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Automatic refractive index profiling of fibers by phase analysis method using Fourier transform

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Abstract

Automatic fringe pattern analysis is a powerful and inexpensive digital image-processing technique. It is used to analyze the fringe pattern obtained by different optical techniques, such as multiple-beam Fizeau fringes. To perform accurate and fast automatic measurement of fiber refractive index profile, phase analysis method has been used with the Fourier transform technique. In this paper, the refractive index profiles of polyethylene fibers with different draw ratios are presented by two methods, fringe shift method and phase analysis method. A comparison between the results obtained is presented. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Automatic fringe analysis; Fourier transform; Phase measurement; Multiple-beam Fizeau fringes; Refractive index profile; Polyethylene fiber

1. Introduction

In the present decade, much progress has been made in the field of optical measurement because of several major advances in its related technologies. Optical measurement is based on the wave characteristics of light when it reflects from or transmits through an object. Light transmission properties through a fiber depend mainly on its refractive index profile and material dispersion. The refractive index

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profile of optical or textile fiber is important not only in assessing the performance of the fiber in a given system but also it helps in fiber fabrication to improve its products. Therefore, there is an increasing need for fast and accurate measurement of refractive index profile of fibers because it provides information for the correlation between their structure and the other properties.

Many different techniques were applied to determine the refractive index profile of fibers. One of the most sensitive and non-destructive technique, used to obtain this information, is multiple-beam Fizeau fringe system. So, this technique has been successfully used to investigate textile and optical fibers [1–6].

In practice, the fiber samples are immersed in a silvered liquid wedge interferometer [3], which act as phase objects displacing the normally straight parallel fringes of the multiple-beam Fizeau fringe interferometer to the fiber region. This fringe displacement or *shift* is proportional to the index difference Δn between the fiber refractive index and the immersion liquid. The methods in which a matching immersion liquid is used to give good results of fiber refractive index profile, especially when both liquid and fiber cladding have refractive indices close to each other.

The principle problem in accurately measuring the refractive index using multiple-beam Fizeau fringe system, is how to determine the contour line of the fringe pattern for; (a) interference fringe shift [3,7] or (b) the area enclosed under the fringe shift, and interfringe spacing. The present paper focuses on the refractive index profile measurement using the phase shift instead of fringe shift (or) the area under fringe shift and interfringe spacing.

Image-processing techniques were used to analyze the fringe pattern to perform automated, high-speed, and accurate analysis [8–10]. To analyze interference fringes accurately, the phase shift and Fourier transform measurement techniques were developed [11–17]. These techniques can be essentially classified into two basic types: the phase shifting and the Fourier transform types. Phase shifting techniques usually require at least three phase shifted interference fringe patterns. On the other hand, Fourier transform method usually requires only one interference fringe pattern for extracting phase information. This technique has been applied to various kinds of interferometric techniques.

2. Theory

2.1. Optical path difference

To produce multiple-beam Fizeau fringes in transmission, a parallel beam of plane polarized light illuminates a wedge interferometer placed on a microscope stage with normal incidence. This wedge interferometer consists of two circular optical flats. The inner surface of each flat is coated with a highly reflecting (75%) and partially transmitting (22%) silver layer. Both the gap thickness and the wedge angle are selected to form the sharpest fringes at right angles to the edge of the wedge. The fiber in the matching liquid acts as a phase object. The amount of the phase shift depends upon the values of refractive indices of the fiber and the immersion liquid used. The phase difference is given by

$$\phi = \frac{2\pi}{\lambda} m\Delta, \tag{1}$$

where m is an integer number equal to 1 in the case of two beam interference and 2 in the case of multiple-beam interference, λ is the wavelength of light used, Δ is the optical path difference. An accurate mathematical expression for the optical path difference (OPD) was given by Hamza et al. [5] in which they considered the refraction of the beam through the fiber due to refractive index change along its radius. Also, they considered that the fiber is assumed to be divided into N circular zones. For large number of layers, each layer can be considered to have a constant refractive index. A general expression is used to calculate the Δ_Q of the Qth layer in the fiber region, in case of multiple-beam Fizeau fringes as follows:

$$\begin{aligned}\Delta_Q = \sum_{j=1}^{Q-1} 2n_j \Bigg\{ & \sqrt{(R-(j-1)a)^2 - (d_Q n_0/n_j)^2}. \\ & - \sqrt{(R-ja)^2 - (d_Q n_0/n_j)^2} \Bigg\} \\ & + 2n_Q \sqrt{(R-(Q-1)a)^2 - (d_Q n_0/n_j)^2} \\ & - n_0 \left\{ \sqrt{R^2 - d_Q^2} + \sqrt{R^2 - X_Q^2} \right\},\end{aligned} \tag{2}$$

where R is the fiber radius, a is the layer thickness ($a = R/N$), $n_0 = n_L$ is the immersion liquid refractive index, X_Q and d_Q are the emerged and incident rays distances from the fiber center, respectively, where

$$d_Q = \frac{n_Q(R-(Q-1/2)a)}{n_0}.$$

And considering that the phase difference is given by

$$\phi = \phi(R) - \phi_0$$

$\phi(R)$ is the phase shift due to the fiber and ϕ_0 is the phase due to the immersion liquid. The value of ϕ_0 must be constant. Because the interferogram has a noise this value is not exactly constant but we can say it is nearly constant so we calculate the main value of ϕ_0.

$$\frac{\lambda \phi_Q}{4\pi} = \Delta_Q$$

$$= \left[\sum_{j=1}^{Q-1} 2n_j \left\{ \sqrt{(R-(j-1)a)^2 - (d_Q n_0/n_j)^2} - \sqrt{(R-ja)^2 - (d_Q n_0/n_j)^2} \right\} + 2n_Q \sqrt{(R-(Q-1)a)^2 - (d_Q n_0/n_j)^2} - n_0 \left(\sqrt{R^2 - d_Q^2} + \sqrt{R^2 - X_Q^2} \right) \right].$$

