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Modulated ion acoustic waves in a plasma with Cairns-Gurevich distribution

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The amplitude modulation of ion acoustic envelope solitary waves in the presence of Cairns-Gurevich distributed electrons has been investigated. Using a reductive perturbation technique, a modified nonlinear Schrödinger equation has been derived. The modulational instability (MI) and its dependence on the system physical parameters and the combined effects of trapped and nonthermal electrons have been analyzed. It is found that the MI maximum growth rate increases (decreases) as the nonthermality (trapping) parameter increases. The present results could be applicable in explaining the basic features of localized electrostatic disturbance in space observations such as the solar energetic particle flows in interplanetary space and the energetic particle events in the Earth's magnetosphere and also in the laser plasma interaction. *Published by AIP Publishing.*
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I. INTRODUCTION

Recently, the nonlinear wave propagation in plasmas has become one of the most important subjects in plasma physics.[1] One of these waves is the nonlinear ion acoustic (IA) wave whose phase velocity lies between the electron and ion thermal velocities. The nonlinear Schrödinger equation (NLSE) could describe the dynamic of the modulated IA wavepacket, where the resulting solutions of the NLSE have an envelope structure called envelope solitons.[2] The nonlinear wave propagation in a dispersive medium is generically subject to amplitude modulation due to the carrier wave self-interaction or intrinsic nonlinearity of the medium. Modulational instability (MI) is an important phenomenon in connection with wavepacket propagation. Few works have been reported in recent years on the MI and formation of envelope solitons in electron ion plasma including simultaneously the effects of nonthermality and trapping of electrons. The nonlinear interaction of the adiabatic particle motion with finite-amplitude IA waves (IAWs) in an unmagnetized plasma is analyzed using a NLSE.[3] It is found that the IAWs remain modulationally stable and the nonlinear wave can propagate in the form of dark envelope solitons.[3] Using a reductive perturbation technique (RPT), Ju-Kui *et al*.[4] have derived a NLSE to study the MI of finite amplitude IAWs in unmagnetized warm plasmas. They[4] reported that the inclusion of the ion temperature enhances the IAW stability and changes the produced soliton structures.

The MI of relativistic IAWs in a plasma with trapped electrons has been studied by Nejoh.[5] He investigated the association between the modified Korteweg de Vries (mKdV) equation and the modulationally unstable NLSE solitary waves. Recently, Shalini *et al*.[1] have studied the amplitude modulation of IA wavepackets in an unmagnetized electron ion plasma in the context of the Tsallis nonextensive statistics by deriving the corresponding NLSE equation. It is remarked that the stable and unstable regions are determined corresponding to different regimes of temperature and density ratios and the thermodynamic state of plasma species. Using a standard RPT, a NLSE that describes the nonlinear evolution of IAWs in a magnetized electron-positron-ion plasma with q-nonextensive distributed electrons and positrons has been discussed by Ghosh and Banerjee.[6] They showed that the excitation of both bright and dark envelope solitary structures is possible in that model.

On the other side, the particle trapping is a commonly observed phenomenon in both space and laboratory plasmas due to the phase space holes formed as a result of the hot electrons trapping in the wave potential.[7,8] Such particles exhibit more complicated shapes showing high-energy tails as in the weakly collisional corona and solar wind acceleration region.[9,10] For instance, the electron's nonthermal distribution which is characterized by high-energy tails is present in astrophysical plasma environments such as solar wind, ionosphere, auroral zone, and also laboratory plasmas.[11–13] The appropriate distribution function for these nonthermal species is known as Cairns distribution.[14]

As possible approximation, many studies assume that the velocities of electrons are in local thermal equilibrium, i.e., Maxwellian distributed species, known to be isotropically distributed around the average velocity, while, for expanded plasmas produced by laser plasma experiments, it is reported that the high mobility light electrons escape faster into vacuum compared to heavier particles, thus generating a self-consistent ambipolar electric field.[15–17] Thus, the electron distribution function is non-Maxwellian. During electron evolution, their interaction with the IA potential is possible, and then, trapped electrons are produced in the wave potential. Clearly, electrons depart from the Boltzmann distribution, and hence, a hybrid nonthermal trapped electron exists.[18,19] This hybrid distribution is observed during the acceleration mechanisms of electrons and ions in the context of laser-plasma acceleration.[20] For that reason, the combination of two effects,

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viz., trapping and nonthermality on the self-similar expansion of the laser plasma, has been studied.[21] Their influence on ion acceleration and production of well collimated multi-MeV ion beams is a very active and interesting topic in laser and plasma physics.[22] Bara *et al.*[21] have depicted that the presence of a sufficient number of non-energetic trapped electrons in the plasma potential wells slows down the plasma expansion, whereas the presence of energetic electrons makes the influence of the trapping effect on the self-similar expansion very weak even in the case where a very small number of energetic electrons are presented.

