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Langmuir oscillations in a nonthermal nonextensive electron-positron plasma

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The high-frequency Langmuir-type oscillations in a pure pair plasma are studied using Vlasov-Poisson's equations in the presence of hybrid nonthermal nonextensive distributed species. The characteristics of the Langmuir oscillations, Landau damping, and growing unstable modes in a nonthermal nonextensive electron-positron (EP) plasma are remarkably modified. It is found that the phase velocity of the Langmuir waves increases by decreasing (increasing) the value of nonextensive (nonthermal) parameter, q (α). In particular, depending on the degree of nonthermality and nonextensivity, both damping and growing oscillations are predicted in the proposed EP plasma. It is seen that the Langmuir waves suffer from Landau damping in two different q regions. Furthermore, the mechanism that leads to unstable modes is established in the context of the nonthermal nonextensive formalism, yet the damping mechanism is the same developed by Landau. The present study is useful in the regions where such mixed distributions in space or laboratory plasmas exist. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4976128]

I. INTRODUCTION

The study of electron-positron (EP) plasma has been highlighted in the past three decades by plasma physicists. Such time-space parity disappears when studying a pure pair plasma. Obviously, pair plasmas consisting of electrons and positrons have attracted a special interest because of their significant applications in astrophysics situations such as active galactic nuclei,[1] pulsar and neutron star magnetosphere,[2] solar atmosphere,[3] accretion disk,[4] black holes,[5] the early universe,[6] and many others. For example, the detections in radio sources suggest that extragalactic radio jets are composed mainly of an EP plasma.[7] Additionally, the creation of EP plasma in pulsars is essentially due to energetic collisions among particles that are accelerated by the electric and magnetic fields exposed to these systems.[8] On the other hand, the successful achievements for the creation of EP plasmas in laboratories have been frequently reported.[9] It has been observed that the annihilation time of EP pairs in typical experiments is often long compared with typical confinement times.[10,11]

Three kinds of electrostatic modes have been observed:[12] the acoustic waves in a relatively low-frequency band, an intermediate-frequency backward like mode, and the Langmuir-type waves in a relatively high frequency band. To our knowledge, a satisfactory and accepted theoretical justification for the explanation of the high-frequency Langmuir oscillations in a pure pair plasma in the presence of nonthermal nonextensive plasma has not been carried out. Here, we will discuss in detail the high-frequency Langmuir oscillations in EP plasma using the kinetic theory model in the context of nonthermal nonextensive species. It is often observed that the physical distribution of particles in space plasmas as well as in laboratory plasmas is not exactly the Maxwellian distribution, and the particles show significant deviations from the thermal distribution.[13–15] The presence of nonthermal particles in space plasmas has been widely confirmed by many spacecraft measurements.[16] In many situations, the velocity distributions show nonMaxwellian tails decreasing as a power-law distribution. Several models for phase-space plasma distributions have deviations from purely the Maxwellian behavior and become rather popular in recent years, such as the nonthermal distribution proposed by Cairns *et al.*[14] which was used to explain the solitary electrostatic structures involving density depletions observed in the upper ionosphere in the auroral zone by the Freja satellite.[17,18] In addition, there are a number of evidences exhibiting the nonextensive statistics which is a good framework for describing certain physical systems, such as galaxy clusters,[19] plasmas,[20] and turbulent systems.[21]

On the other hand, in the presence of nonextensive statistics:[22] Lima *et al.*[23] have studied the Langmuir oscillations and Landau damped waves in a collisionless electron ion plasma in the context of nonextensive statistics. In particular, they have stressed that, due to the long-range nature of Coulombic interactions in the plasma, the standard Maxwell-Boltzmann distribution may provide only a very crude description of such systems, even in the collisionless limit. Also, the nonlinear Landau damping of the electrostatic waves in an unmagnetized collisionless electron ion plasma has been investigated numerically using a semi-Lagrangian Vlasov-Poisson code.[24]

Landau[25] has shown that the resonant interaction of a wave with particles resulted in the collisionless damping of the observed electrostatic waves. Also, Landau damping increases significantly when the distribution functions contain superthermal particles and shoulders in the distribution function profile.[26,27] Furthermore, Liyan and Jiulin[28] have discussed the dispersion relation and Landau damping of