2.2. Phase measurement

Mapping the phase of the fringes displayed on an interferogram is an important problem in many areas of optical measurement. In the 1970s Bruning et al. [18] introduced to interferometry a phase-detection technique for testing optical

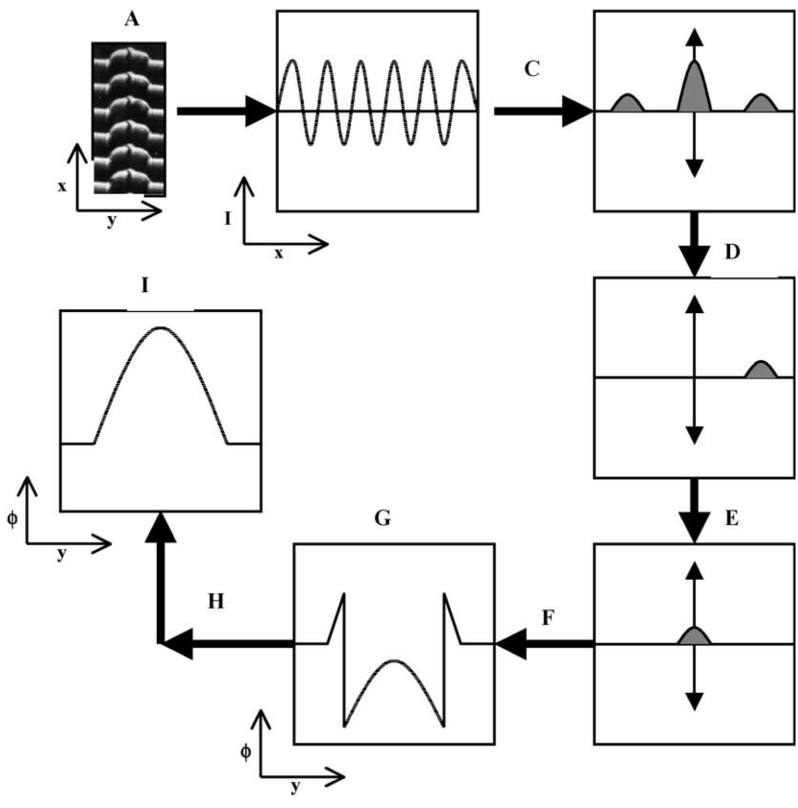


Fig. 1. Fourier transform process; A—original pattern; B—1-D intensity distribution; C—FFT process; D—filter process; E—shifting process; F—IFFT process; G—wrapped phase obtained; H—unwrapped process; I—unwrapped phase obtained.

components that uses a solid-state detector. In this method, an interference pattern was phase shifted and the digitized intensity values were then correlated with sines and cosines to determine the optical phase. This method requires at least three phase-shifted interferograms. In 1982 Takeda et al. [16] proposed a novel method of phase analysis using Fourier transform. The phase can be retrieved from a single interferogram by using this method. This technique has been used and modified by several authors [15,19,20]. This technique is summarized as follows.

Generally, the intensity distributions in such interference fringe pattern can be written as

$$g(x,y) = a(x,y) + b(x,y)\cos(2\pi f_0 x + \phi(x,y)), \tag{4}$$

where $a(x,y)$ represents the background illuminations of the intensity distribution $g(x,y)$, $b(x,y)$ describes the amplitude of the corresponding interference fringe, f_0 is the carrier frequency, $\Phi(x,y)$ is the phase of the object that we have to analyze at any point (x,y) on the interferogram. In most cases $a(x,y)$, $b(x,y)$, and $\Phi(x,y)$ are very slowly varying functions compared with the variation introduced by the spatial

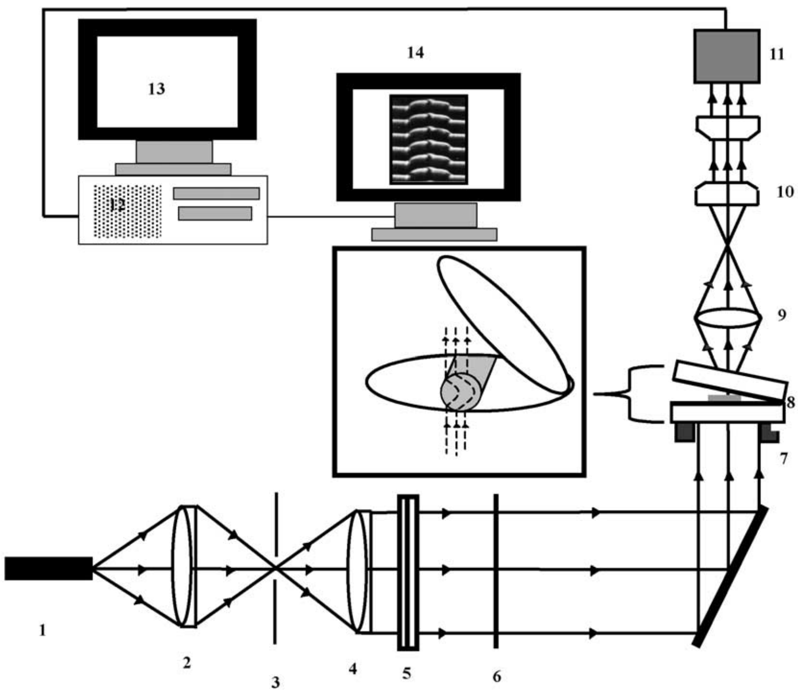


Fig. 2. The optical setup for producing multiple-beam Fizeau fringes in transmission: 1—Mercury lamp; 2—condenser lens; 3—Iris diaphragm; 4—Collimating lens; 5—Polarizer; 6—Monochromatic interference filter; 7—Microscope stage; 8—Silvered liquid wedge interferometer; 9—Microscope objective; 10—Microscope ocular; 11—CCD camera; 12—Frame grabber; 13—Graphic and text screen; 14—Multisync monitor.

carrier frequency f_0. This equation can be rewritten as

$$g(x,y) = a(x,y) + c(x,y)\exp(i2\pi f_0 x) + c^*(x,y)\exp(-i2\pi f_0 x), \quad (5)$$

where

$$c(x,y) = \frac{1}{2}\, b(x,y)\exp(i\phi(x,y)) \quad (6)$$

and $c^*(x,y)$ denotes complex conjugation of $c(x,y)$.