Until now, the properties of nonlinear IAW envelope structures in two component plasmas considering the Cairns-Gurevich (CG) distribution of electrons through a NLSE framework have not been investigated. Furthermore, a particular question is to be answered: how do MI, angular frequency, group velocity, and the IA envelope solitons are influenced by the incorporated new effects: the combined effects of nonthermal and trapped electrons. Therefore, it is expected that the answer of this question could lead to a significant improvement for understanding the observed nonlinear wave MI.[21] Therefore, the purpose of the present paper is to make a detailed study of MI of IAWs in an electron ion plasma, including the CG distribution of electrons.

This paper is organized in the following fashion. In Sec. II, we present the relevant equations governing the dynamics of nonlinear IAWs. Accounting for the CG distribution[21] and using a RPT, a modified NLSE is derived in Sec. III. In Sec. IV, we investigate the effects of CG density distribution on the MI of the IAW envelopes. The conclusion is presented in Sec. V.

II. GOVERNING EQUATIONS

We consider the nonlinear propagation of IAWs in a collisionless unmagnetized plasma consisting of electrons obeying the CG distribution and cold fluid ions.[21] The dynamics of the IAWs in such a plasma are governed by the following equations:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u)=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{\partial \varphi}{\partial x}=0, \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2}=n_e-n_i, \tag{3}$$

where n_i and u represent the normalized ion density and velocity, respectively. φ is the electrostatic potential. n_i is normalized by the unperturbed density n_o and u scaled by the ion sound speed $C_s=\left(\frac{k_B T_e}{m}\right)^{1/2}$, φ by $\frac{k_B T_e}{e}$, and the space and time variables are in units of the Debye length $\lambda_D=\left(\frac{\epsilon_o k_B T_e}{n_o e^2}\right)^{1/2}$, and the inverse plasma frequency $\omega_p^{-1}\left[\omega_p=\left(\frac{n_o e^2}{\epsilon_o m}\right)^{1/2}\right]$. Here, T_e is the electron temperature and m is the ion mass. The CG equation for electrons in the small amplitude limit, viz., $\varphi\ll 1$, takes the following form:[21]

$$n_e=\left(1-b\varphi+2b\varphi^2\right)\left[1+\varphi-\frac{4(1-\beta)}{3\sqrt{\pi}}(\varphi)^{\frac{3}{2}}+\frac{1}{2}\varphi^2\right]. \tag{4}$$

It is noted that based on the Gurevich distribution function,[18,23] Bennaceur-Doumaz *et al.*[24] have investigated the effect of trapped electrons on the electrostatic potential arising during the plasma expansion process. Their work is used to explain the salient features of the plasma expansion produced by laser ablation. The CG distribution involves two important indices b and β; β is a parameter determining the number of trapped electrons and its magnitude is defined as the ratio of the free electron temperature, T_{ef}, to the trapped electron temperature T_{et}, i.e., $\beta=T_{ef}/T_{et}$. $\beta=0$ $(\beta=1)$ corresponds to a flat-topped (Maxwellian) electron distribution,[25,26] and the nonthermal parameter, b, defined as $b=4\alpha/(3\alpha+1)$, where α is a parameter determining the population of nonthermal electrons in our plasma model.

III. AMPLITUDE MODULATION OF IA NONLINEAR WAVES

In this section, we study the amplitude modulation of nonlinear IAWs propagating in the proposed plasma medium. For this purpose, we introduce the following slow variables:

$$\xi=\epsilon(x-\lambda t) \quad \text{and} \quad \tau=\epsilon^2 t, \tag{5}$$

where ϵ is a small positive parameter characterizing the bandwidth of superposed waves. λ is the group velocity to be determined later. The field variables are assumed to be functions of the fast variables (x, t) and the slow variables (ξ, τ). It appears convenient to write $n_i=1+n$. Now, we expand the field quantities in a power series of ϵ as follows:[27]

$$\begin{pmatrix} n \\ u \\ \varphi \end{pmatrix}=\epsilon^4\begin{pmatrix} n^{(1)}+\epsilon n^{(2)}+\epsilon^2 n^{(3)}+\cdots, \\ u^{(1)}+\epsilon u^{(2)}+\epsilon^2 u^{(3)}+\cdots, \\ \varphi^{(1)}+\epsilon\varphi^{(2)}+\epsilon^2\varphi^{(3)}+\cdots. \end{pmatrix}. \tag{6}$$

