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DUSTY PLASMA

Stability of Dust Acoustic Wavepackets Suffering from Polarization Force Due to the Presence of Trapped Ions[1]

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Abstract—The combined effects of the polarization force, free and trapped ions, and dust charge variation are incorporated in a rigorous study of the nonlinear dust acoustic waves (DAWs) propagating in an unmagnetized dusty plasma. Owing to the departure from the Boltzmann ion distribution, it is found that the nonlinear DAWs are governed by a modified Korteweg–de Vries (mKdV) equation. The association between the mKdV solitary wave and the DAW envelope in the system under consideration is discussed. A modified nonlinear Schrödinger equation appropriate for describing the modulated DAWs is derived. The modulation instability (MI) and the dependence of the system physical parameters on the polarization force, trapped ions, and dust charge variation have been analyzed. It is found that the critical curve separating the stable/unstable regions is strongly influenced by both of the polarization and the ion trapping parameters. Moreover, increasing the polarization leads to an increase of the critical wave number, while increasing the trapping parameter yields the opposite effect. The MI maximum growth rate decreases (increases) as the polarization (trapped ion) increases. The obtained results may be helpful in better understanding of space observations of the solar energetic particle flows in interplanetary space and the energetic particle events in the Earth's magnetosphere.

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1. INTRODUCTION

Dusty plasma physics concerns the properties of charged dust in the presence of electrons and ions [1]. These media have been observed in lower and upper mesosphere, planetary magnetosphere, cometary tails, planetary rings, interplanetary spaces, interstellar media, etc. [2–4]. The importance of studying dusty plasmas appears also in the laboratory and plasmas technology, such as low-temperature physics (radio frequency), plasma discharge [5], and dusty crystals [6]. It has been shown both theoretically and experimentally that the presence of these extremely massive and highly charged dust grains in a plasma can either modify the behavior of the usual waves and their associated instabilities or introduces new eigenmodes [1, 7–10]. The dust acoustic wave (DAW) arises due to the restoring force is provided by the plasma thermal pressure (electrons and ions), while the inertia is due to the dust mass [1].

The highly charged massive dust grains present in a dusty plasma may exhibit charge fluctuations due to a variety of intrinsic plasma charging mechanisms [11]. Moreover, it has been shown that dust charge variation affects the characteristic of the collective behavior of plasmas [12]. Xie et al. [13] investigated dust acoustic solitons with varying dust charge and they showed that only rarefactive solitary waves exist when the Mach number lies within an appropriate regime, depending on the system parameters. Ivlev and Morfill [14] have considered two limiting cases of ion distribution (Boltzmann and highly energetic cold ions) in studying dust acoustic solitons with variable charged dust grains. It is found that the charge variation is crucial if the particle number density is sufficiently high. Taken noctilucent clouds and polar mesosphere summer echoes as proper applications containing vast amounts of charged dust or aerosols, Kopnin et al. [15] have studied the effect of the dust charge sign on the properties of dust acoustic solitons propagating in this dusty ionosphere. It is shown that when the dust charge is negative, dust acoustic solitons correspond to a well in the electron density and a hill in the ion density. When the dust is charged positively, the situation is opposite.

If streaming particles are injected into plasmas, we often find that they evolve toward a coherent trapped particle state, instead of developing into a turbulent one. This has been confirmed by computer simulations [16, 17] and experiments [18]. The wave propagation characteristics in collisionless plasmas are significantly modified by the presence of the trapped particles [19]. In most laboratory dusty plasmas, ions

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are not always isothermal because they could be trapped by the DAW potential. Accordingly, they follow the trapped/vortex-like distribution [20]. Some recent theoretical studies focused on the effects of ion and electron trapping which are common not only in space plasmas but also in laboratory experiments [21–24]. In addition, El-Labany et al. [25] have studied DAWs having trapped ion distribution and dust charge fluctuation in unmagnetized dusty plasmas. They found [25] that the dynamics of DAWs are governed by a modified Korteweg–de Vries (mKdV) equation. The dust charge fluctuation leads to a decrease of the soliton amplitude.

Mamun et al. [26] stated that the nonlinear propagation of DAWs in a strongly coupled liquid state dusty plasma with a vortex-like ion distribution is governed by an mKdV equation. Later, Alinejad [27] found that the dust temperature, resonant ions, and equilibrium free electron density significantly change the regions of the existence of large-amplitude DAWs. Younsi and Tribeche [28] have investigated the nonlinear localized dust acoustic waves in a charge varying dusty plasma with trapped ions [28]. They calculated the trapped ion charging current based on the orbit motion limited (OML) approach.