Applying Fourier transform (FT) algorithm, we compute the one-dimension (1-D) Fourier transform of Eq. (5) for the variable x only, with y being fixed.

$$G(f,y) = A(f,y) + C(f - f_0, y) + C^*(f - f_0, y), \quad (7)$$

where the capital letters denote Fourier spectra, f is the variable in spatial frequency space and $C(f - f_0, y)$ is the FT of $c(x,y)$ with respect to x. From Eq. (7) one can notice that, the FT of the fringe pattern exhibit three distinct peaks. The Fourier transform of the term $A(f,y)$ is placed in the center of the spectrum and represents the zero frequency (or DC). And the Fourier transforms of $C(f - f_0, y)$ and $C^*(f + f_0, y)$ then will be symmetric with respect to the center and placed at a distance that is determined by f_0. We make use of either of the two peaks spectra on the carrier, say

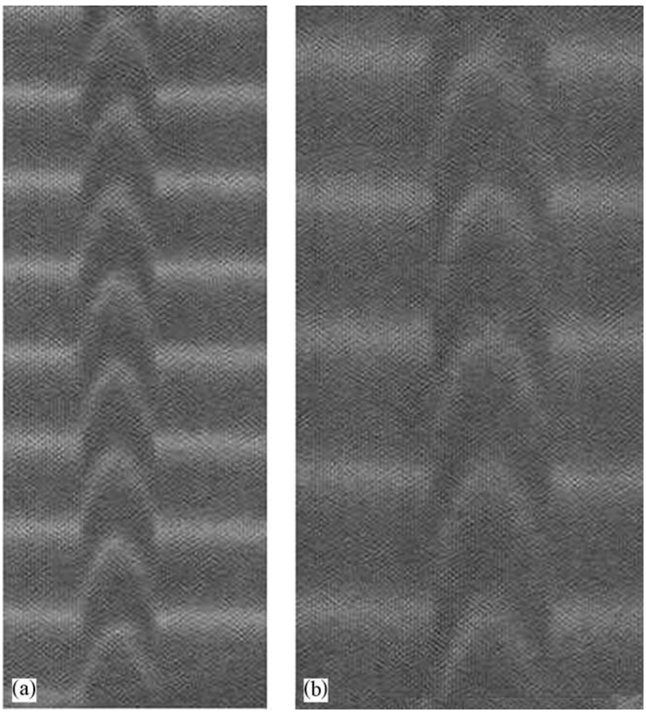


Fig. 3. Microinterferogram of polyethylene fiber with the draw ratio 7.5 using monochromatic light vibrating parallel to the fiber axis, immersion liquid used is 1.5787 (A) pixel size is 1.316614 μm, (B) pixel size is 0.8665226 μm.

$C(f-f_0,y)$. In other words, the unwanted peaks have been filtered out in this step. After $C(f-f_0,y)$ is shifted a distance of f_0 toward the original in the frequency domain, we compute the inverse Fourier transform (IFT) of $C(f-f_0,y)$ with respect to f then the term $c(x,y)$ itself will be obtained. Then we calculate a complex logarithm of Eq. (6)

$$\log[c(x,y)] = \log\left[\frac{1}{2}\, b(x,y)\right] + \mathrm{i}\phi(x,y).$$

From this equation it is easy to obtain the phase. In most cases, a computer-generated function subroutine gives a principal value ranging from $-\pi$ to π. So, the phase map obtained is wrapped in the range between $-\pi$ and π, and the phase unwrapping procedure is generally required to produce a continuous phase distribution. The relation between the wrapped and the unwrapped phase can be stated as

$$\begin{aligned}\phi(x_i,y_j) &= \phi_{\mathrm{w}}(x_i,y_j) + 2\pi m(x_i,y_j),\\ &1 \leqslant i \leqslant N, \quad 1 \leqslant j \leqslant M,\end{aligned} \tag{8}$$

where $\phi_{\mathrm{w}}(x,y)$ is the wrapped phase, $\phi(x,y)$ is the unwrapped phase, and $m(x,y)$ is an integer-valued number called the field number. The unwrapping problem is trivial for phase maps calculated from good quality fringe data when the following two conditions are satisfied [21]:

(1) The signal is free of noise.
(2) The Nyquist condition is not violated, which means that the absolute value of the phase difference between any two consecutive phase samples (pixels) is $<\pi$.

The whole Fourier transform analysis process is illustrated in Fig. 1.

3. Experimental setup

Schematic diagram represents the optical setup for producing multiple-beam Fizeau fringes in transmission is shown in Fig. 2. The fringes are characterized by sharp bright fringe on a dark background. A parallel beam of monochromatic light falls on the plane-mirror of the microscope that reflects the light in a direction perpendicular to the wedge interferometer. The wedge interferometer is adjusted in such a way that the fiber is exactly perpendicular to the interference fringes in the liquid region. The accuracy of the refractive index profile and hence the structural parameters measurement are affected by the deviation of the fiber and fringes from the right angle. Deviation from right angle by one degree or less causes a negligible change in the measured quantities. The intensity of the multiple-beam Fizeau image is converted to an electric video signal and sampled to yield a digital picture made up of 512×512 sample points, each of which is quantized to 256 discrete gray levels. Using two-dimensional intensity sensor (Panasonic CCD micro-camera attached directly to the microscope), the digitized picture is stored in a digital frame grabber memory. The stored picture transferred to PC with microprocessor 500 MHz and