Introducing Eqs. (5) and (6) into the basic Eqs. (1)–(3) and setting the coefficients of like powers of ϵ equal to zero, the following sets of differential equations are obtained. To the lowest order of ϵ for the field equations; O(ϵ^4),

$$\begin{aligned} &\frac{\partial n^{(1)}}{\partial \tau_o}=-\frac{\partial u^{(1)}}{\partial \xi_o}, \quad \frac{\partial u^{(1)}}{\partial \tau_o}=-\frac{\partial \varphi^{(1)}}{\partial \xi_o}, \\ &\frac{\partial^2 \varphi^{(1)}}{\partial \xi_0^2}=(1-b)\varphi^{(1)}-n^{(1)}. \end{aligned} \tag{7}$$

To the next order in ϵ; O(ϵ^5), we get

$$\frac{\partial n^{(2)}}{\partial \tau_o}+\frac{\partial u^{(2)}}{\partial \xi_o}=\lambda\frac{\partial n^{(1)}}{\partial \xi}-\frac{\partial u^{(1)}}{\partial \xi}, \tag{8}$$

$$\frac{\partial u^{(2)}}{\partial \tau_o}+\frac{\partial \varphi^{(2)}}{\partial \xi_o}=\lambda\frac{\partial u^{(1)}}{\partial \xi}-\frac{\partial \varphi^{(1)}}{\partial \xi}, \tag{9}$$

$$\frac{\partial^2 \varphi^{(2)}}{\partial \xi_0^2}-(1-b)\varphi^{(2)}+n^{(2)}=-2\frac{\partial^2 \varphi^{(1)}}{\partial \xi_0 \partial \xi}. \tag{10}$$

If we continue to the next order in ϵ; $O(\epsilon^6)$, we obtain

$$\frac{\partial n^{(3)}}{\partial \tau_o}+\frac{\partial u^{(3)}}{\partial \xi_o}=\lambda\frac{\partial n^{(2)}}{\partial \xi}-\frac{\partial u^{(2)}}{\partial \xi}-\frac{\partial n^{(1)}}{\partial \tau}, \tag{11}$$

$$\frac{\partial u^{(3)}}{\partial \tau_o}+\frac{\partial \varphi^{(3)}}{\partial \xi_o}=\lambda\frac{\partial u^{(2)}}{\partial \xi}-\frac{\partial \varphi^{(2)}}{\partial \xi}-\frac{\partial u^{(1)}}{\partial \tau}, \tag{12}$$

$$\frac{\partial^2 \varphi^{(3)}}{\partial \xi_0^2}-(1-b)\varphi^{(3)}+n^{(3)} = -\left[\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2}+2\frac{\partial^2 \varphi^{(2)}}{\partial \xi_0 \partial \xi}+\frac{4(1-\beta)}{3\sqrt{\pi}}\left(\varphi^{(1)}\right)^{3/2}\right]. \tag{13}$$

Regarding Eq. (7), we seek for a solution of the form

$$\begin{pmatrix} n^{(1)} \\ u^{(1)} \\ \varphi^{(1)} \end{pmatrix} = \begin{pmatrix} N^{(1)}(\xi,\tau) \\ U^{(1)}(\xi,\tau) \\ \phi^{(1)}(\xi,\tau) \end{pmatrix} \exp(i\theta)+c.c, \text{ where } \theta=\omega t-kx; \tag{14}$$

here, ω is the angular frequency and k is the wave number. $N^{(1)}, U^{(1)}$, and $\phi^{(1)}$ are complex functions of the slow variables (ξ, τ), and $c.c$ stands for the complex conjugate of the corresponding quantities. Solving Eq. (7) for $U^{(1)}$ and $N^{(1)}$ with the aid of Eq. (14), we get

$$\begin{pmatrix} U^{(1)} \\ N^{(1)} \end{pmatrix} = \frac{k}{\omega}\begin{pmatrix} 1 \\ \frac{k}{\omega} \end{pmatrix}\phi^{(1)}. \tag{15}$$