On the other hand, the amplitude modulation of waves propagating in nonlinear dispersive media is a unique nonlinear phenomenon that is relevant to many areas including physics and technology [29].The term "modulational instability" (MI) has been firstly described by Gailitis [30] and Vedenov and Rudakov [31]. Actually, Gailitis [30] was the first to obtain the threshold for MI and demonstrated that the isotropic spectra of Langmuir oscillations are unstable with respect to density modulation, though the instability growth rate was not derived. This was done by Vedenov and Rudakov [31] for a gas of plasmons. The self-modulation involving second harmonic generation is responsible for the modulational instabilities and possibly the localization of energy via the formation of the envelope of excitations (solitons). The MI of ion-acoustic waves in a relativistic plasma including trapped electrons has been studied [32]. The association between the mKdV solitary wave and the modulationally unstable solitary wave envelope in such plasma model has been discussed. It is found that the trapped electrons modify the nonlinearity behavior of the nonlinear Schrödinger equation (NLS) and gives rise to the propagation of the modulationally unstable ion-acoustic solitary wave. For more details about the history of the MI of plasma waves, we consult the reader to have a look on Vladimirov et al. [29] or Vladimirov and Popel [33]. Popel et al. [34] have developed a universal nonlinear formalism for description of the MI nonlinear effects of random plasma wavepackets. They found that their equations play the same role as the set of coupled equations for the fields of modulational perturbations in the case of a single monochromatic pump wave and the instability of the broad wave spectrum is significantly suppressed comparing with narrow spectrum of the same energy. Later on, Popel [35] has demonstrated that the "long-scale" instability of the wave spectra is generally weaker than the instability of a monochromatic pump wave.

Although dusty plasma physics has much interests and few papers have considered the effect of trapped electrons [36, 37] and dust charge fluctuations [22, 25, 28], no one has considered the effects of the polarization force, trapped ions, and dust charge variation on the MI of DAWs. Therefore, it is worthwhile to present a first study for the MI of these waves propagating in dusty plasma.

The paper is organized in the following fashion. In Section 2, we present the relevant equations governing the dynamics of nonlinear DAWs. Furthermore, an mKdV equation is derived by employing a reductive perturbation technique (RPT) in Section 3. In Section 4, accounting for the trapped ions distribution and using RPT, a modified NLS equation is derived. We discuss the MI of the DAW envelopes in Section 5. Section 6 presents numerical illustrations and discussion.

2. GOVERNING EQUATIONS

We consider a dusty plasma model which consists of extremely massive and highly negatively charged warm dust grains and Boltzmann distributed electrons, together with free and trapped ions. The charge neutrality condition requires $n_{i0} = n_{e0} + Z_{d0}n_{d0}$, where n_{e0}, n_{i0}, and n_{d0} are the unperturbed electron, ion, and dust number densities, respectively, and Z_{d0} is the unperturbed number of charges residing on the dust grain measured in units of the electron charge.

In the presence of low phase velocity of DAWs, the ions number density follows the vortex-like/trapped distribution [19, 20]. The polarization force (F_P) acting on the dust grain is defined as [38–41] $F_P = -Z_d eR(n_i/n_{i0})^{1/2}\nabla\varphi$, where $R = Z_d e^2/(4T_i\lambda_{Di0})$ is a parameter determining the effect of the polarization force and $\lambda_{Di0} = \left[T_i/\left(4\pi e^2 n_{i0}\right)\right]^{1/2}$, with T_i being the ion temperature. For one-dimensional low-frequency DAWs, we have the following dimensionless basic equations [26, 41]:

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial x} + 3\sigma_d n_d\frac{\partial n_d}{\partial x} + Z_d^2 R\left(\frac{n_i}{\mu}\right)^{1/2}\frac{\partial \phi}{\partial x} - Z_d\frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = Z_d n_d + \nu\exp(\sigma\phi) - n_i, \tag{3}$$

$$n_i = \mu\left[1 - \gamma_1\phi - \frac{4}{3}\frac{(1-\beta)}{\sqrt{\pi}}(-\gamma_1\phi)^{\frac{3}{2}} + \frac{1}{2}(\gamma_1\phi)^2\right], \quad (4)$$

where n_d and u_d refer to the number density and the fluid velocity, respectively, and ϕ is the electrostatic potential. Here, $Z_d > 0$ is the variable charge number of the dust grains in units of electron charge, β is a parameter determining the number of trapped ions and its magnitude is defined as the ratio of the free hot ion temperature T_{if} to the hot trapped ion temperature T_{it} (i.e., $|\beta| = T_{if}/T_{it}$), $\beta = 0$ ($\beta = 1$) corresponds to a flat-topped (Maxwellian) ion distribution [22, 25, 42]. Note that a hole could be created in the trapped region, corresponding to an under population of trapped ions which is represented by negative β's (a vortex-like excavated trapped ion distribution) [45]. The following notations are used $\mu = \frac{n_{i0}}{Z_{d0}n_{d0}}, \nu = \frac{n_{e0}}{Z_{d0}n_{d0}}, \quad \sigma_d = \frac{T_d}{Z_{d0}T_{if}}, \quad \sigma = \frac{T_{if}}{T_e}$, and $\gamma_1 = \frac{T_{if}}{T_i}$.

