l_i + l_{i+1}}{2} \right) \tag{22}$$

This summation gives the value of the fiber cross-sectional area A, where l_i is the length of line representing the vertical path length into the fiber at position i given by Eq. (10), x_i the distance between two successive lines (see Fig. 6) and m the number of these lines.

With the help of a computer program we can draw the cross-sectional shape of the fiber using the calculated values of the vertical path l. This method is applicable only for drawing the shapes of fibers with cross-sectional shapes having at least one axis of symmetry.

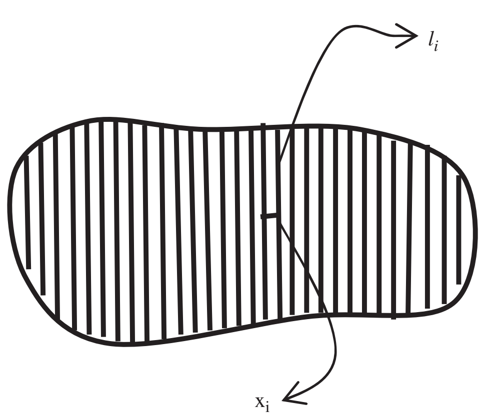

Fig. 6. The lines representing the light paths through the fiber.

3. Experimental results and discussion

Using the suggested model, the refractive index profile, cross-sectional area, and the shape are calculated for three fibers of different types and shapes. The first fiber is the Cortelle fiber, which represents a model of homogeneous and regular fiber. The second one is a single-mode optical fiber. This fiber represents a skin-core regular fiber. The third fiber is the Dralon fiber, which represents a model of multi-layer irregular fiber.

3.1. Cortelle fiber (with regular cross-section)

The Pluta polarizing interference microscope [19,20] attached to a CCD camera, which are connected to a PC, is used to determine the refractive index profile for this fiber. About 3 cm of the fiber is fixed at two ends on a microscope slide, and a drop of a suitable liquid is placed on it. Fig. 7(a) and (b) shows the microinterferogram of the totally duplicated image of the Cortelle fiber and its contour lines using immersion liquid of refractive index $n_0 = 1.5213$. Also Fig. 7(c) and (d) shows the microinterferogram and its contour lines using immersion liquid of refractive index $n_0 = 1.5158$. These microinterferograms are obtained at temperature 29.5 °C using monochromatic light of wavelength 546.1 nm. Using Eqs. (11) and (12), the refractive index profile of the Cortelle fiber in case of light vibrating parallel and perpendicular to the fiber axis are calculated. Fig. 8(a) and (b) shows these profiles, respectively. It is clear that the fiber has constant refractive index along its diameter. The measured refractive index of the parallel component is $n^{\parallel} = 1.514 \pm 1 \times 10^{-3}$ and that for the perpendicular component is $n^{\perp} = 1.517 \pm 1 \times 10^{-3}$. Using Eq. (11) and the prepared computer program, the shape of the cross-section is drawn. Fig. 9(a) shows the shape of the cross-section of the Cortelle fiber, which is nearly circular. Using Eqs. (11) and (22) with the aid of a computer program, the area of the Cortelle fiber is calculated. The area of the Cortelle fiber is then calculated using an optical microscopy method, where the cross-section of a bundle of the Cortelle fibers is enlarged using the optical microscope (see Fig. 9(b)). Using this figure, the average area of 25 cross-sections is calculated. The calculated values for the area and the radius of the Cortelle fiber using the two methods are given in Table 1. It is clear that there is a slight difference between values obtained from the two methods. So the conventional optical microscopy method confirms our suggested method.

3.2. Single-mode optical fiber (skin-core regular fiber)

It is well known that the single-mode optical fibers have small cores with radius $2 < r < 5\,\mu m$. So it represents a difficult task to detect the profile of this small core