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ion-acoustic waves in a collisionless magnetic-field-free plasma including nonextensive statistics. Particularly, they have emphasized that the physical state described by the q distribution (Tsallis's distribution) is not the thermodynamic equilibrium. Recently, Saberian *et al.*[11,13] have discussed the properties of the Langmuir oscillations[11] and the low-frequency acoustic-like modes,[13] Landau damping, and growing unstable modes in an EP plasma using Vlasov-Poisson's equations in the context of Tsallis's nonextensive statistics. They found that decreasing the nonextensive index q leads to an increment of the phase velocity of the Langmuir waves. Qureshi *et al.*[29] have studied the effect of non-Maxwellian distribution function on Landau damping rates of Langmuir waves when a small hot electron population is present. They found that the departure from Maxwellian distributions significantly alters the damping rates. Strong damping is detected for highly nonMaxwellian distributions as well as for plasmas with a higher density and hot electron population.[29]

In the present work, we will investigate the Langmuir oscillations in a magnetic field-free collisionless EP plasma in the context of the nonthermal nonextensive (α, q) statistics,[15] emphasizing the possible damping and instability. In Sec. II, a kinetic theory model based on Vlasov-Poisson's equations is applied for deriving the dielectric function, $D(k,\omega)$, for longitudinal waves in an unmagnetized pair plasma. We calculate the dispersion relation for the Langmuir oscillations in Sec. III. Also, the real and imaginary parts of the frequency of Langmuir oscillations are obtained and the Landau damping and instability of the Langmuir modes are discussed. Finally, a summary of our results is given in Sec. IV.

II. THE MODEL EQUATIONS

To describe the electrostatic collective modes in an EP plasma, we consider a spatially uniform EP plasma at the equilibrium state. If at a given time $t = 0$, a small amount of charge is displaced in the plasma, the initial perturbation may be described by

$$f_s(t=0) = f_{s0}(\vec{v}) + f_{s1}(\vec{x},\vec{v},t=0), \quad f_{s1} \ll f_{s0}, \tag{1}$$

where f_{s0} corresponds to the unperturbed and time independent stationary distribution. However, f_{s1} is the corresponding perturbation about the equilibrium state, where s stands for electrons and positrons ($s = e$ or p). Assuming that the perturbation is electrostatic and the displacement of charge gives rise to a perturbed electric field but no magnetic field released. A one-dimensional Vlasov-Poisson system is given by[25,30]

$$\frac{\partial f_{e1}}{\partial t} + \vec{v}.\frac{\partial f_{e1}}{\partial \vec{x}} + \frac{e}{m}\vec{\nabla}\phi_1.\frac{\partial f_{e0}}{\partial \vec{v}} = 0, \tag{2}$$

$$\frac{\partial f_{p1}}{\partial t} + \vec{v}.\frac{\partial f_{p1}}{\partial \vec{x}} - \frac{e}{m}\vec{\nabla}\phi_1.\frac{\partial f_{p0}}{\partial \vec{v}} = 0, \tag{3}$$

$$\nabla^2\phi_1 = 4\pi ne\int (f_{e1} - f_{p1})d\vec{v}, \tag{4}$$

where e, m, and n denote, respectively, the absolute charge, mass, and number density of the electron and ϕ_1 is the electrostatic potential produced by the perturbation. This set of linearized equations for perturbed quantities may be solved simultaneously to investigate the plasma properties for the time intervals shorter than the binary collision times. The standard technique for simultaneously solving the differential equations, Eqs. (2)–(4), is the method of integral transforms.[25,30] Another simplified method for studying the longitudinal waves, with the frequency ω and the wave vector $\vec{k}$, is to assume that the solution has the form,[11]

$$f_{s1}(\vec{x},\vec{v},t) = f_{s1}(\vec{v})e^{i(\vec{k}.\vec{x}-\omega t)}, \text{ and } \phi_1(\vec{x},t) = \phi_1 e^{i(\vec{k}.\vec{x}-\omega t)}. \tag{5}$$

We consider the x-axis to be along the direction of the wave vector $\vec{k}$ and thus $v_x = u$. Then, applying Eq. (5) to Eqs. (2)–(4), we get the dispersion relation for longitudinal waves in the proposed EP plasma as follows:

$$D(k,\omega) = 1 - \frac{4\pi ne^2}{mk^2}\int \frac{\frac{\partial}{\partial u}\left(f_{e0}(u) + f_{p0}(u)\right)}{u - \frac{\omega}{k}}du = 0, \tag{6}$$

where $D(k,\omega)$ is the dielectric function of the longitudinal oscillations propagating in the EP plasma. We can investigate the response of the EP plasma to an arbitrary perturbation via the response occurred in the dielectric function $D(k,\omega)$. In general, the frequency ω that satisfies the dispersion relation, Eq. (6), is complex, i.e., $\omega = \omega_r + i\omega_i$. However, in many cases $Re[\omega(k)] \gg Im[\omega(k)]$, and the plasma responds to the perturbation a long time after the initial disturbance with oscillations at a range of the well-defined frequencies. We can determine the normal modes corresponding to Eq. (6). It should be further mentioned that when Vlasov-Poisson's equations are solved as an initial value problem, it is possible to obtain the solutions with negative or positive values of ω_i. This can be explicitly seen from the electrostatic potential associated with the wave number k of the excitation as follows:

$$\phi_1(x,t) = \phi_1 e^{i(kx-\omega_r t)}e^{\omega_i t}, \tag{7}$$

where a solution with negative (positive) ω_i displays a damped wave (unstable mode).