In the standard orbit-limited probe model for the dust grain surface [25, 28, 32, 43–45], the latter is charged by the plasma species currents. Accordingly, the variable dust charge $q_d = -eZ_d$ is determined self consistently by

$$v_d\frac{dq_d}{dx} = I_e + I_i, \quad (5)$$

where I_e and I_i are the average microscopic electron and ion currents grazing the dust grains surfaces. Equation (5) is the additional dynamical equation that is coupled self-consistently to the plasma equations through the plasma currents. It is noted that the characteristic time for dust motion is of the order of tens of milliseconds for micrometer sized grains, while the dust charging time is typically of the order of 10^{-8} s [26, 44]. Within the time of charging, the displacement of the grain is thus negligible compared to the spatial scale of the problem. It follows that the charging process can be treated as a local phenomenon, and the convective term on the left-hand side of charging equation (5) can be neglected [28]. It follows that we have $I_e + I_i \simeq 0$, which can be presented in dimensionless form as

$$e^{\gamma_1\phi+\sigma Z_d} + (-\beta_a + \beta_b Z_d)e^{-\beta\phi} - \beta_c Z_d - \beta_d\phi + \mu^*\left(\frac{1}{\beta}-1\right)\left(-\frac{1}{\beta}-1\right) = 0, \quad (6)$$

where $\beta_a = \frac{\mu^*}{\beta^2}, \quad \beta_b = \frac{\mu^*\Psi_o}{\beta\sigma}, \quad \beta_c = \frac{\mu^*\Psi_0}{\sigma}\left(\frac{1}{\beta}-1\right)$, $\beta_d = \mu^*\left(\frac{1}{\beta}-1\right)$, $\mu^* = \frac{\mu}{\nu}(\sigma\mu_i)^{1/2}$, $\mu_i = \frac{m_e}{m_i}$, and Ψ_0 is the equilibrium surface potential on the dust grain. Also, it is noted that, we will restrict ourselves to the case of $1 > \beta > 0$.

3. NONLINEAR DAWs AND DERIVATION OF THE mKdV EQUATION

In order to study the dynamics of small-amplitude DAWs attached to variable charge grains, we derive an evolution equation from the system of Eqs. (1–4) and (6) using a RPT [22, 25, 42]. Introducing the stretched coordinates [42] $\xi = \varepsilon^{1/4}(x - \lambda t)$ and $\tau = \varepsilon^{3/4}t$, where ε is a small parameter and the physical variables expanded as $n_d = 1 + \varepsilon n_{d1} + \varepsilon^{3/2}n_{d2} + \ldots$, $u_d = \varepsilon u_{d1} + \varepsilon^{3/2}u_{d2} + \ldots$, $Z_d = 1 + \varepsilon Z_{d1} + \varepsilon^{3/2}Z_{d2} + \ldots$, and $\phi = \varepsilon\phi_1 + \varepsilon^{3/2}\phi_2 + \ldots$, we get

$$\lambda = \sqrt{3\sigma_d + \left[(1-\tilde{R})/(\nu\sigma + \mu\gamma_1 + \gamma^*)\right]}, \quad (7)$$

where $\gamma^* = \left[\beta(\beta_b - \beta_a) + \beta_d - \sigma\right]/\left[\sigma(1+\sigma) + \beta_b - \beta_c\right]$ and $\tilde{R} = \frac{R}{\sqrt{\mu}}$. Putting $\sigma_d = 0$ and $\beta = 1$, Eq. (7) agrees exactly with Xie et al. [13] and with El-Labany and El-Taibany [22] by ignoring the polarization force effect. It is obvious that the inclusion of the polarization force decreases the velocity and the contrary occurs with the effect of the ion trapping parameter.

To next order of ε, the charging current equation leads to $Z_{d2} = \gamma^*\phi_2$. Following the regular procedure of the RPT [22, 25, 42], one can easily obtain the mKdV given by

$$\frac{\partial\phi_1}{\partial\tau} + A\sqrt{-\phi_1}\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} = 0, \quad (8)$$

where $A = -\frac{2B\mu(1-\beta)\gamma_1^{3/2}}{\sqrt{\pi}}$ and $B = \frac{1}{2\sqrt{(\nu\sigma + \mu\gamma_1 + \gamma^*)\left[3\sigma_d(\nu\sigma + \mu\gamma_1 + \gamma^*) + (1-\tilde{R})\right]}}$.

The solution of Eq. (8) is a solitary wave solution with the form

$$\phi_1 = -\varphi_0 \operatorname{sech}^4(\grave{\eta}/W_1), \quad (9)$$

where $\varphi_0 = (15u_0/8A)^2$ is the amplitude of the solitary wave, $W_1 = (16B/u_0)^{1/2}$ is the width of the solitary wave, $\grave{\eta} = \xi - u_0\tau$, and $\grave{\eta}$ is the transformed coordinate with respect to a frame moving with a constant velocity u_0. It is obvious from Eq. (9) that the allowed DAW is of rarefactive type independent of the sign of the nonlinear coefficient A.