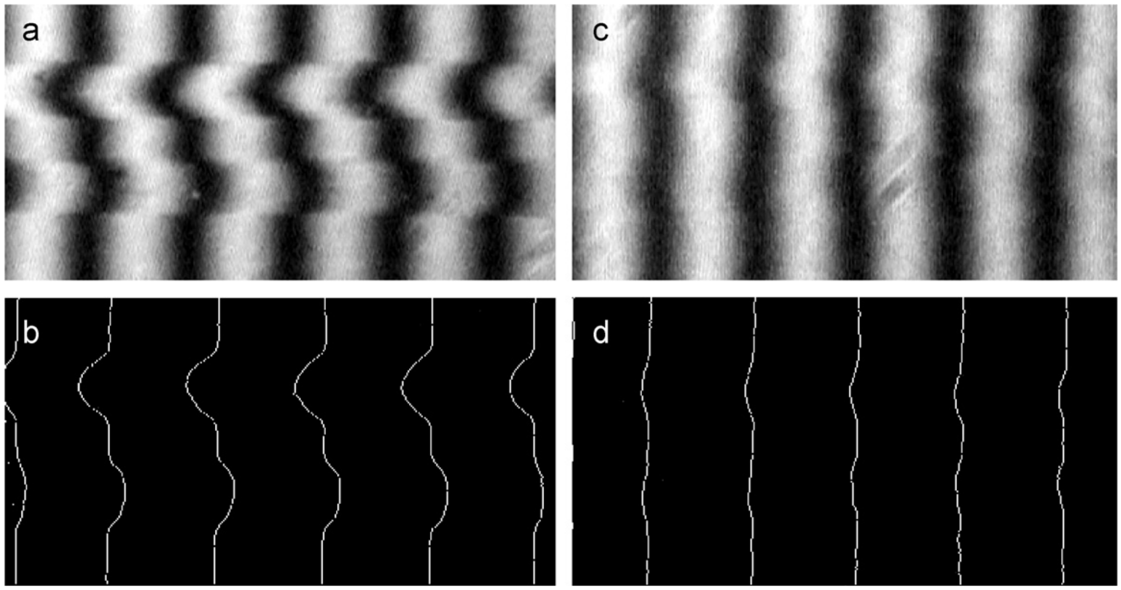

Fig. 7. Microinterferograms of two-beam interference fringes crossing Cortelle fiber using Pluta microscope and its contour lines using two immersion liquids: (a, b) using liquid of refractive index $n_0 = 1.5213$; (c, d) using liquid of refractive index $n_0 = 1.5158$.

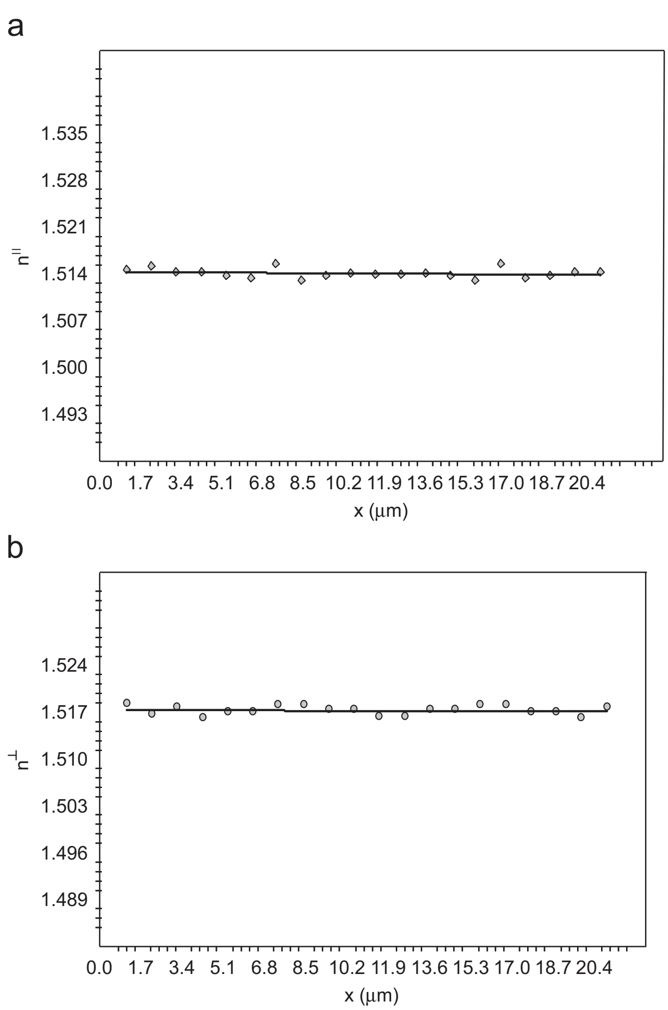

Fig. 8. (a) Refractive index profile of Cortelle fiber for light vibrating parallel to the fiber axis ($n^{\parallel}$). (b) The refractive index profile of Cortelle fiber for light vibrating perpendicular to the fiber axis ($n^{\perp}$).