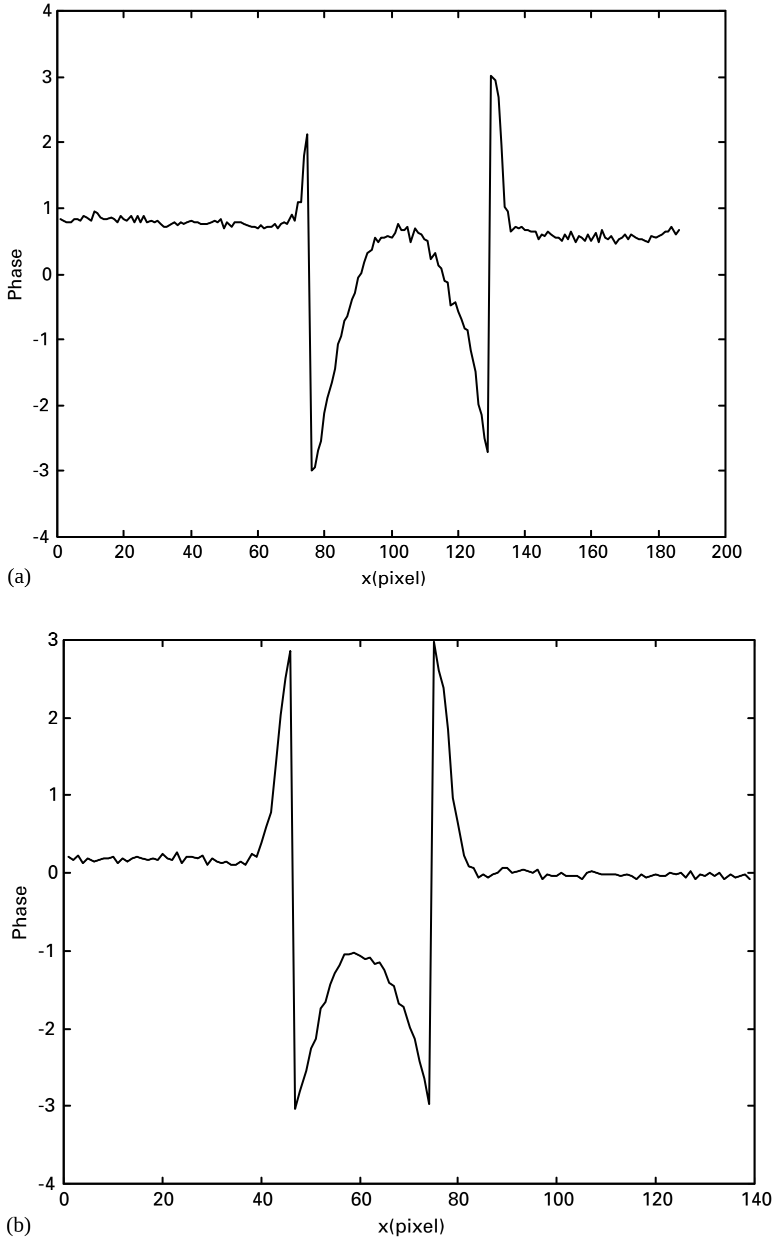
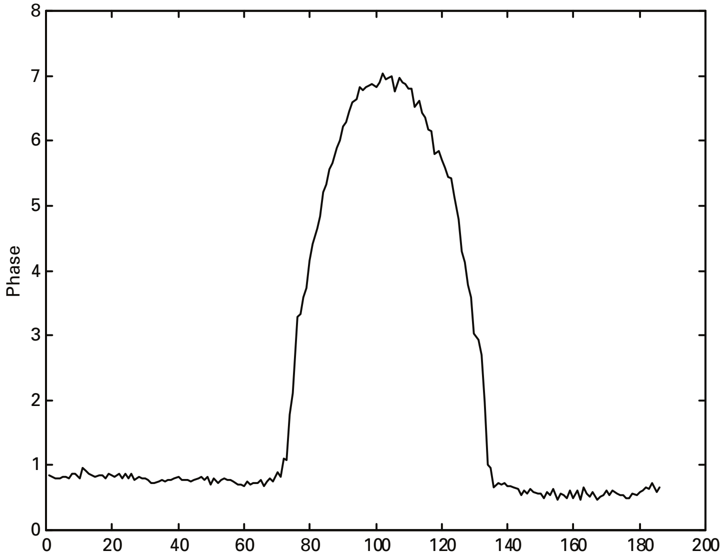


Fig. 4. The 1-D wrapped phase profile (A) of Fig. 3A, (B) of Fig. 3B.

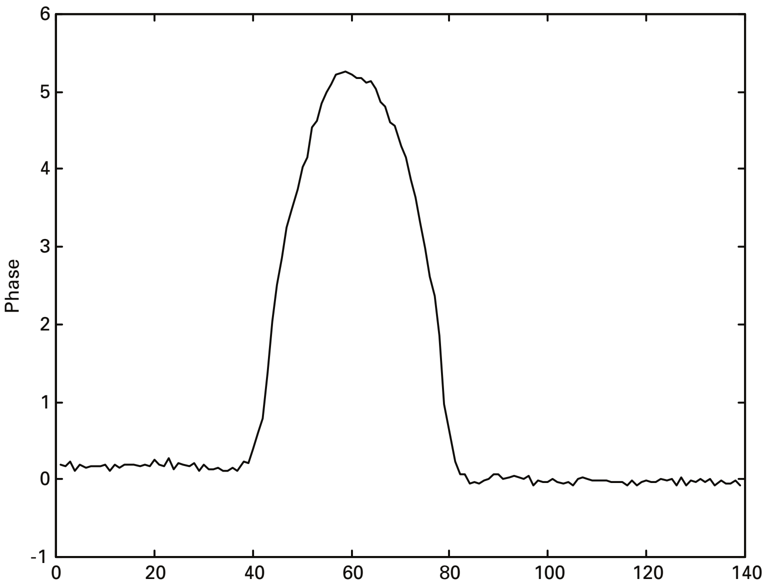
recorded on the mass storage device of a disk. A cross hair is used to adjust the fiber to be perpendicular to the fringes. A relay lens is inserted between the microscope and the CCD camera to sharpen the interferogram.