With the constraint of the weak damping or growth, i.e., $\omega_i \ll \omega_r$, the dielectric function $D(k,\omega)$ given in Eq. (6) can be Taylor expanded in the small quantity ω_i, and then we explicitly find the real and imaginary parts of the dielectric function as follows:[11]

$$D_r(k,\omega_r) = 1 - \frac{4\pi ne^2}{mk^2}P.V.\int \frac{\frac{\partial}{\partial u}\left(f_{e0}(u) + f_{p0}(u)\right)}{u - \frac{\omega_r}{k}}du, \tag{8}$$

$$D_i(k,\omega_r) = -\pi\left(\frac{4\pi ne^2}{mk^2}\right)\left[\frac{\partial}{\partial u}\left(f_{e0}(u) + f_{p0}(u)\right)\right]_{u=\frac{\omega_r}{k}}. \tag{9}$$

Here, the analytic continuation of the velocity integral in Eq. (6) has been made over u, along the real axis, which passes under the pole at $u = \frac{\omega}{k}$ with the constraint of weakly damped waves, where $P.V.\int$ denotes the Cauchy principal value.[11,13] By these relations and neglecting the terms of order $(\frac{\omega_i}{\omega_r})^2$, ω_r and ω_i can be computed, respectively, from the relations[30]

$$D_r(k, \omega_r) = 0 \tag{10}$$

and

$$\omega_i = -\frac{D_i(k, \omega_r)}{\partial D_r(k, \omega_r)/\partial \omega_r}. \tag{11}$$

III. LANGMUIR OSCILLATIONS WITH NONTHERMAL NONEXTENSIVE DISTRIBUTION

To obtain the features of the Langmuir waves in the EP plasma in the context of the (α, q) statistics, we assume that the stationary state of the plasma obeys the (α, q) distribution function that is given by[15]

$$f_{s0}(u) = C_{q,\alpha}\left(1 + \alpha\frac{u^4}{u_{ts}^4}\right)\left\{1 - (q-1)\frac{u^2}{2u_{ts}^2}\right\}^{1/(q-1)}, \tag{12}$$

where $u_{ts} = (k_B T_s/m_s)^{1/2}$ is the thermal velocities, $T_s(m_s)$ is the s species temperatures (mass), and

$$C_{q,\alpha} = \begin{cases} n_{s0}\sqrt{\dfrac{m_s}{2\pi T_s}}\dfrac{\Gamma\left(\dfrac{1}{1-q}\right)(1-q)^{\frac{5}{2}}}{\Gamma\left(\dfrac{1}{1-q}-\dfrac{5}{2}\right)\left[3\alpha + \left(\dfrac{1}{1-q}-\dfrac{3}{2}\right)\left(\dfrac{1}{1-q}-\dfrac{5}{2}\right)(1-q)^2\right]}, & for\ -1 < q < 1, \quad (13a) \\ n_{s0}\sqrt{\dfrac{m_s}{2\pi T_s}}\dfrac{\Gamma\left(\dfrac{1}{q-1}+\dfrac{3}{2}\right)(q-1)^{5/2}\left(\dfrac{1}{q-1}+\dfrac{3}{2}\right)\left(\dfrac{1}{q-1}+\dfrac{5}{2}\right)}{\Gamma\left(\dfrac{1}{q-1}+1\right)\left[3\alpha + (q-1)^2\left(\dfrac{1}{q-1}+\dfrac{3}{2}\right)\left(\dfrac{1}{q-1}+\dfrac{5}{2}\right)\right]} & for\ q > 1 \quad (13b) \end{cases}$$

is the constant of normalization. Here, α is a parameter determining the number of nonthermal particles present in our plasma model, q stands for the strength of nonextensivity, and Γ denotes the standard Gamma function. For $q = 1$, the distribution of Cairns *et al.*[14] is recovered. For $q > 1$ and as the nonextensive character of the nonthermality increases, the distribution (12) shoulders become less prominent and high-energy states are less probable than in the extensive nonthermal case.[15] For $q < 1$, high-energy states are more probable than in the extensive case.

