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Differential Subordination and Superordination Results Associated with Mittag–Leffler Function

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Abstract: In this paper, we derive a number of interesting results concerning subordination and superordination relations for certain analytic functions associated with an extension of the Mittag–Leffler function.

Keywords: analytic function; Mittag–Leffler function; differential subordination; differential superordination

MSC: 30C45; 33E12

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1. Definitions and Preliminaries

Let $\mathbb{H}$ be the class of analytic functions in the open unit disc $\mathbb{U} = \{z : |z| < 1\}$. Also, let $\mathbb{H}[a, n]$ denote the subclass of the functions $f \in \mathbb{H}$ of the form

$$f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \dots \quad (a \in \mathbb{C}). \tag{1}$$

Furthermore, let

$$A_m = \left\{ f \in \mathbb{H} \mid f(z) = z + a_{m+1} z^{m+1} + a_{m+2} z^{m+2} + \dots \right\}.$$

Moreover, assume that $A = A_1$ which is the subclass of the functions $f \in \mathbb{H}$ of the form

$$f(z) = z + a_2 z^2 + \dots \quad . \tag{2}$$

For $f, g \in \mathbb{H}$, we say that the function f is subordinate to g, written symbolically as follows:

$$f \prec g \quad \text{or} \quad f(z) \prec g(z),$$

if there exists a Schwarz function w, which (by definition) is analytic in $\mathbb{U}$ with $w(0) = 0$ and $|w(z)| < 1$ $(z \in \mathbb{U})$, such that $f(z) = g(w(z))$ for all $z \in \mathbb{U}$. In particular, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence relation (cf., e.g., [1,2]; see also [3]):

$$f(z) \prec g\,(z) \Leftrightarrow f(0) \prec g(0) \ \ and \ \ f(\mathbb{U}) \subset g(\mathbb{U}).$$