4. Result and discussion

Mapping the phase and refractive index profiles of polyethylene fiber, which was drawn to different draw ratios (7.5 and 10), were obtained automatically. The draw



(a)



(b)

Fig. 5. The 1-D unwrapped phase profile (A) of Fig. 3A, (B) of Fig. 3B.

ratio is the ratio of the fiber length after drawing to the fiber original length. Monochromatic light of wavelength 546.1 nm vibrating parallel to the fiber axis is used. Fig. 3A shows the original interferogram of multiple-beam Fizeau fringe pattern of polyethylene fiber with the draw ratio 7.5 when using immersion liquid of refractive index 1.5787 and the pixel size is 1.32 μm. Fig. 3B shows the same but with the pixel size 0.87 μm. FT of the fringe pattern was taken to the interferograms which

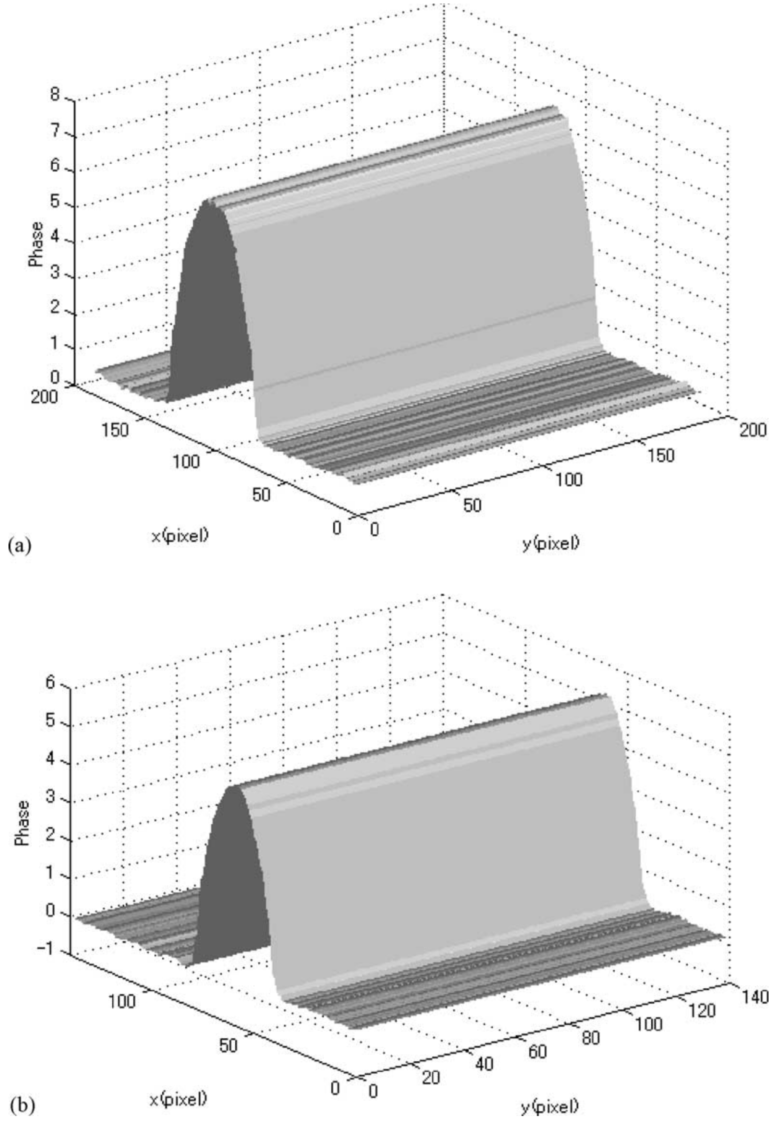


Fig. 6. Plot of the 2-D unwrapped continuous phase distribution (A) of Fig. 3A, (B) of Fig. 3B.