It is remarked that the Vasyliunas–Cairns distribution presented in Ref. 31 involves the indices α and κ (which is the superthermality parameter in the plasma system). The Vasyliunas–Cairns distribution function must satisfy $\kappa > 3/2$ and $0 < \alpha < 1$. The empirically derived kappa distribution function in space plasmas is equivalent to the q-distribution function in Tsallis nonextensive formalism, in the sense that the spectrum of the velocity distribution function in both models shows similar behavior. In fact, both the kappa distribution and the Tsallis q-nonextensive distribution describe deviations from the thermal distribution.[13] Particularly, Leubner[32] showed that the nonextensive distribution is very close to the kappa distribution which is a consequence of the generalized entropy favored by the nonextensive statistics, and he proposed a link between the Tsallis nonextensive and the kappa distributions. The relation between the two parameters q to κ is $\kappa = 1/(1-q)$.[32] Furthermore, Livadiotis and McComas[33] examined how kappa distributions arise naturally from the Tsallis statistical mechanics.

Here, we are interested in the high-frequency oscillations with the phase velocity much greater than the thermal speed of the electrons and positrons $\left(\frac{\omega}{k} \gg u\right)$. Then, the Cauchy principal value of Eq. (8) could be evaluated as follows:

$$\begin{aligned} -\int_{-u_{max}}^{+u_{max}} \frac{\frac{\partial}{\partial u}\left(f_{e0}(u) + f_{p0}(u)\right)}{u - \frac{\omega_r}{k}} du &= \frac{k}{\omega_r}\int_{-u_{max}}^{+u_{max}}\left(\frac{\partial f_{e0}(u)}{\partial u} + \frac{\partial f_{p0}(u)}{\partial u}\right) \\ &\quad \times \left(1 + \frac{k}{\omega_r}u + \frac{k^2}{\omega_r^2}u^2 + \frac{k^3}{\omega_r^3}u^3 + \cdots\right) du. \end{aligned} \tag{14}$$

Here, in order to include both $q < 1$ (superextensivity) and $q > 1$ (subextensivity) cases, we have denoted the integration limits in Eq. (14) by $\pm u_{max}$. In fact, the integration limits are unbounded, i.e., $\pm u_{max} = \pm\infty$ for $q < 1$, and they are given by the q dependent thermal cutoff $u_{max} = \sqrt{\frac{2k_B T_s}{m_s(q-1)}}$ for $q > 1$.[13]

In the present study, we shall consider $T_e = T_p = T$ which is in agreement with the experimental works of the EP plasma comprised of particles with the same dynamics.[12] In a different manner, comparing the electron-electron, positron-positron, and electron-positron relaxation time scales reveals that the creation of a pure EP plasma with a considerable difference in the temperature of the pairs is not possible in practice.[34]

Using Eq. (14), the real part of the dielectric function included in Eq. (8) reads as

$$D_r(k,\omega_r) = 1 + \frac{8\pi n e^2}{m}\left(\frac{I_1}{\omega_r^2} + \frac{k^2 I_2}{\omega_r^4}\right), \tag{15}$$

where

$$I_1 = -\frac{\sqrt{\pi}\,C_{q,\alpha}}{4\left(\frac{1-q}{2u_{te}^2}\right)^{5/2}}\left[\frac{\frac{3\alpha}{u_{te}^4}\Gamma\left(-\frac{5}{2}+\frac{1}{1-q}\right) + \frac{(q-1)^2}{u_{te}^4}\Gamma\left(\frac{1+q}{2-2q}\right)}{\Gamma\left(\frac{1}{1-q}\right)}\right]$$

and

$$I_2 = \frac{3\sqrt{\pi}\,C_{q,\alpha}}{8\left(\frac{1-q}{2u_{te}^2}\right)^{7/2}}\frac{\left[\frac{15\alpha}{u_{te}^4}(-1+q)^2 + \frac{(q-1)^2}{4u_{te}^4}(-3+5q)(-5+7q)\right]\Gamma\left(-\frac{7}{2}+\frac{1}{1-q}\right)}{(-1+q)q\,\Gamma\left(\frac{q}{1-q}\right)}.$$