compared with the large cladding. Our suggested method succeeds in measuring the refractive index profile of this small core. The automated multiple-beam Fizeau interferometer in transmission [11] is used. In this interferometer 2 cm of the fiber under study is immersed in a liquid of suitable refractive index in a silvered wedge. We use plane-polarized light of wavelength $\lambda = 546.1$ nm at temperature 29.5 °C. Fig. 10(a) and (b) shows the microinterferogram of Fizeau fringes crossing the single mode optical fiber and its contour lines using an immersion liquid of refractive index $n_0 = 1.4589$. Also, Fig. 10(c) and (d) shows the micro-interferogram and its contour lines using immersion liquid of refractive index $n_0 = 1.4574$. A prepared computer program based on Eqs. (10), (19), and (20) is used to calculate the refractive index profile for the fiber. Fig. 11 gives the refractive index profile of the fiber under study where we have cladding with refractive index 1.4596 and core with refractive index 1.463. With the aid of a computer program we can draw the cross-sectional shape of the used optical fiber using the calculated values of the vertical path l which is given from Eq. (10). Fig. 12 shows the drawn shape of the cross-section of the single-mode optical fiber. Using Eq. (22), the area of this fiber is calculated. The area of this fiber is $A = 11563 \pm 327.29\,\mu m^2$ and its radius $r = 60.6 \pm 0.756\,\mu m$.

3.3. Dralon fiber (with irregular cross-section)

Using the Pluta polarizing interference microscope with the aid of CCD camera, the microinterferograms for Dralon fiber (where 3 cm of the fiber is immersed in suitable liquid) are captured. Fig. 13(a) and (b) shows the microinterferogram of a totally duplicated image of this Dralon fiber and its contour lines using an immersion

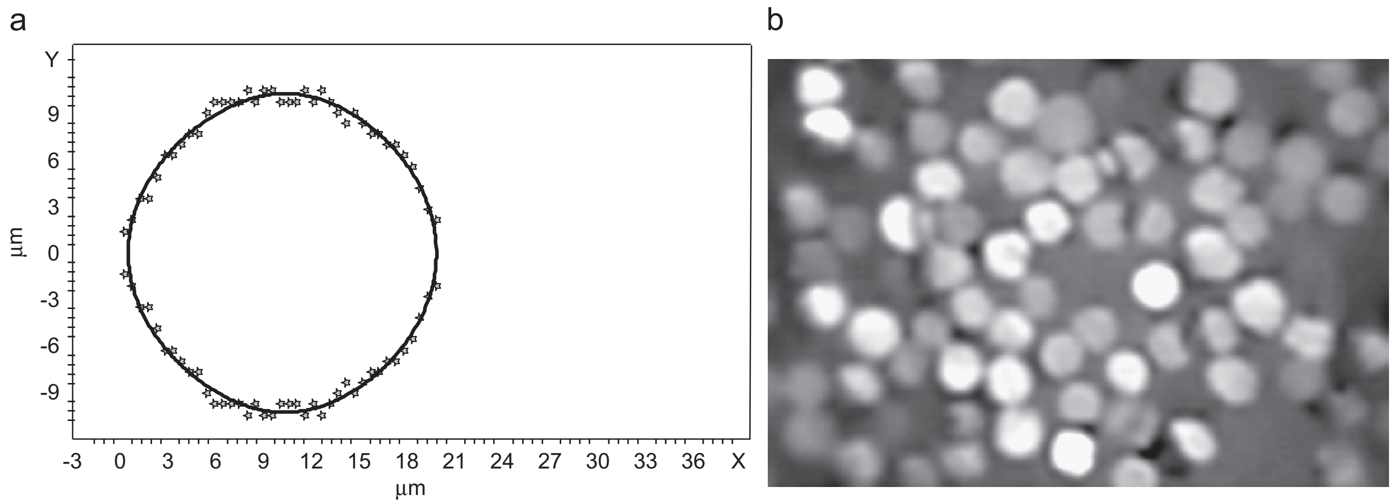

Fig. 9. (a) The drawn cross-sectional shape of Cortelle fiber using the suggested method. (b) The shape of the cross-section of Cortelle fiber using the optical microscope.