The linear dispersion relation is obtained as

$$\omega = k\sqrt{\frac{1}{k^2+1-b}}. \tag{16}$$

In Fig. 1, the variation of the angular frequency, ω, against k is plotted for different values of the nonthermal parameter, b. ω increases with the increase in the nonthermal parameter b which can be extracted also from Eq. (16). ω approaches unity at higher values of k. Equation (16) tells us that the linear phase velocity, $v_{ph} = \omega/k$, is independent of the electron trapped parameter, β. With the exclusion of the ion temperature ($\equiv \sigma$) in Ref. 4 and omitting b in the present work, one can find that the two results are in well agreement. Furthermore, in the presence of nonthermal electron distribution, Eq. (16) coincides exactly with that obtained in Ref. 28 by neglecting σ_i. Until now, $\phi^{(1)}(\xi,\tau)$ is an unknown complex function whose governing equation will be obtained later. With the aid of the field solutions proposed in Eq. (14) and Eqs. (8)–(10), we get

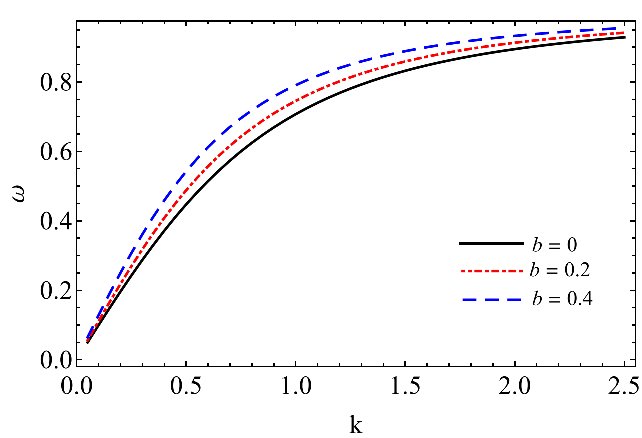


FIG. 1. Variation of the angular frequency, ω, against the wave number k for different values of the nonthermal parameter, b.

$$\frac{\partial n^{(2)}}{\partial \tau_o}+\frac{\partial u^{(2)}}{\partial \xi_o}+c.c.=\frac{\lambda k^2}{\omega^2}\frac{\partial \phi^{(1)}}{\partial \xi}e^{i\theta}-\frac{k}{\omega}\frac{\partial \phi^{(1)}}{\partial \xi}e^{i\theta}, \tag{17}$$

$$\frac{\partial u^{(2)}}{\partial \tau_o}+\frac{\partial \phi^{(2)}}{\partial \xi_o}+c.c.=\frac{\lambda k}{\omega}\frac{\partial \phi^{(1)}}{\partial \xi}e^{i\theta}-\frac{\partial \phi^{(1)}}{\partial \xi}e^{i\theta}, \tag{18}$$

$$\frac{\partial^2 \varphi^{(2)}}{\partial \xi_0^2}-(1-b)\varphi^{(2)}+n^{(2)}+c.c.=2ik\frac{\partial \phi^{(1)}}{\partial \xi}e^{i\theta}. \tag{19}$$

For the second perturbed quantities, $n^{(2)}, u^{(2)}$, and $\varphi^{(2)}$, we seek a solution in the form

$$\begin{pmatrix} n^{(2)} \\ u^{(2)} \\ \varphi^{(2)} \end{pmatrix} = \begin{pmatrix} N^{(2)}(\xi,\tau) \\ U^{(2)}(\xi,\tau) \\ \phi^{(2)}(\xi,\tau) \end{pmatrix} \exp(i\theta)+c.c. \tag{20}$$

