


## Article

# Two-Qubit Local Fisher Information Correlation beyond Entanglement in a Nonlinear Generalized Cavity with an Intrinsic Decoherence

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**Abstract:** In this paper, we study a Hamiltonian system constituted by two coupled two-level atoms (qubits) interacting with a nonlinear generalized cavity field. The nonclassical two-qubit correlation dynamics are investigated using Bures distance entanglement and local quantum Fisher information under the influences of intrinsic decoherence and qubit–qubit interaction. The effects of the superposition of two identical generalized coherent states and the initial coherent field intensity on the generated two-qubit correlations are investigated. Entanglement of sudden death and sudden birth of the Bures distance entanglement as well as the sudden changes in local Fisher information are observed. We show that the robustness, against decoherence, of the generated two-qubit correlations can be controlled by qubit–qubit coupling and the initial coherent cavity states.

**Keywords:** nonclassical correlation; intrinsic decoherence  $SU(1,1)$ ;  $SU(2)$ -algebraic treatment



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## 1. Introduction

Nonclassical correlations (NCs) and quantum entanglement (QE) are substantial as tools for quantum information [1–5]. Therefore, the generation of two-qubit correlated states has been extensively investigated [3,6–8]. QE is an important type of NC, but it is not a unique resource in nonclassical correlations [9]. Other types of NCs beyond QE were defined via quantum Fisher information (QFI) [10], local quantum Fisher information (LQFI) [11], quantum discord [12], and other geometrical correlation quantifiers based on skew information [13] and distance norms [14].

Quantum entanglement and purity are recognized as primarily important in developing modern quantum technologies [2,15,16]. The entanglement can be created and preserved between completely separated qubits inside the cavity [17–22]. Quantum Fisher information is the most used to describe absolute accuracy in parameter estimation scenarios [23].

Recently, several suggestions have been introduced based on QFI dynamics to demonstrate the importance of quantum entanglement especially for quantum metrology [24] and for parameter estimation precision [25]. On the other hand, Bures distance entanglement (BDE) was used to measure correlations between the parts of a quantum system. The Gaussian entanglement in an identical two-mode Gaussian cavity state was evaluated in terms of its Bures distance with the set of separable Gaussian states [26]. The robustness of the