ω_r corresponding to Langmuir waves in a (α, q) EP plasma is obtained by setting $D_r(k,\omega_r) = 0$ that leads to

$$\omega_r = \omega_p\sqrt{\frac{I_1}{2}\left(1 + 3(k\lambda_D)^2\frac{2Z_2}{I_1 Z_1(1-q)}\right)}, \tag{16}$$

where $\omega_p = \left(\frac{8\pi n e^2}{m}\right)^{1/2}$, $Z_1 = \frac{I_1}{\frac{-C_{q,\alpha}\sqrt{\pi}}{4\left(\frac{1-q}{2u_{te}^2}\right)^{5/2}}}$, and $Z_2 = \frac{I_2}{\frac{3\sqrt{\pi}C_{q,\alpha}}{8\left(\frac{1-q}{2u_{te}^2}\right)^{7/2}}}$; λ_D is the Debye length that becomes, in a charge-neutral EP plasma, $\lambda_D^{-2} = \frac{4\pi n e^2}{k_B}\left(\frac{1}{T_e}+\frac{1}{T_p}\right)$. On the other hand, using Eq. (9), we can obtain the imaginary part of the dielectric function as follows:

$$D_i(k,\omega_r) = -\frac{\pi\omega_p^2}{k^2}\left\{\frac{4\alpha C_{q,\alpha}\omega_r^3}{u_{te}^4 k^3}\left[1 - \frac{(q-1)}{2u_{te}^2}\frac{\omega_r^2}{k^2}\right]^{1/(q-1)} - \frac{C_{q,\alpha}\omega_r}{u_{te}^2 k}\left(1 + \frac{\alpha}{u_{te}^4}\frac{\omega_r^4}{k^4}\right)\left[1 - \frac{(q-1)}{2u_{te}^2}\frac{\omega_r^2}{k^2}\right]^{(2-q)/(q-1)}\right\}. \tag{17}$$

Equation (17) is quite compatible with the results of Saberian *et al.*,[11] by considering $\alpha \to 0$ and $q \to 1$. With the help of Eqs. (15) and (17), we can calculate ω_i as follows:

$$\omega_i = -\frac{\pi}{k^2 I_1}\left[\frac{\alpha C_{q,\alpha}\omega_p^6 I_1^3}{4k^3 u_{te}^4}\left(1 - \frac{(q-1)}{2k^2u_{te}^2}\frac{\omega_p^2 I_1}{2}\left\{\left[1 + 3(k\lambda_D)^2\frac{2Z_2}{I_1Z_1(1-q)}\right]\right\}\right)^{1/(q-1)} - \frac{C_{q,\alpha}\omega_p^4 I_1^2}{8ku_{te}^2}\left(1 + \frac{\alpha\omega_p^4 I_1^2}{4k^4u_{te}^4}\right)\right.$$
$$\left.\times\left\{1 - \frac{(q-1)}{2k^2u_{te}^2}\frac{\omega_p^2 I_1}{2}\left[1 + 3(k\lambda_D)^2\frac{2Z_2}{I_1Z_1(1-q)}\right]\right\}^{(2-q)/(q-1)}\right]. \tag{18}$$

Equations (17) and (18) have been derived for the high-frequency oscillations with $\frac{\omega}{k} \gg \left(\frac{2k_BT_s}{m}\right)^{\frac{1}{2}}$. One basic feature of our analysis is the inclusion of the nonthermality, α, and nonextensivity, q, of the system. Therefore, depending on these parameters, both the damping and growth of the electrostatic oscillations may be happening in the proposed EP plasma.

IV. DISCUSSION

Equations (16) and (18) describe the Langmuir oscillations in an EP plasma. Hereafter, we will discuss the dispersion relation and damping or growth of the Langmuir oscillations by analyzing these expressions.

A. Dispersion relation

In Figs. 1(a) and 1(b), we depict the dispersion curve (real part, ω_r) of the Langmuir oscillations showing the influence of the spectral index q and the nonthermal parameter α. It can be seen that, for a given α value, the phase velocity of the