4. DERIVATION OF THE MODIFIED NLS EQUATION AND THE OSCILLATORY WAVE SOLUTION

In order to seek for the oscillatory wave solution and study the stability of the wave envelope in the small wavenumber region, we apply the following expansion. The real dependent variable ϕ_1 is expanded as [32]

$$\phi_1 = \sum_{n=0}^{\infty} \tilde{\mu}^n \phi^{(n)}(\eta,\zeta), \qquad (10)$$

$$\phi^{(n)}(\eta,\zeta) = \sum_{l=-\infty}^{\infty} \phi_l^{(n)}(\eta,\zeta)\exp[il(k\xi-\omega\tau)], \qquad (11)$$

where the fact that $\phi^{(n)}(\eta,\zeta)$ is real requires that the condition $\phi_l^{(n)} = \phi_{-l}^{(n)*}$ be satisfied and $\phi_l^{(n)}(\eta,\zeta)$ presents a slowly varying complex amplitude. We select a scaling

$$\eta = \tilde{\mu}^{1/4}(\xi - \chi\tau), \quad \zeta = \tilde{\mu}^{1/2}\tau, \qquad (12)$$

and apply Eqs. (10) and (11) to mKdV equation (8), where $\tilde{\mu}$ is a smallness parameter. If we expand the frequency $\omega(k)$ in a Taylor series around the wavenumber k_0, where $k = k_0 + \Delta k$, the phase factor of the modulational envelope wave depends only on Δk. If we select the order of $\Delta k \sim O(\varepsilon^{1/4})$, the new scaling, Eq. (12), is valid in this system. The frequency ω and $\chi = d\omega/dk$ are determined later.

Since we are interested in the modulation of the plane wave with the frequency ω and the wavenumber k, $\phi_l^{(1)}$ can set equal to zero for all l except $l = \pm 1$. When $n = 0$, then $\phi_l^{(0)} = 0$, because there are no higher harmonic wave components. The $l = 0$ component implies that the plane wave does not exist from Eq. (11).

To the first order in $\tilde{\mu}$, the terms $l = \pm 1$ require,

$$\omega = -Bk^3, \qquad (13)$$

and to the second order in $\tilde{\mu}$, the terms $l = \pm 1$ leads to,

$$\chi = -3k^2 B. \qquad (14)$$

To the third order in $\tilde{\mu}$, the $l = 1$ term implies a modified NLS equation (MNLSE)

$$i\frac{\partial \phi_1^{(1)}}{\partial \zeta} - 3kB\frac{\partial^2 \phi_1^{(1)}}{\partial \eta^2} - Ak\left|\phi_1^{(1)}\right|^{1/2}\phi_1^{(1)} = 0. \qquad (15)$$

In order to obtain the solution of Eq. (15), we rewrite it using the following transformations:

$$\phi_1^{(1)} = \varphi/B_1^2, \quad \eta = \sqrt{(3k/2)}\eta', \quad \zeta = \zeta'/(2B), \qquad (16)$$

where $B_1 = (Ak)/(2B)$. Substitution of expressions (16) into Eq. (15) yields

$$i\frac{\partial \varphi}{\partial \zeta'} - \frac{\partial^2 \varphi}{\partial \eta'^2} - |\varphi|^{1/2}\varphi = 0. \qquad (17)$$

In order to obtain the solution of (17), we replace φ by

$$\varphi = \Psi(v')\exp[i(K\eta' - \Omega\zeta')], \qquad (18)$$

where $\Psi(v')$ is a slowly varying real amplitude and $v' = \eta' - s\zeta'$. Then, the solution of Eq. (17) is

$$\varphi = \Phi_0^2 \operatorname{sech}^4\left[\sqrt{\frac{\Phi_0}{20}}(v' - \nu_0)\right]\exp[i(K\eta' - \Omega\zeta')], \qquad (19)$$

where $\Phi = \sqrt{\Psi}$ and $\Phi_0 = \frac{5}{4}\left(\Omega + K^2\right)$.

Returning to original expressions (16), we hence obtain an envelope solitary wave as the oscillatory solution

$$\phi_1^{(1)}(\Psi_0, v') = \frac{\Psi_0}{B_1^2}\operatorname{sech}^4\left\{\sqrt{\frac{\sqrt{\Psi_0}}{30k}}\left[\eta - sB\sqrt{6k}\zeta - \nu_0\right]\right\} \times \exp\left\{i\sqrt{\frac{2}{3k}}\left[Kx - B\sqrt{6k}\Omega\zeta\right]\right\}, \qquad (20)$$

where $s = -2\sqrt{-\Omega + \frac{4}{5}\Psi_0^{1/2}}$.