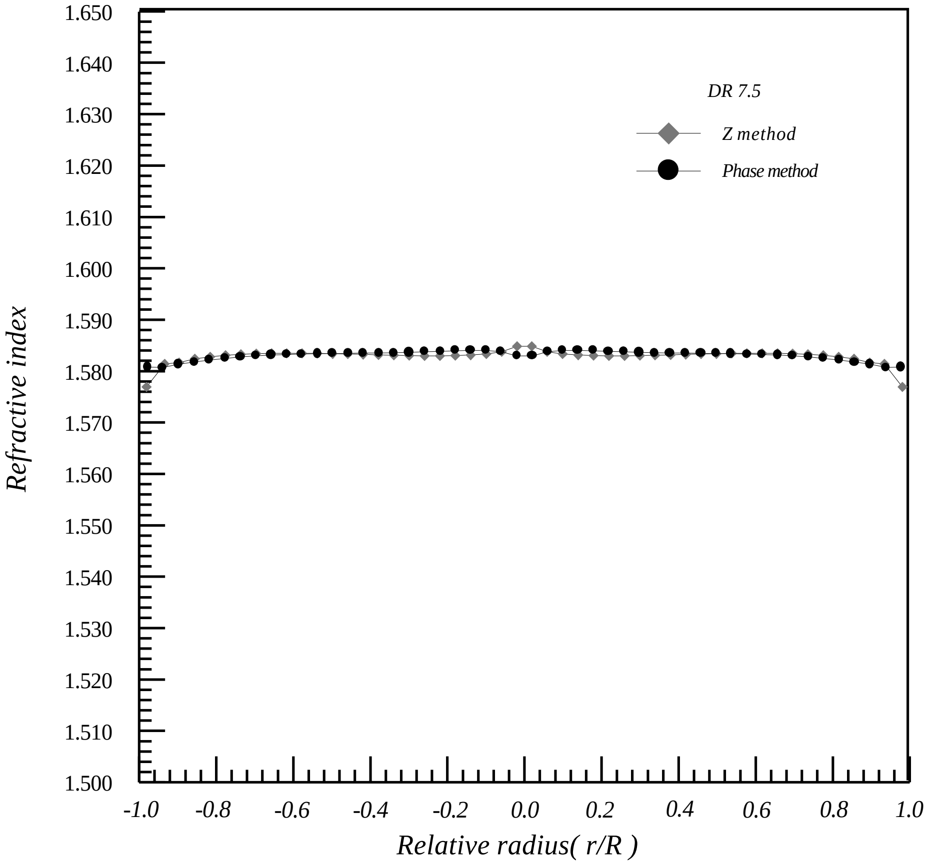


Fig. 7. Refractive index profile of polyethylene fiber with the draw ratio 7.5 using phase method and Z-method. Monochromatic light vibrating parallel to the fiber axis is used and the pixel size is found to be 1.31661 μm and the immersion liquid used is 1.5787.

are shown in Figs. 3A and B to obtain the wrapped phase, shown in Figs. 4A and B, respectively. Figs. 5A and B show the unwrapped phase in one dimension (1-D) while the unwrapped phase in two dimensions (2-D) was shown in Figs. 6A and B. Eq. (3) was used with software prepared by us to determine the refractive index along the fiber radius (refractive index profile). These results were compared with the refractive index profiles, which are obtained using fringe shift method (Z-method), shown in Figs. 7 and 8.

Fig. 9 shows the original interferogram of polyethylene fiber with the draw ratio 10 immersed in a liquid of refractive index 1.5849 and the pixel size is found to be 0.625 μm. The 1-D and the 2-D unwrapped phase were shown in Figs. 10 and 11, respectively. Fig. 12 shows the refractive index profile compared with that calculated

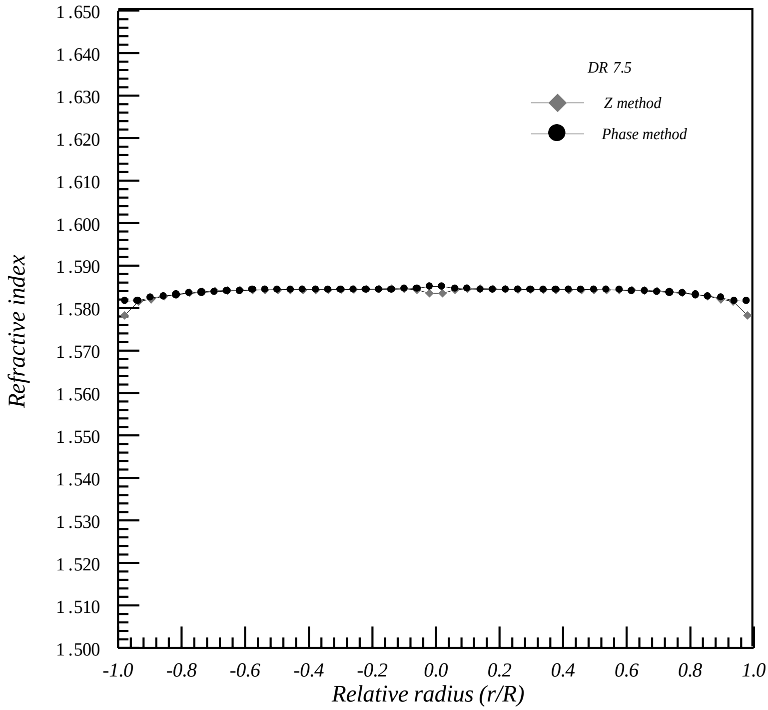


Fig. 8. Refractive index profile of polyethylene fiber with the draw ratio 7.5 using phase method and Z-method. Monochromatic light vibrating parallel to the fiber axis is used and the pixel size is found to be 0.8665 μm and the immersion liquid used is 1.5787.

using Z-method. It is clear that, when the immersion liquid and fiber refractive indices are close to each other, a small error in the measurements of both method (phase method and Z-method) is obtained and the two curves coincide with each other and tend to be one curve.

In fact, the fibers used in this study have nearly constant refractive index profile. Referring to Figs. 3, 7, 8 and 12 one can easily notice that: (a) the applied phase with FT method gives a stable profile which is more reliable than that uses the fringe shift method, (b) as the difference in fiber and immersion liquid refractive indices increases the errors in the measurement increases, but still the phase with Fourier method has a good presentation of refractive index profile, (c) the smaller value of pixel size the more accurate are the obtained results.

Table 1 lists a statistical comparison between phase method and Z-method. From which, it can be seen that the refractive index profile measured using phase method is more accurate than that obtained by Z-method. Also, it shows that at low pixel size a small error in the results of both method is obtained.