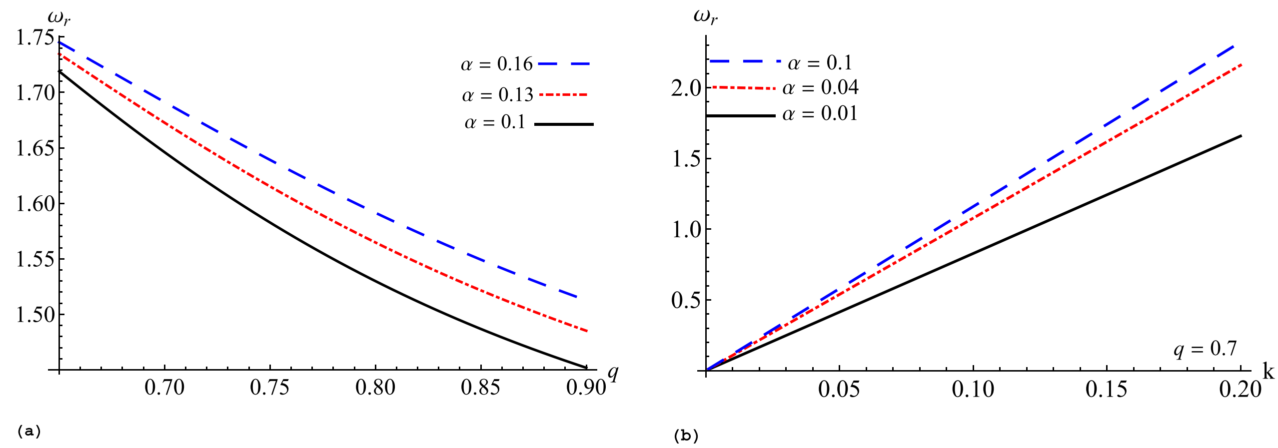

FIG. 1. The effect of the nonextensivity on the dispersion relation of Langmuir waves with $k = 0.05$ in (a), after which (b) the effect of the nonthermality on the dispersion relation of Langmuir waves.

Langmuir waves increases as q decreases. Next, keeping $q = 0.7$, ω_r increases with the increase of α as seen in Fig. 1(b). This coincides with what presented in Refs. 11 and 35. The q distribution function with $q < 1$ indicates that the systems contain more superthermal particles (superextensivity). However, with $q < 1$, the distribution is strongly suggested for the real superthermal plasmas. It is expected that the illustration with $q < 1$ (cf. Fig. 1) is more probable for space plasma systems.[11]

B. Landau damping and growing oscillations

1. Superextensive or superthermal plasmas ($q < 1$)

The Landau damping and growing Langmuir modes in a superextensive EP plasma can be discussed via analyzing the imaginary part of the frequency, given in Eq. (18), for the values of $q < 1$. In Fig. 2(a), we have plotted ω_i against the nonextensivity index q. It is explicitly seen that both the damped ($\omega_i < 0$) and the growing unstable oscillations ($\omega_i > 0$) are predicted in the present EP plasma. Our numerical analysis shows that in two q domains, i.e., $0.31 \lesssim q \lesssim 0.335$ and $0.52 \leq q \leq 0.6$, the longitudinal oscillations are unstable, due to the fact that $\omega' s$ have the positive imaginary parts and then the associated electrostatic modes will grow in time [cf. Eq. (7)].

To interpret the physical mechanism which leads to this instability, we have the nonthermal nonextensive distribution with $q < 1$ which describes a system with a large number of superthermal particles. Therefore, our solution for Vlasov-Poisson's equations with $q < 1$ indicates an evolution which started from a stationary state with a large portion of superthermal particles. The Langmuir modes may gain energy from these superthermal particles and result in growing oscillations in time. In other words, this instability arises from a stationary state that described by a superthermal plasma.

On the other hand, the Langmuir waves have Landau damping in two q domains; $0.335 \leq q \leq 0.52$ and $0.6 \leq q \leq 0.9$, with the nonthermality parameter $\alpha(\equiv 0.4)$. The Landau damping is a resonant phenomenon among the plasma particles (electrons and positrons) and the wave attached to the particles.[25,34] Our analysis reveals that the damping rate in the first q domain, i.e., $0.335 \leq q \leq 0.52$, is heavy and it would disappear after a few periods of time. But in the other region, $0.6 \leq q \leq 0.9$, the Langmuir oscillations are weakly damped. These are the normal modes of the plasma which would persist in several oscillations periods.