From Eqs. (17)–(19) and with the help of Eq. (20), we get the following equations:

$$i\left(\omega N^{(2)}-kU^{(2)}\right)=\frac{k}{\omega}\left(\frac{\lambda k}{\omega}-1\right)\frac{\partial \phi^{(1)}}{\partial \xi}, \tag{21}$$

$$i\left(\omega U^{(2)}-k\phi^{(2)}\right)=\left(\frac{\lambda k}{\omega}-1\right)\frac{\partial \phi^{(1)}}{\partial \xi}, \tag{22}$$

$$-(k^2+1-b)\phi^{(2)}+N^{(2)}=2ik\frac{\partial \phi^{(1)}}{\partial \xi}, \tag{23}$$

whose solutions for the second order perturbed quantities are given by

$$N^{(2)}=\frac{k^2}{\omega^2}\phi^{(2)}+\frac{2ik}{\omega^2}\left(-\frac{\lambda k}{\omega}+1\right)\frac{\partial \phi^{(1)}}{\partial \xi}, \tag{24}$$

$$U^{(2)}=\frac{k}{\omega}\phi^{(2)}+\frac{i}{\omega}\left(-\frac{\lambda k}{\omega}+1\right)\frac{\partial \phi^{(1)}}{\partial \xi}, \tag{25}$$

$$2ik\left(-\frac{\lambda k}{\omega^3}+\frac{1-b}{k^2}\right)\frac{\partial \phi^{(1)}}{\partial \xi}=0. \tag{26}$$

The coefficient of $\frac{\partial \phi^{(1)}}{\partial \xi}$ appeared in Eq. (26) must vanish in order to have a non-zero solution for $\phi^{(1)}$. Therefore, the group velocity, λ, is given by

$$\lambda=\frac{\omega^3(1-b)}{k^3}=\frac{\partial \omega}{\partial k}. \tag{27}$$

Figure 2 shows the variation of λ against k for different values of the nonthermal parameter b. For smaller values of k, λ decreases with the increase in k, but it increases with an

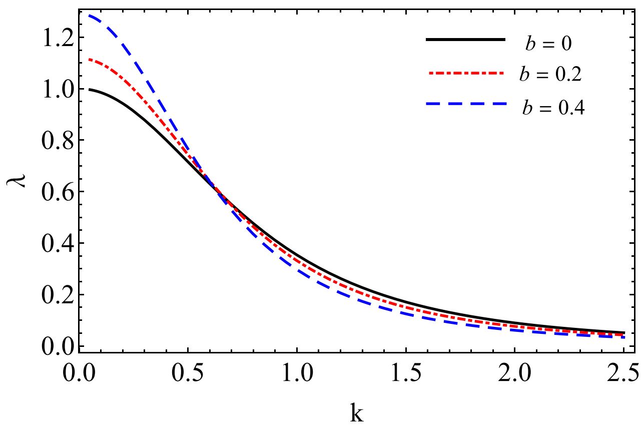


FIG. 2. Variation of the group velocity, λ, against the wave number k for different values of the nonthermal parameter, b.

increment of b. This behavior continues until reaching a certain value of $k(\cong 0.6)$, and then, λ turns to decrease by increasing b.

To obtain the solution of $\mathrm{O}(\epsilon^6)$ equations, we use Eq. (16) and the solutions (24) and (25) into Eqs. (11)–(13) to obtain

$$\frac{\partial n^{(3)}}{\partial \tau_o}+\frac{\partial u^{(3)}}{\partial \xi_o}+\frac{k}{\omega}\left(-\frac{\lambda k}{\omega}+1\right)\frac{\partial \phi^{(2)}}{\partial \xi}e^{i\theta} +\frac{i}{\omega}\left(-\frac{\lambda k}{\omega}+1\right)\left(1-\frac{2\lambda k}{\omega}\right)\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}e^{i\theta} +\frac{k^2}{\omega^2}\frac{\partial \phi^{(1)}}{\partial \tau}e^{i\theta}+c.c.=0, \tag{28}$$

$$\frac{\partial u^{(3)}}{\partial \tau_o}+\frac{\partial \varphi^{(3)}}{\partial \xi_o}-\frac{i\lambda}{\omega}\left(-\frac{\lambda k}{\omega}+1\right)\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}e^{i\theta} +\left(-\frac{\lambda k}{\omega}+1\right)\frac{\partial \phi^{(2)}}{\partial \xi}e^{i\theta}+\frac{k}{\omega}\frac{\partial \phi^{(1)}}{\partial \tau}e^{i\theta} +c.c.=0, \tag{29}$$

$$\frac{\partial^2 \varphi^{(3)}}{\partial \xi_0^2}+\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}e^{i\theta}-2ik\frac{\partial \phi^{(2)}}{\partial \xi}e^{i\theta}-(1-b)\varphi^{(3)} +n^{(3)}+\frac{4(1-\beta)}{3\sqrt{\pi}}(\phi^{(1)})^{3/2}+c.c=0. \tag{30}$$