Table 1
The mean cross-sectional area and radius of the Cortelle and Dralon fibers

	Cortelle fiber		Dralon fiber	
	Cross-sectional area A (µm^2)	Radius r (µm)	Cross-sectional area A (µm^2)	Radius r (µm)
Optical microscopy method	320.82±4.38	10.105±0.118	453±10.15	–
The suggested interferometric method	316±3.37	10.03±0.103	448.08±9.53	–

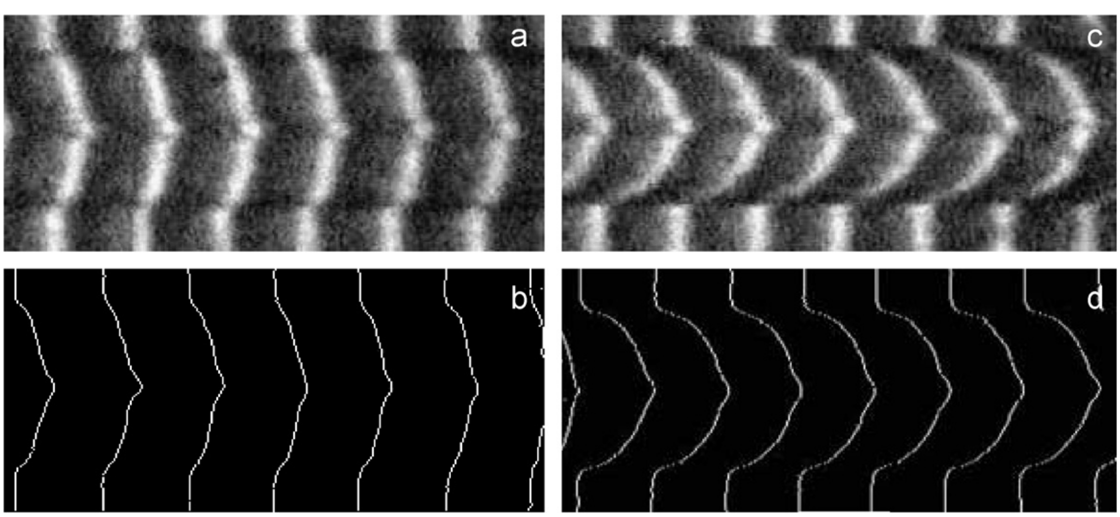

Fig. 10. Microinterferograms of Fizeau fringes crossing a single-mode optical fiber and its contour lines using two immersion liquids: (a, b) using liquid of refractive index $n_0 = 1.4589$; (c, d) using liquid of refractive index $n_0 = 1.4574$.

liquid of refractive index $n_0 = 1.5348$. Also, Fig. 13(c) and (d) shows that microinterferogram and its contour lines using an immersion liquid of refractive index $n_0 = 1.5189$. Using Eq. (21) and prepared computer program with the aid of the obtained microinterferograms, the refractive index profile of Dralon fiber is calculated. Fig. 14 shows the calculated refractive index profile of the Dralon fiber for light vibrating parallel to the fiber axis with accuracy $\pm 1 \times 10^{-3}$. With the aid of a computer program based on Eq. (11) the shape of the fiber cross-section is drawn. Fig. 15(a) shows the shape of the Dralon fiber cross-section. It is clear that the transverse cross-section of the fiber has the dog bone shape. Eqs. (11) and (22) with the aid of a computer program are used to calculate the

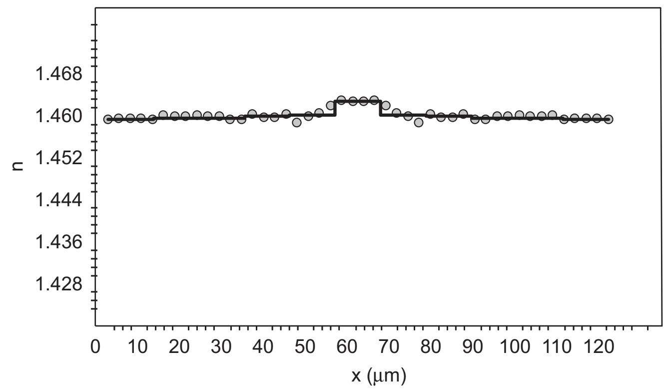

Fig. 11. Refractive index profile of the single mode optical fiber.