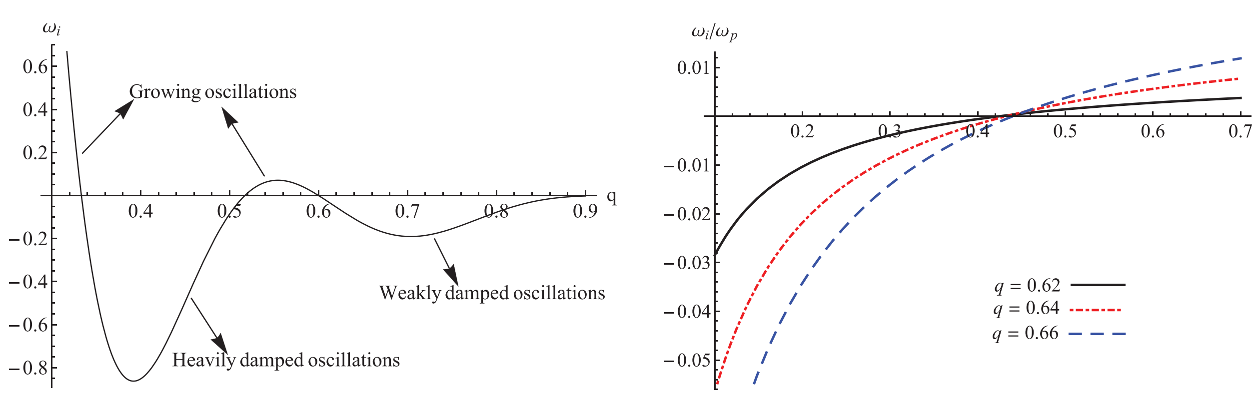

FIG. 2. The Landau damping rates (imaginary part of the frequency) (a) with respect to the nonextensivity index for $q < 1$ (superextensivity), which shows the q regions for damped and growing oscillations with $\alpha = 0.4$ and (b) with respect to the nonthermality parameter α for $q < 1$ (superextensivity).

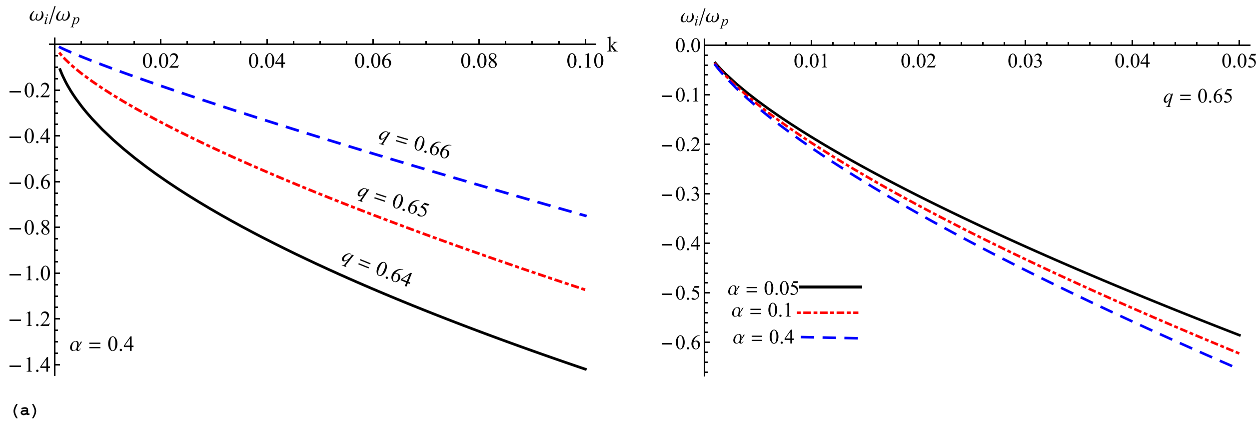

FIG. 3. The imaginary part of the frequency (a) with respect to the wave number k for $q < 1$ (superextensivity) with $\alpha = 0.4$ and (b) with respect to wave number k for some values of nonthermality parameter α.

Also, the influence of the nonthermal parameter on the damping rate is depicted in Figure 2(b) for certain values of the nonextensivity parameter q. In Fig. 2(b), it is seen that the nonthermality parameter affects strongly the behavior of ω_i that determines the stable/unstable regions of the Langmuir oscillations. Furthermore, the corresponding damping rates are shown in Fig. 3 for different values of q and α in Fig. 3(b). Figure 3(a) shows that by increasing q, the amplitude of the damping rate decreases. But, in Fig. 3(b), the damping rate increases with the increase of α for $q(\equiv 0.65)$.

2. Subextensive plasmas ($q > 1$)

We can investigate the Langmuir oscillations in a subextensive EP plasma where a number of low-speed particles are present. In Fig. 4, the imaginary part of the wave frequency is plotted against q for fixed values of α. One can recognize that the Langmuir oscillations suffer from damping only. It is remarked here that our present results agree exactly with those of Ref. 11. Also, it is seen that both the nonextensivity and nonthermality characters affect the damped Langmuir oscillations . In Fig. 5, we have plotted the damping rate versus the wave number k which reveals that the Landau damping increases with q increment.