The phase factor in the sech^4 function of expression (20) gives the velocity V of the envelope solitary wave. Hence, we obtain

$$V = -\frac{1}{\lambda R^2}\sqrt{6k\left(-\Omega + \frac{4}{5}\sqrt{\Psi_0}\right)}, \qquad (21)$$

and, thereby, the velocity of the solitary wave is proportional to $\Psi_0^{1/4}$. On the basis of expansion (10), the perturbed DAW component for $n = 1$ and $l = 1$ is reduced to

$$\phi_1 = \phi_1^{(1)}\left\{\tilde{\mu}^{1/4}\left[x - \left(\lambda + 3k^2B\right)t\right], \tilde{\mu}^{1/2}t\right\} \times \exp\left\{ik\left[x - \left(\lambda - k^2B\right)t\right]\right\}. \qquad (22)$$

The linear dispersion relation is obtained as (in the low-frequency limit):

$$\omega = k\left\{\lambda - k^2B\right\}. \qquad (23)$$

5. THE STABILITY OF THE OSCILLATORY WAVE SOLUTION

In order to investigate the stability of the envelope solitary wave, we define the coefficients and the potential of the MNLS equation (15) as

$$p = \frac{1}{2}\frac{\partial^2 \omega}{\partial k^2} = -3kB, \quad q = -Ak\sqrt{\left|\phi_1^{(1)}\right|}, \quad \phi \equiv \phi_1^{(1)},$$

where ω is given by Eq. (23). We consider the sum of three waves, i.e., carrier wave $\phi_0(\zeta) = \phi_0 \exp(-i\Omega_0\zeta)$, and two small sidebands at $k \pm K$ and $\omega \pm \Omega$ [32]. Then, ϕ is represented as

$$\phi = \phi_0(\zeta)\left\{1 + \phi_+ \exp[i(K\eta - \Omega\zeta)] + \phi_-^* \exp[-i(K\eta - \Omega^*\zeta)]\right\}. \quad (24)$$

Substituting expression (24) into Eq. (15), we find

$$\begin{bmatrix} \Omega - K\frac{\partial\omega}{\partial k} - pK^2 - \frac{1}{2}|\phi_0|\frac{\partial q}{\partial|\phi_0|} & -\frac{1}{2}|\phi_0|\frac{\partial q}{\partial|\phi_0|} \\ \frac{1}{2}|\phi_0|\frac{\partial q}{\partial|\phi_0|} & \Omega - K\frac{\partial\omega}{\partial k} + pK^2 + \frac{1}{2}|\phi_0|\frac{\partial q}{\partial|\phi_0|} \end{bmatrix}\begin{bmatrix}\phi_+ \\ \phi_-\end{bmatrix} = 0. \quad (25)$$

Setting the determinant equals to zero, we obtain the nonlinear dispersion relation

$$\left[\Omega - K\left(\frac{\partial\omega}{\partial k}\right)\right]^2 = p\left(-|\phi_0|\frac{\partial q}{\partial|\phi_0|} + pK^2\right)K^2. \quad (26)$$

When the nonlinearity is stronger than the dispersion, which corresponds to $|\phi_0|\frac{\partial q}{\partial|\phi_0|} > pK^2$, $p|\phi_0|\frac{\partial q}{\partial|\phi_0|}$ is positive. In this case, the right hand side of Eq. (26) is negative, Ω becomes complex, and, thereby, sidebands turn out to be unstable.

When K and $\mathrm{Re}(\Omega)$ satisfy the conditions $K_{\max} = \sqrt{\frac{|\phi_0|}{2p}\frac{\partial q}{\partial|\phi_0|}}$ and $\Omega_{\max} = K_{\max}\left(\frac{\partial\omega}{\partial k}\right)$.

The maximum growth rate is obtained as $\Gamma_{\max} = \frac{1}{2}|\phi_0|\frac{\partial q}{\partial|\phi_0|}$, where $K_{\max}$ and $\Omega_{\max}$ are the maximum of wavenumber and the maximum of frequency, respectively.

6. DISCUSSION AND CONCLUSION

A study of small but finite amplitude DAWs in a system consisting of three components: extremely massive and highly negatively charged warm dust grains, Boltzmann distributed electrons, and free and trapped ions has been carried out using the RPT. We have analyzed the MI and the dependences of the system physical parameters on the polarization term R, the trapping ion parameter, β and the dust charge variation presented through the parameter γ^*.

The results obtained from this investigation, can be summarized as listed below.

Figure 1 shows the variation of the angular frequency, ω, against β, R, and γ^* changes. They reveal that ω increases (decreases) as β (R or γ^*) increases.