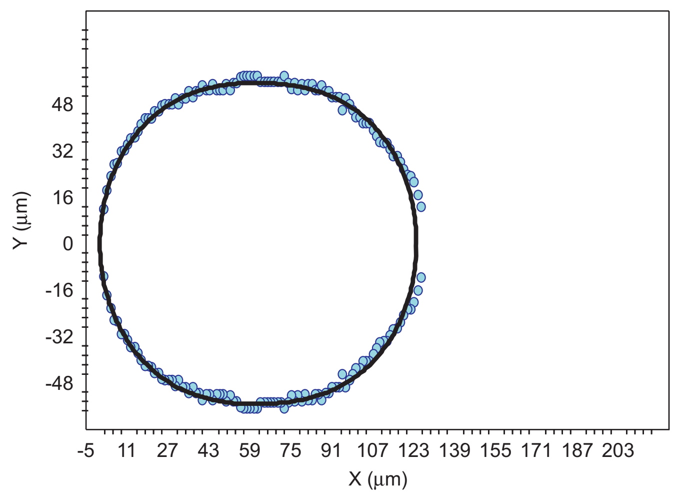

Fig. 12. The cross-sectional shape of the single mode optical fiber.

cross-sectional area of the Dralon fiber. Also, an optical microscopy method is used to calculate the cross-sectional area of the fiber. In this method, a bundle of the Dralon fibers are enlarged using the optical microscope (see Fig. 15(b)). Using this figure, the average area of 25 cross-sections is calculated. The calculated values for the transverse cross-sectional area of the Dralon fiber using the two methods are given in Table 1. It is clear from Fig. 15(a) and (b) that the cross-sectional shape drawn using the suggested method is similar to the one using the conventional optical microscope. Also, the calculated values of cross-section area using the two methods are nearly the same. These results confirm our suggested model.

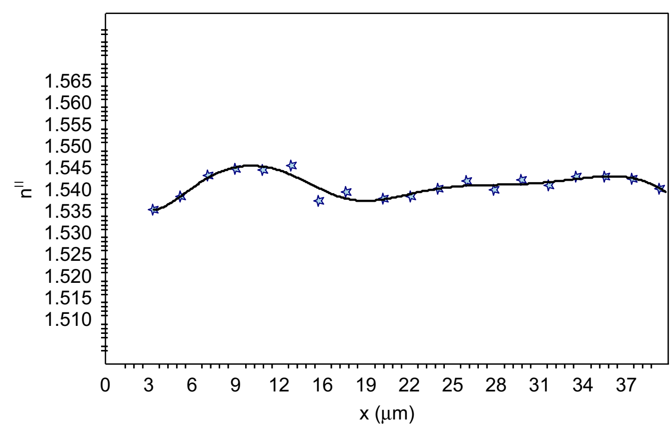

Fig. 14. The refractive index profile of Dralon fiber for light vibrating parallel to the fiber axis.