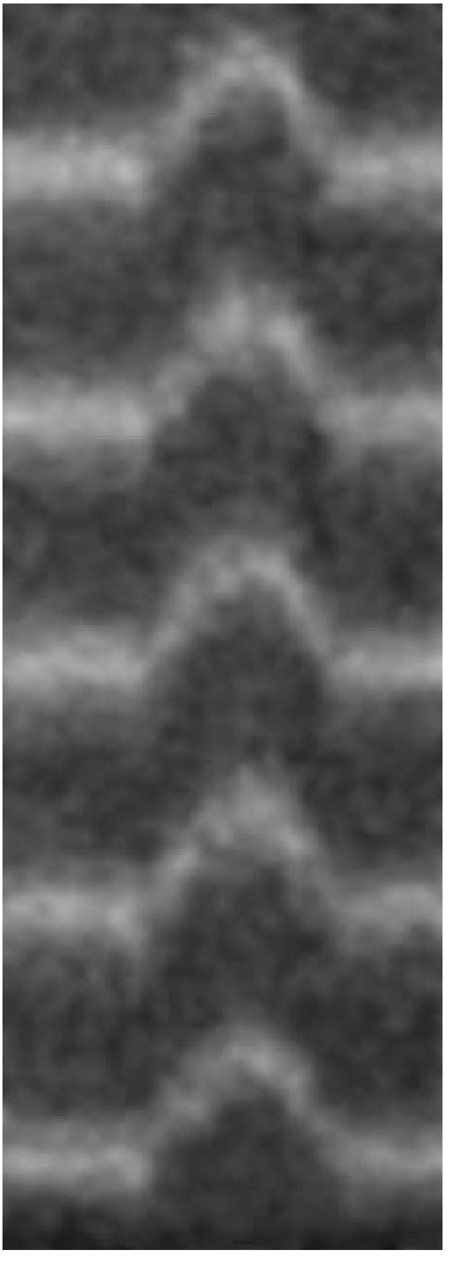


Fig. 9. Microinterferogram of polyethylene fiber with the draw ratio 10 using monochromatic light vibrating parallel to the fiber axis, immersion liquid used is 1.5849, and pixel size is 0.625 μm.

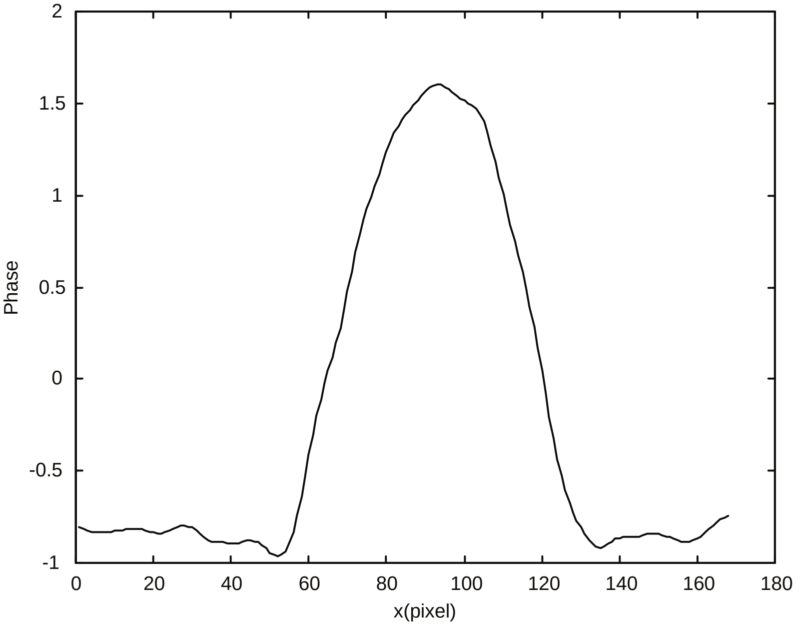


Fig. 10. The 1-D unwrapped phase profile of Fig. 9.

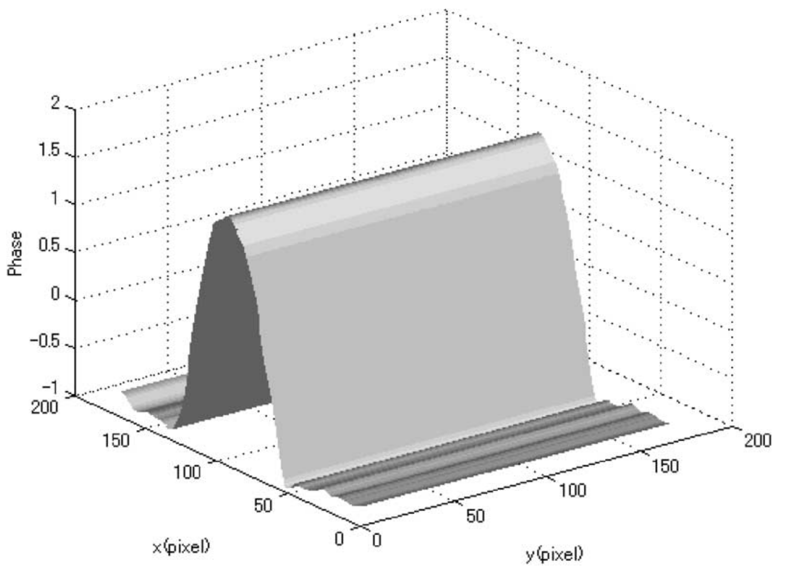


Fig. 11. Plot of the 2-D unwrapped continuous phase distribution of Fig. 9.