The variations of the coefficient of the dispersion term, $p(= -3kB)$, of the MNLSE, Eq. (15), against k, β, and R changes are displayed in Fig. 2. It shows that p is always negative, which agrees exactly with previous studies [26] and [45]. By increasing β, p increases. Whereas, p decreases as R increases as illustrated in Fig. 2b. On the other side, the variation of the term

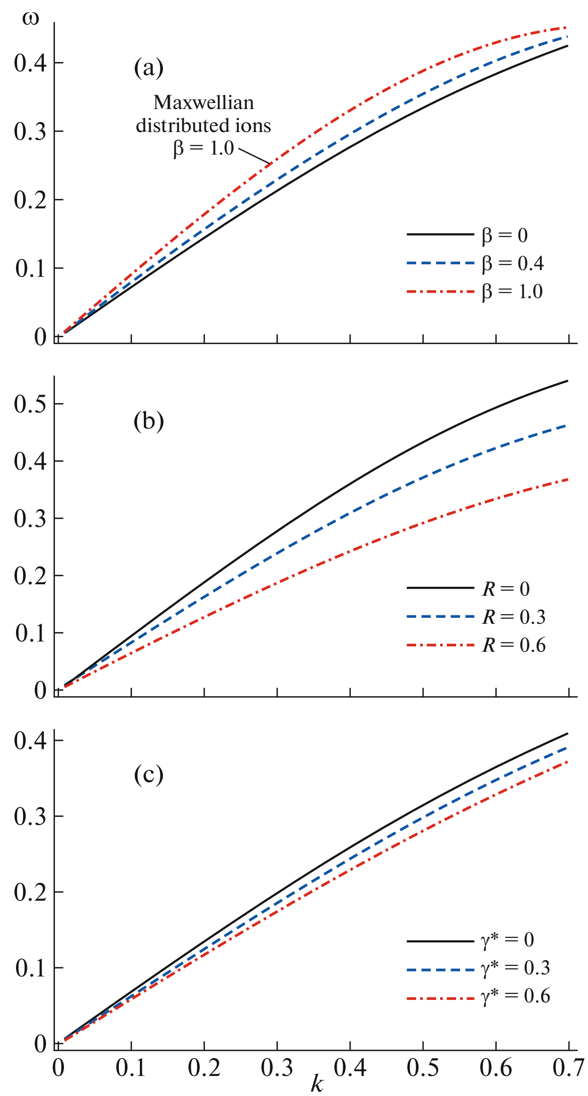

Fig. 1. (Color online) Variation of ω, against k (a) for R = 0.4, $\mu = 3$, $v = 2$, $\sigma_d = 0.02$, and different values of β; (b) for $\mu = 3$, $v = 2$, $\sigma_d = 0.02$, $\beta = 0.3$, and different values of R; (c) for $\mu = 3$, $v = 2$, $\sigma_d = 0.02$, $\beta = 0.3$, $R = 0.4$, and different values of γ^*.

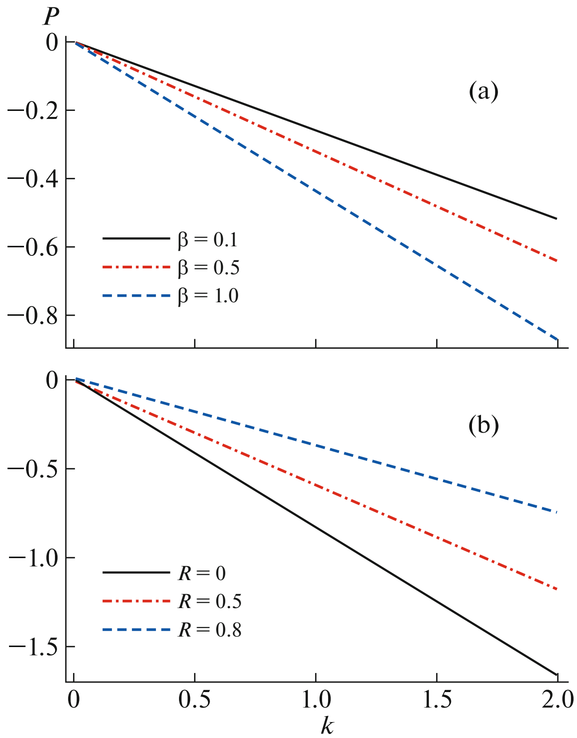

Fig. 2. (Color online) Variation of the dispersion coefficient, $P(\equiv p)$, against k (a) for $R = 0.4$, $\mu = 3$, $v = 2$, $\sigma_d = 0.02$, and different values of β; (b) for $\mu = 3$, $v = 2$, $\sigma_d = 0.02$, $\beta = 0.3$, and different values of R.