Since we concern with the equations containing the coefficients of $\exp(i\theta)$ terms only, we have to examine the term $(\phi^{(1)})^{3/2}$, which can be rewritten as suggested by Demiray[27]

$$(\phi^{(1)})^{\frac{3}{2}}=|\phi^{(1)}|^{\frac{3}{2}}|e^{i(s+\theta)}+e^{-i(s+\theta)}|^{\frac{3}{2}} =2\sqrt{2}|\phi^{(1)}|^{3/2}|\cos^{3/2}(s+\theta)|, \tag{31}$$

where $|\phi^{(1)}|$ refers to the $\phi^{(1)}$ modulus and s is the argument of the complex variable $\phi^{(1)}$. In order to ensure that $(\phi^{(1)})^{3/2}$ remains real, we must have the condition $|(s+\theta)|\leq \pi/2$.

Now, we return back to Eqs. (28)–(30) and introduce a solution in the form $(n^{(3)},u^{(3)},\varphi^{(3)})=\sum_{n=1}^{\infty}\left(N^{3n},U^{3n},\phi^{3n}\right)$ $\exp(in\theta)$. Then, we can extract the following equations (related to the coefficients of $\exp(i\theta)$ terms) as follows:

$$i(\omega N^{31}-kU^{31})=\frac{k}{\omega}\left(\frac{\lambda k}{\omega}-1\right)\frac{\partial \phi^{(2)}}{\partial \xi}+\frac{i}{\omega}\left(\frac{\lambda k}{\omega}-1\right) \times\left(1-\frac{2\lambda k}{\omega}\right)\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}-\frac{k^2}{\omega^2}\frac{\partial \phi^{(1)}}{\partial \tau}, \tag{32}$$

$$i(\omega U^{31}-k\phi^{31})=\frac{i\lambda}{\omega}\left(-\frac{\lambda k}{\omega}+1\right)\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}+\left(\frac{\lambda k}{\omega}-1\right) \times\frac{\partial \phi^{(2)}}{\partial \xi}-\frac{k}{\omega}\frac{\partial \phi^{(1)}}{\partial \tau}, \tag{33}$$

$$-(k^2+1-b)\phi^{31}+N^{31}=-\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}+2ik\frac{\partial \phi^{(2)}}{\partial \xi} -0.87\frac{(1-\beta)}{\sqrt{\pi}}|\phi^{(1)}|^{\frac{1}{2}}\phi^{(1)}. \tag{34}$$

Eliminating N^{31}, U^{31}, ϕ^{31} among Eqs. (32)–(34), with the use of dispersion relation, Eq. (16), and the definition of λ, Eq. (27), we finally reduce these equations to the following modified NLSE:

$$i\frac{\partial \phi^{(1)}}{\partial \tau}+P\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}+Q|\phi^{(1)}|^{\frac{1}{2}}\phi^{(1)}=0, \tag{35}$$

where

$$P=\frac{\omega^3}{2k^2}\left[\frac{\lambda k}{\omega^3}\left(-\frac{\lambda k}{\omega}+1\right)-\frac{1}{\omega^2}\left(-\frac{\lambda k}{\omega}+1\right)\left(1-\frac{2\lambda k}{\omega}\right)+1\right], \tag{36}$$

and the nonlinearity coefficient

$$q=0.43\frac{\omega^3}{k^2}\frac{(1-\beta)}{\sqrt{\pi}}\sqrt{|\phi^{(1)}|}\,(\equiv Q|\phi^{(1)}|^{\frac{1}{2}}). \tag{37}$$

The variations of the coefficient of the dispersion term, P, against k and b are displayed in Fig. 3. It is shown that P is always positive irrespective of the values of k and the plasma parameters. This result agrees exactly with that done by Demiray.[27] Also, at smaller values of k, it is depicted that P increases as b increases until reaching a certain value of

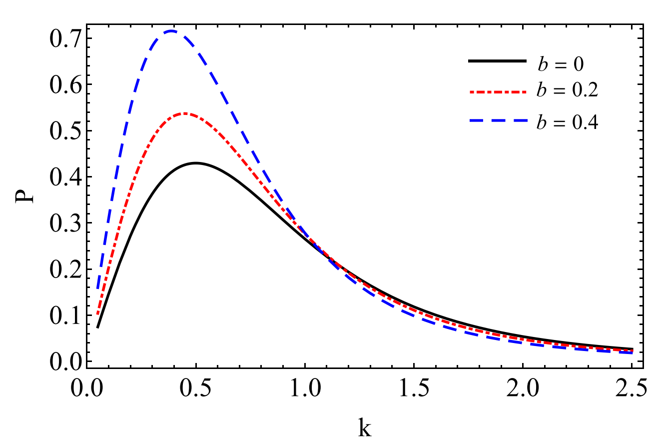


FIG. 3. Variation of the dispersion coefficient, P, against the wave number k for different values of the nonthermal parameter, b.