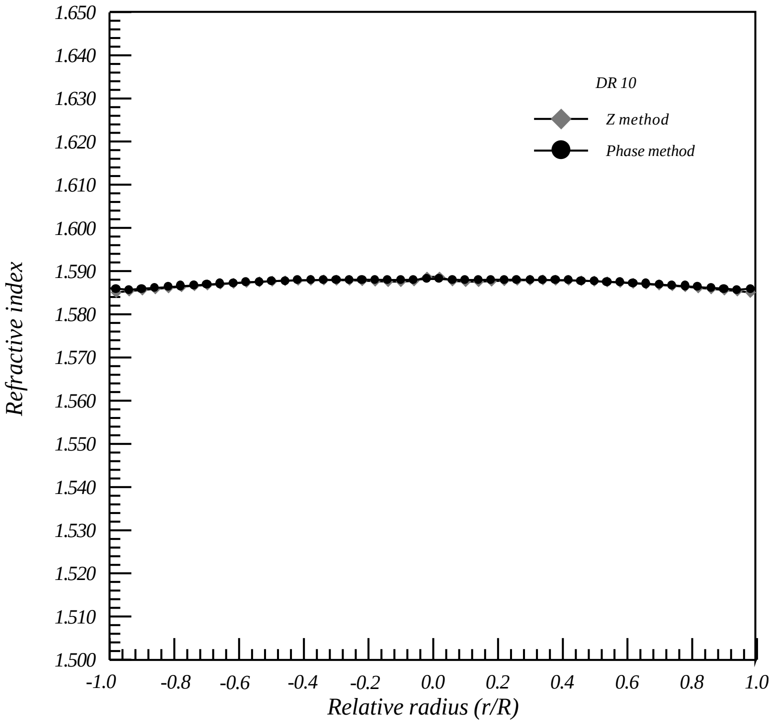


Fig. 12. Refractive index profile of polyethylene fiber with the draw ratio 10 using phase method and Z-method. Monochromatic light vibrating parallel to the fiber axis is used and the pixel size is found to be 0.625 μm and the immersion liquid used is 1.5849.

Table 1
The Statistical comparisons between phase method and Z-method for the measured refractive index profile of polyethylene fiber having different draw ratios

Draw ratio	Pixel size (μ)	Standard deviation		Sigma error	
		Phase method	Fringe shift	Phase method	Fringe shift
7.5	1.317	9.560×10^{-4}	13.77×10^{-4}	9.331×10^{-7}	19.37×10^{-7}
7.5	0.867	8.781×10^{-4}	13.51×10^{-4}	7.871×10^{-7}	18.63×10^{-7}
10	0.625	8.016×10^{-4}	9.251×10^{-4}	6.560×10^{-7}	8.740×10^{-7}

5. Conclusions

The development of precise and efficient technique for refractive index profile measurement is an important technical task. Multiple-beam interference Fizeau fringes system is applied to polyethylene fiber to determine its refractive index profile. FT method has been used for high-precision evaluation of the phase distribution of multiple-beam Fizeau fringes. By using Figs. 3 and 9, the measured radius of drawn polyethylene fibers 23.725 and 20.44 μm correspond to draw ratios 7.5 and 10, respectively. The refractive index profiles of drawn polyethylene fibers were obtained using the phase and fringe shift methods. We can conclude from the results that the phase method is a powerful technique to analyze the multiple-beam Fizeau fringes pattern and to measure the refractive index profile of fibers. Both the techniques perform equally well when applied to any interferogram having a low pixel size. The phase technique strategy presents two main benefits; (i) it is simple to automate and (ii) it performs better under certain circumstances, such as the case in high accuracy of refractive index profile measurement. Moreover, for large difference in fiber and immersion liquid refractive indices, it gives more stability of refractive index profile (see Table 1).

References

[1] Zanger H, Zanger C. Fibre optics communication and other applications. New York: Macmillan, 1991.
[2] Tolansky S. Multiple-beam interferometry. Oxford: Claredon Press, 1948.
[3] Barakat N, Hamza AA. Interferometry of fibrous materials. Bristol: Hilger, 1990.
[4] Barakat N, Hamza AA, Gonied AS. Multiple-beam interference fringe applied to GRIN optical waveguides to determine fibre characteristics. Appl Opt 1985;24:4383–6.
[5] Hamza AA, Sokkar TZN, Mabrouk MA, Ghandar AM, Ramadan WA. On the determination of the refractive index of a fibre: II graded index fibre. Pure Appl Opt 1995;4:161–77.
[6] Mabrouk MA, Shams-Eldin MA. Interferometric measurement of some structural parameters of drawn polyethylene fibres. Pure Appl Opt 1996;5:929–40.
[7] Mabrouk MA, El-Bawab HF. Refractive index profile of GR-IN optical fibre considering the area under the interference fringe shift: I. The matching case. Pure Appl Opt 1997;6:247–56.
[8] Yatagai T. Automated fringe analysis techniques in Japan. Opt Laser Eng 1991;15:79–91.
[9] Morimoto Y. Fujisawa, Fringe pattern analysis by a phase-shifting method using Fourier transform. Opt Eng 1994;33:3709–14.
[10] Hamza AA, Sokkar TZN, Mabrouk MA, El-Morsy MA. Refractive index profile of polyethylene fiber using interactive multiple-beam Fizeau fringe analysis. J Appl Polym Sci 2000;77:3099–106.
[11] Bone DJ, Bachor HA, Sandemen RJ. Fringe-pattern analysis using a 2-D Fourier transform. Appl Opt 1986;25:1653–60.
[12] Creath K. Phase measurement interferometry techniques. Prog Opt 1988;26:350–93.
[13] Omura K, Yatagai T. Phase measuring Ronchi test. Appl Opt 1988;27(3):523–8.
[14] Lai G, Yatagai T. Use of the fast Fourier transform method analyzing linear and equispaced Fizeau fringes. Appl Opt 1994;33(25):5935–40.
[15] Nicola SD, Ferraro P. Fourier transform method of fringe analysis for Moire interferometry. Pure Appl Opt 2000;2:228–33.
[16] Takeda M, Ina H, Kobayashi S. Fourier-transform method of fringe-pattern analysis for computer-based topography and interferomtry. J Opt Soc Am 1982;72:156–60.
[17] Takeda M, Mutoh K. Fourier transform profilometry for the automatic measurement of 3-D object shapes. Appl Opt 1983;22:3977–82.
[18] Bruning HA. Fringe scanning interferometers. In: Malacara D, editor. Optical shop testing. New York: Wiley, 1978.

[19] Green JR, Walker GJ, Robinson WD. Investigation of the Fourier-transform method of fringe pattern analysis. Opt Laser Eng 1988;8:29–44.

[20] Liu BJ, Ronney DP. Modified Fourier transform method for interferogram fringe pattern analysis. Appl Opt 1997;36(25):6231–41.

[21] Malacara D, Servin M, Malacara Z. Interferogram analysis for optical testing. New York: Marcel Dekker, 1998. p. 328–83.