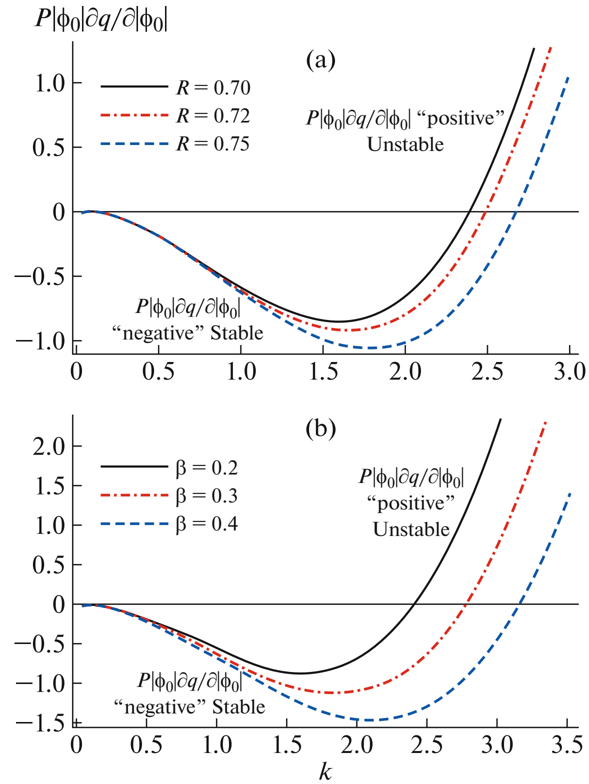

Fig. 3. (Color online) Variation of the product $\left(p|\phi_0|\frac{\partial q}{\partial|\phi_0|}\right)$ against k (a) for $\mu = 3$, $v = 2$, $\sigma_d = 0.01$, and different values of R; (b) for $\mu = 3$, $v = 2$, $\sigma_d = 0.01$, and different values of β.

$p|\phi_0|\frac{\partial q}{\partial|\phi_0|}$ elucidates the existence of transition points (stability boundary) where $p|\phi_0|\frac{\partial q}{\partial|\phi_0|}$ changes its sign. These points are the critical points that determine the stable/unstable regions as shown in Fig. 3. The term $\left(p|\phi_0|\frac{\partial q}{\partial|\phi_0|}\right)$ has an essential role in determining the stability boundaries; $\left(p|\phi_0|\frac{\partial q}{\partial|\phi_0|}\right) < [>]\ 0$ corresponds to stable [unstable] region for DAW envelope [32]. As depicted in Fig. 4, the left region is enhanced by increasing R and β. However, the right region is shifted to be appeared at larger k when raising either R or β. The critical wave number, k_c, is plotted against k for different values of R and β. This is shown in Fig. 5. It is obvious that increasing R leads to an increase of k_c, while increasing β has the opposite effect.

Moreover, an illustration of variation of the MI maximum growth rate, Γ_{max}, against k for R and β changes is provided in Fig. 6. It illustrates that Γ_{max} decreases as R increases, while Γ_{max} increases with the increase of the trapping parameter β as illustrated in Fig. 6b.

To conclude, we have shown that there is a link between the mKdV solitary wave and a MNLSE solitary wave envelop. The effect of trapped ions is dominant in the relation between the frequency and wave number in the dispersion relation. We have examined the dependence of the MI on the system physical parameters; the ion trapping parameter β, and dust charge variation γ^*. The numerical illustrations show that these parameters have strong influences on the nature of DAWs.

Concerning the MI of the MNLSE, it is found that:

(i) The wave angular frequency increases (decreases) as the trapping parameter (polarization term or dust charge number) increases.

(ii) The dispersion coefficient of the MNLSE is always negative and by increasing the trapping parameter, it increases. Though, it decreases as the polarization term increases.

(iii) The transition borders (stability boundary) that determine the stable/unstable regions are strongly

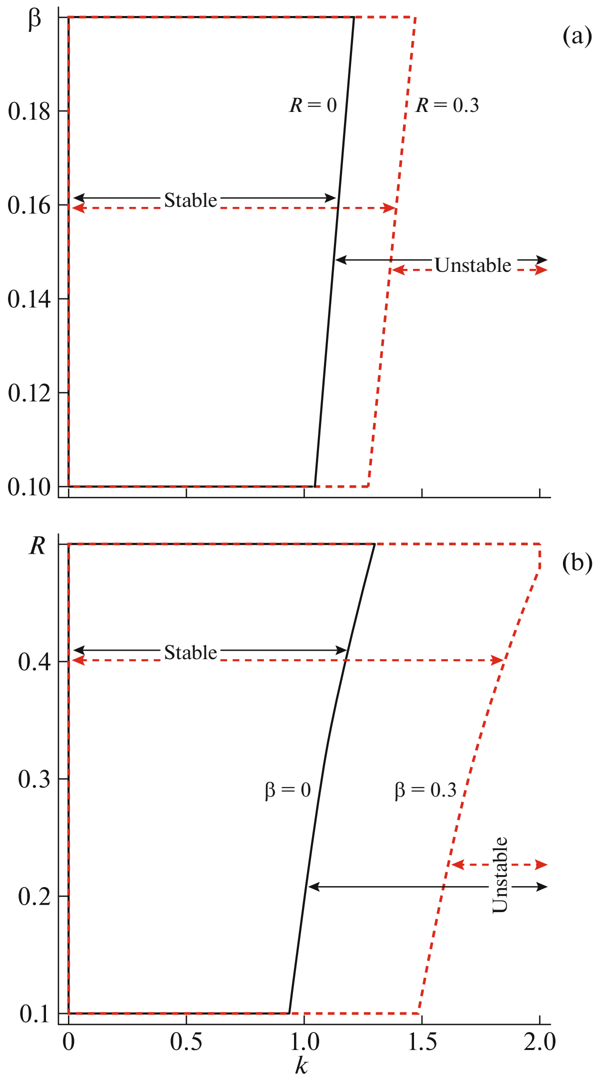

Fig. 4. (Color online) Contour plot $\left(p|\phi_0|\frac{\partial q}{\partial|\phi_0|}\right)$ in the "k–β" domain for different values of R in panel (a) and in the "k–R" domain for different values of β is presented in panel (b).