$k(\cong 1.1)$, and then, P begins to decrease with an increase in b. In addition, P increases as k (for smaller values of k) until reaching a maximum peak, and then, it decreases corresponding to raising the values of k (for higher k values).

IV. STABILITY OF THE OSCILLATORY WAVE SOLUTION

In order to investigate the stability of the produced envelope solitons, we follow the analysis presented by Nejoh[5] and El-Labany *et al.*[29] Considering the sum of three waves, i.e., carrier wave $\phi_0(\xi) = \phi_0 \exp(-i\Omega_0\xi)$, with frequency shift Ω_0 and two small sidebands at $k \pm K$ and $\omega \pm \Omega$.

$$\phi^{(1)} = \phi_o(\xi)\{1 + \phi_+ \exp[i(K\xi - \Omega\tau)] + \phi_-^* \exp[-i(K\xi - \Omega^*\tau)]\}. \tag{38}$$

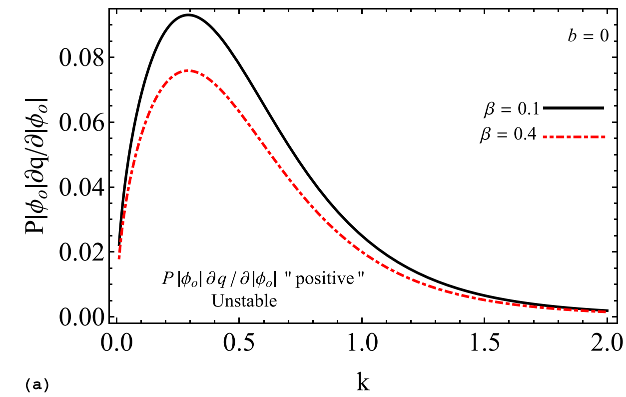
Substituting (38) into the modified NLSE, we obtain the nonlinear dispersion relation

$$\left[\Omega - K\left(\frac{\partial\omega}{\partial k}\right)\right]^2 = P\left(-|\phi_o|\frac{\partial q}{\partial|\phi_o|} + PK^2\right)K^2. \tag{39}$$

When the nonlinearity is stronger than the dispersion $(|\phi_o|\frac{\partial q}{\partial|\phi_o|} > PK^2)$, $P|\phi_o|\frac{\partial q}{\partial|\phi_o|}$ is positive. In this case, the right hand side of (39) is negative, Ω becomes complex, and thereby, sidebands turn out to be unstable. As seen, the term $\left(P|\phi_o|\frac{\partial q}{\partial|\phi_o|}\right)$ has an essential role in determining the stable boundaries; for $\left(P|\phi_o|\frac{\partial q}{\partial|\phi_o|}\right) < [>]\ 0$, the stable (unstable) region for the IAW envelope exists,[5,29] while, in the present investigation, the variation of the term $P|\phi_o|\frac{\partial q}{\partial|\phi_o|}$ elucidates the existence of the unstable region as shown in Fig. 4. As depicted in Fig. 4, β and b affect strongly the amplitude of the term $P|\phi_o|\frac{\partial q}{\partial|\phi_o|}$ not the stability domain. When K and $Re(\Omega)$ satisfy the conditions, $K_{max} = \sqrt{\frac{|\phi_o|}{2P}\frac{\partial q}{\partial|\phi_o|}}$ and $\Omega_{max} = K_{max}\left(\frac{\partial\omega}{\partial k}\right)$, the maximum growth rate is obtained as $\Gamma_{max} = \left[\frac{1}{2}|\phi_o|\frac{\partial q}{\partial|\phi_o|}\right]$, where K_{max} and Ω_{max} are the maximum wave number and the maximum frequency, respectively. Figure 5 illustrates the dependence of the instability growth rate, Γ_{max}, on b and β. It is clear that Γ_{max} increases with the increase in the nonthermality of electrons, but the increment of the electron trapped parameter reduces the instability growth rate. These results are coincident with that obtained by Ghosh *et al.*[28]