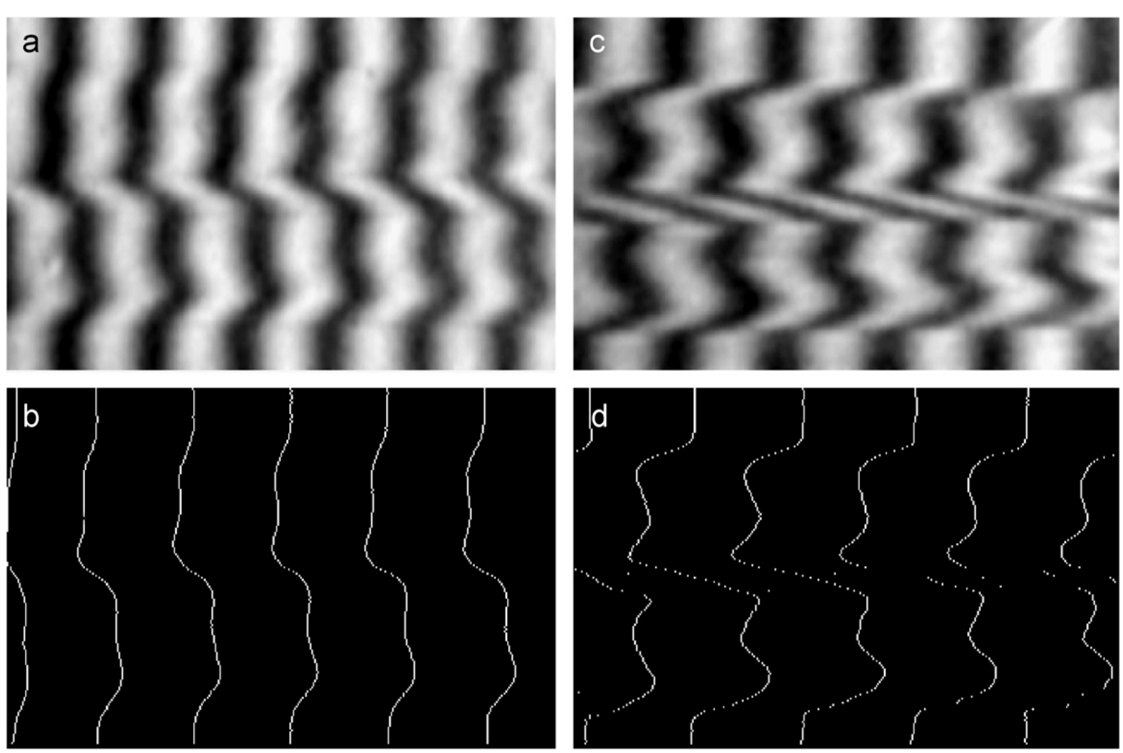

Fig. 13. Microinterferograms of two-beam interference fringes crossing Dralon fiber using Pluta microscope and its contour lines using two immersion liquids: (a, b) Using immersion liquid of refractive index $n_0 = 1.5348$. (c, d) Using immersion liquid of refractive index $n_0 = 1.5189$.

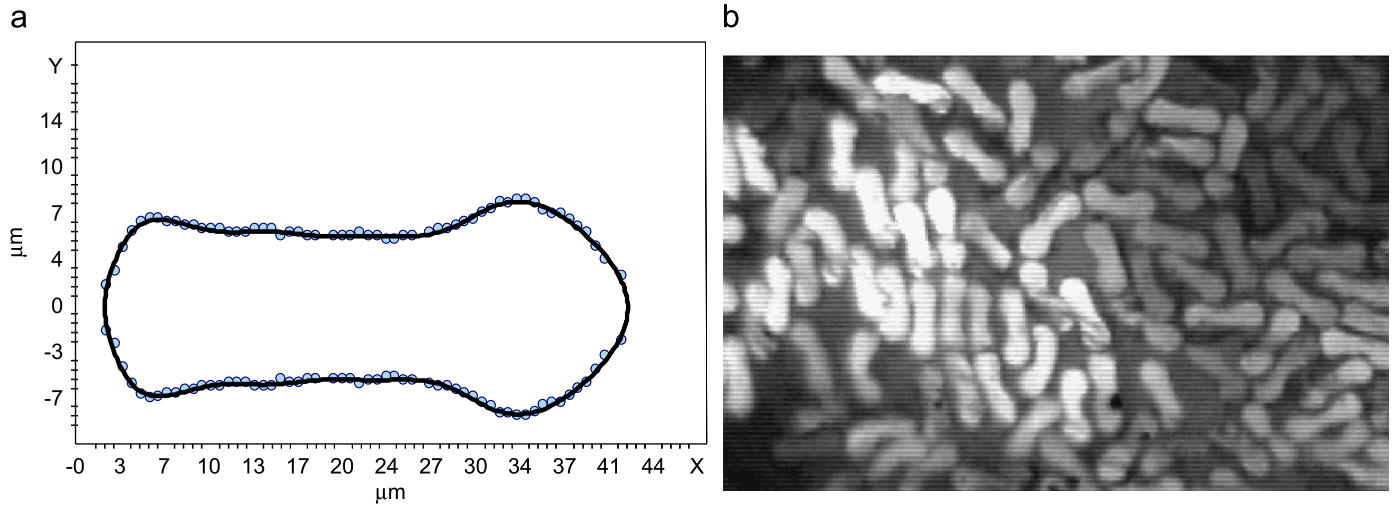

Fig. 15. (a) Cross-sectional shape of Dralon fiber using the suggested method. (b) The shape of the cross-section of Dralon fiber using the optical microscope.

4. Conclusion

We have successfully developed a method based on mathematical model for measuring the refractive index profile of fibers having regular and/or irregular transverse cross-sectional shape. Using this method, we are also able to measure the cross-sectional area and draw the cross-section shape with real dimensions. All calculations and results are performed automatically using prepared computer programs. We applied the suggested method for three different fibers of different types and shapes. We obtained accurate results compared with the conventional methods. For instance, the measured cross-sectional areas of the Dralon and Cortelle fiber are 448.08 ± 9.53 and 316 ± 3.37 μm^2, respectively, while the measured values using the conventional optical microscopy method are 453 ± 10.15 and $320.82 \pm 4.38\,\mu m^2$, respectively. Also, the conventional methods give only the mean refractive indices for irregular fibers, but our method succeeded in measuring the refractive index profile of the irregular Dralon with accuracy $\pm 1 \times 10^{-3}$. High accuracy and high speed of data analysis are the most important advantages of the developed method.

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