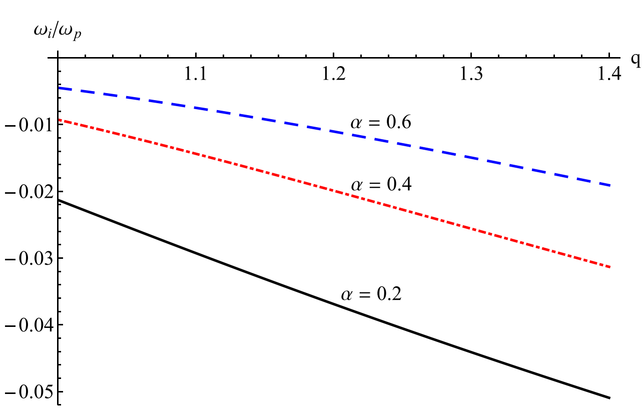

FIG. 4. The imaginary part of the frequency with respect to the nonextensivity index for $q > 1$ (subextensivity) for different values of nonthermality parameter α. For these values of the nonextensivity index q, the Langmuir waves have the damping but no growth behavior.

V. SUMMARY AND CONCLUSIONS

In this work, we have investigated the Langmuir waves in a collisionless and magnetic field free quasineutral plasma composed of electrons and positrons following the hybrid nonthermal nonextensive statistics. We have thereby used the kinetic theory model on Vlasov-Poisson's equations to obtain the corresponding dielectric function of the proposed EP plasma. The dispersion relation and the properties of the Langmuir waves are discussed in detail, and it is shown that the phase velocity of the Langmuir waves increases as the spectral index value decreases. But it increases as the nonthermal value increases.

Furthermore, it is found that depending on the degree of nonextensivity of the plasma appeared through q and nonthermality presented through α, both the damping and growth occur in the present EP plasma, arising from a resonance phenomenon between the wave and the plasma particles. In the case of $q < 1$ (superextensivity), both the damping and growing unstable oscillations have been detected, while in the case of $q > 1$ (subextensivity), the Langmuir oscillations suffering from damping only . Moreover (for $q < 1$), the Langmuir waves have Landau damping in two q domains $0.335 \le q \le 0.52$ and $0.6 \le q \le 0.9$, because $\omega's$ have the negative imaginary parts for these degrees of nonextensivity where $\alpha(\equiv 0.4)$ is kept. The damping rate in the first q region,

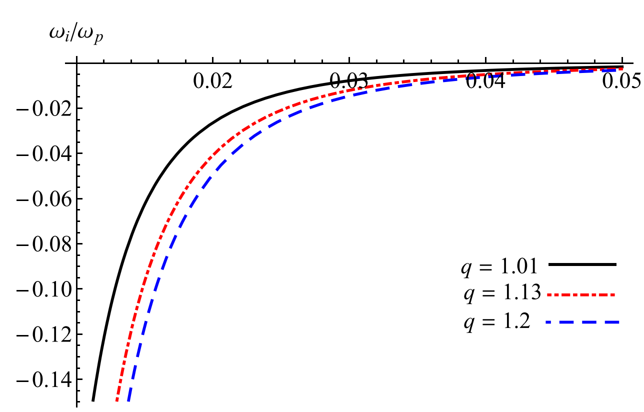

FIG. 5. The imaginary part of the frequency with respect to the wave number k for different values of the nonextensivity index for $q > 1$ (subextensivity).

i.e., $0.335 \leq q \leq 0.52$, is heavy, and the wave disappears after a few periods. But in the other region, $0.6 \leq q \leq 0.9$, the Langmuir oscillations are weakly damped. These are the normal modes of the plasma which would persist in several oscillations periods.

The mechanism which leads to the damping is the same as that developed by Landau,[25] arising from a decreasing distribution function with velocity. The concerned instability can be associated with the presence of numerous superthermal particles (in the case $q < 1$), which may give energy to the wave in the resonance process and results in growing oscillations in time.[11] Additionally, the damping rate in the case $q > 1$ is smaller than that in the case of $q < 1$ because of the difference in the number of the particles participating in the resonance process with the wave. We emphasize that in the present work, we have considered a plasma in a nonequilibrium thermal state by considering the (α, q) distribution for the stationary state of the plasma. Therefore, it is reasonable that our results should differ from those of a homogeneous EP plasma. In fact, the properties of the Langmuir oscillations derived here are suitable for plasmas in a nonequilibrium stationary state with homogeneous temperature which contain many superthermal or low-speed particles. This study would be useful for the explanation of typical modes observed in an EP plasma containing nonMaxwellian species that obey the nonthermal nonextensive statistics. As mentioned in Ref. 36, the product Cairns-Tsallis distribution could potentially be applicable to a wider range of situations involving high-energy nonMaxwellian tails. The present study is useful in the regions where such mixed distributions in space or laboratory plasmas exist.[35,37]

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