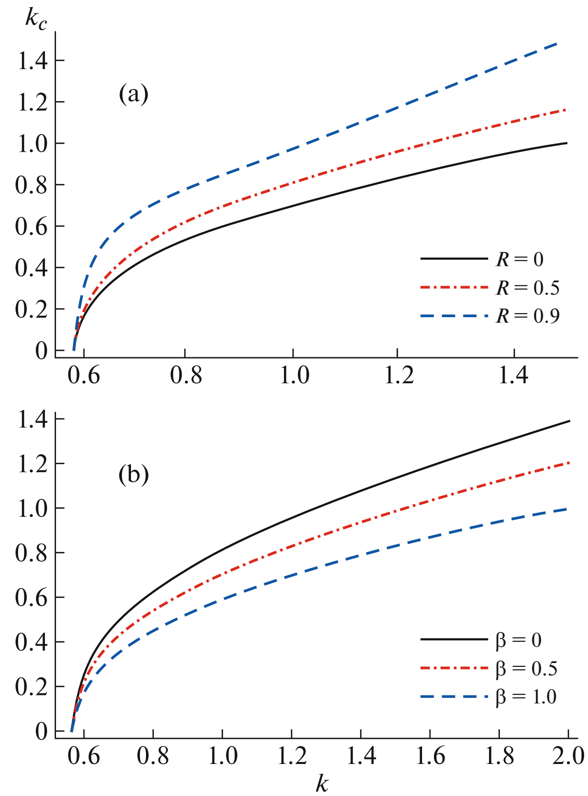

Fig. 5. (Color online) Variation of the critical wave number k_c against k (a) for $\mu = 3$, $v = 2$, $\beta = 0.15$, and different values of R; (b) for $R = 0.2$ and different values of β.

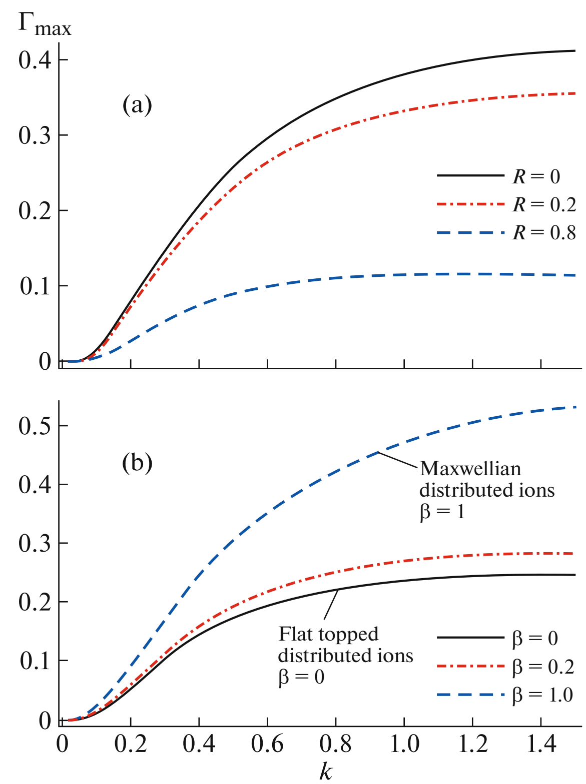

Fig. 6. (Color online) Variation of Γ_{max} against k (a) for $\mu = 3$, $v = 2$, $\beta = 0.5$, and different values of R; (b) for $R = 0.2$ and different values of β.

influenced by both of the polarization and the ion trapping parameter, i.e., the stable region (corresponding to smaller wavenumber values) is enhanced by increasing R and β. However, the unstable region (appeared at higher wavenumber values) is shifted to appear at larger wavenumber by raising either polarization or trapping parameter.

(iv) Increasing the polarization force leads to an increase of the critical wave number while increasing the trapping parameter has the opposite effect.

(v) Finally, the MI maximum growth rate decreases as the polarization force increases, whereas, it increases with the increase of the trapping parameter.

The present results should also help us to understand the basic features of localized DAW propagating in the space and laboratory dusty plasmas. Also these results may be applicable to the investigation of nonlinear modulation phenomena of energetic DAWs associated with trapped ions propagating in space plasmas [19, 23]. Moreover, Popel [46] have studied recently a related problem, where he got the theoretical values of the lower hybrid wave energy threshold that agreed well with the results of the Freja experiment. His result concludes that the formation of localized wave structures in the Earth's magnetosphere is indeed associated with the development of modulational processes.

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