V. CONCLUSION

In the present work, we have investigated in detail the MI of IAW envelopes in an unmagnetized collisionless plasma consisting of ions and electrons obeying a hybrid CG distribution. A modified NLSE with a fractional power of electrostatic potential governing the evolution of these IAW envelopes is derived. It is found that unstable IAW modes only exist in the plasma system under consideration. Concerning the MI of the modified NLSE, we can conclude that



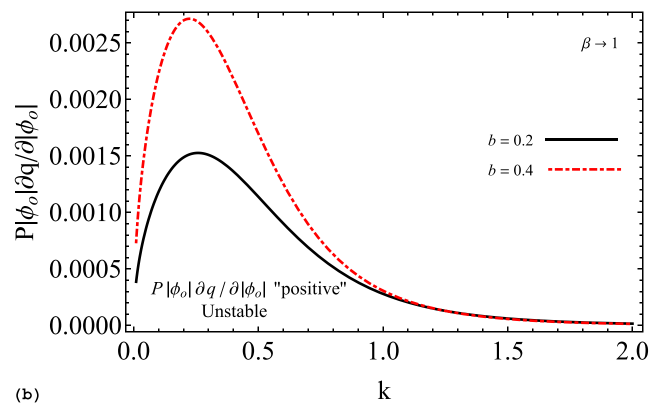


FIG. 4. Variation of the product $(p|\phi_o|\frac{\partial q}{\partial|\phi_o|})$ against k (a) for different values of β where the nonthermal effect is ignored ($b = 0$) and in panel (b) against k for different values of b where $\beta \to 1$ is presented.

(i) The wave angular frequency increases as the nonthermality parameter increases. The group velocity decreases with the increase in k, and it increases (decreases) with an increment of b (for smaller values of k) [(for higher values of k)], while the trapping parameter does not contribute to both the linear wave frequency and the group velocity.

(ii) In this model, the dispersion coefficient of the modified NLSE is always positive. The sign of the term $P|\phi_o|\frac{\partial q}{\partial|\phi_o|}$ (which determines the unstable wave regime) is always

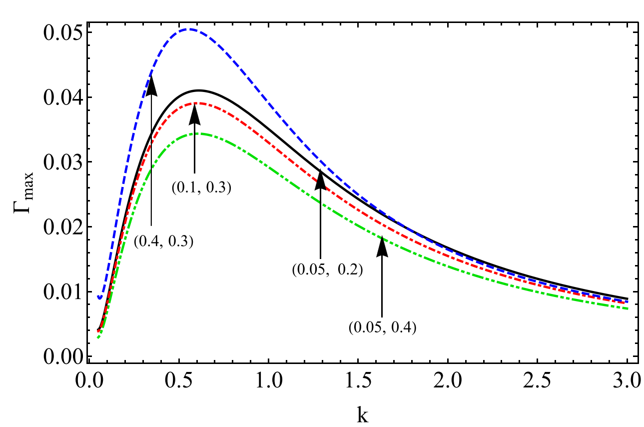


FIG. 5. The growth rate of instability Γ_{max} is plotted against the wave number of modulation k for fixed values of nonthermal and trapped parameters (b, β), respectively.

positive. Therefore, the unstable mode only exists in the presence of CG distribution. In the presence of the trapped (non−energetic) electrons, it is found that increasing the trapping parameter leads to a decrease in the amplitude $p|\phi_o|\frac{\partial q}{\partial|\phi_o|}$. Whereas, in the presence of the energetic (nonthermal) electrons, it is depicted that the nonthermal parameter has the inverse effect on the amplitude of $p|\phi_o|\frac{\partial q}{\partial|\phi_o|}$. Both nonthermal and trapping parameters in the CG distribution have no effect on the domain of the unstable region.

(iii) The MI maximum growth rate increases (decreases) as the nonthermality (trapping) parameter increases.

Finally, the present investigation would be helpful to understand the basic features of localized IA perturbations propagating in laboratory plasmas such as laser-plasma interaction[21] and also in the space plasma: pulsar magnetosphere, the auroral zone, and the upper ionosphere, where plasmas with trapped and energetic electrons are often present.[6,15]

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