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ORIGINAL ARTICLE

Higher-order corrections to nonlinear dust-ion-acoustic shock waves in a degenerate dense space plasma

S.K. El-Labany · W.F. El-Taibany · A.E. El-Samahy · A.M. Hafez · A. Atteya

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Abstract A reductive perturbation technique is employed to investigate the contribution of higher-order nonlinearity and dissipation to nonlinear dust-ion-acoustic (DIA) shock waves in a three-component degenerate dense space plasma. The model consists of degenerate electron (being either ultrarelativistic or nonrelativistic), nonrelativistic ion fluid and stationary heavy dust grains. A nonlinear Burger equation and a linear inhomogeneous Burger-type equation are derived. The present model admits only compressive DIA shocks. Including these higher-order corrections results in creating new solitary wave structures "*humped DIA shock*" waves. For the case of ultrarelativistic (nonrelativistic) electrons, one (two) humped DIA shock is (are) created. The DIA shock wave amplitude and velocity is larger in case of ultrarelativistic electrons than of nonrelativistic electrons. It is shown that the effects of kinematic viscosity, heavy dust grains number density, and equilibrium ion number density have important roles in the basic features of the produced DIA shocks and the associated electric fields. The implications of our results to dense plasmas in astrophysical objects (e.g., non-rotating white dwarf stars) are briefly discussed.

Keywords Higher-order nonlinearity · Dissipation · Dust-ion acoustic shock wave · Degenerate plasma · Ultrarelativistic limit

S.K. El-Labany · W.F. El-Taibany (✉)
Department of Physics, Faculty of Science, Damietta University, P.O. 34517, New Damietta, Egypt
e-mail: eltaibany@du.edu.eg

W.F. El-Taibany
e-mail: eltaibany@hotmail.com

S.K. El-Labany
e-mail: skellabany@hotmail.com

W.F. El-Taibany
Department of Physics, College of Science for Girls in Abha, King Khalid University, P.O. 142, Abha, Saudi Arabia

A.E. El-Samahy · A.M. Hafez · A. Atteya
Department of Physics, Faculty of Science, Alexandria University, Alexandria, Egypt

A. Atteya
e-mail: ahmed_ateya2002@yahoo.com

1 Introduction

The dynamics of ion acoustic (IA) wave, which is a fundamental electrostatic plasma wave mode, have been studied for several decades both theoretically and experimentally (Sagdeev 1966; Washimi and Taniuti 1966). In a dissipative system, an IA shock wave is created due to the balancing between the nonlinearity and different dissipation processes. These IA shocks have received a great deal of attention in the recent plasma studies because of its connection with space-plasma physics observations (Masters et al. 2013). The dissipation in plasmas may be caused by several mechanisms, e.g., the inter-particle interaction, the multi-ion streaming, the Landau damping, the anomalous viscosity, etc. The IA shock was firstly observed in a Q-machine where ion-ion collisions are included (Andersen et al. 1968), and later in a double plasma device where the shock thickness is governed by Landau damping (Taylor et al. 1970). Recently, collisionless IA shocks have been also observed in a Laser-Plasma experiment (Romagnani et al. 2008) and in a double plasma device (Bailung et al. 2008). However, in complex plasma, anomalous dissipation gives the possibility of existence new kind of shock waves that distinguished from those observed in ordinary plasma (Popel et al. 1998). Dust IA (DIA) shock waves were detected in various laboratory plasma experiments (Luo et al. 1999; Nakamura et al. 1999; Nakamura 2002; Fortov et al. 2005; Sarma and Nakamura 2009). For instance, Nakamura et al. (1999) and Nakamura (2002) have observed DIA shock waves in collisional dominated dusty plasmas. Both of monotonic shock [double layer (DL)] and oscillatory shock structures are detected due to the presence of dust grains. It reveals that with increasing the dust particle number density the oscillatory wave structure behind the shock becomes less in number and finally completely disappears. In addition, as the dust number density increases, the shock speed increases. It is predicted that the DIA shock wave would be arising due to ion viscosity process (Nakamura et al. 1999, Ghosh and Bharuthram 2008). On the other side, it is well known that the presence of these static charged dust grains modifies the existing plasma wave spectra (D'Angelo 1990; Shukla and Silin 1992; El-Labany et al. 2012; Samanta et al. 2013; Saha and Chatterjee 2014a). So, dust grains have the vital role in plasma systems and some important aspects can be found when studying the propagation of nonlinear wave structures. Including these heavy particles in ordinary plasma leads to its importance in understanding many related astrophysical situations, such as planetary rings, asteroids, the ionosphere and auroras (Shukla et al. 1999; Yasmin et al. 2012; Mamun and Zobaer 2014; Saha and Chatterjee 2014a). For this reason, we will include an additional static heavy charged particles to make our present investigation more general (Zobaer et al. 2013b, 2013c).

A great deal of interest has been noticed in exploring the basic properties of degenerate or extremely dense matter because of its occurrence in many cosmic environments and compact astrophysical objects (Michel 1982; Tandberg-Hansen and Emslie 1988). Nowadays, most of the recent theoretical investigations are focused on understanding the environment of the compact objects having their interiors supporting themselves via degenerate pressure. Generally the degenerate pressure, arising due to the combine effect of Pauli's exclusion principle and Heisenberg's uncertainty principle, depends only on the number density of constituent particles, but independent on its own temperature (Mamun and Shukla 2010a, 2010b; Roy et al. 2012). This degenerate pressure has an important role in studying the electrostatic perturbation in matters that exist in extreme conditions (Chandrasekhar 1931a, 1931b, 1935; Koester and Chanmugam 1990; Berro et al. 2010). The degenerate state of matter arises at extraordinarily high density (in compact stars) or at extremely low temperatures in the laboratories (Andrew et al. 2001; Zobaer et al. 2013a). Generally the extreme conditions of degenerate matter are caused by significant compression in which high density of degenerate matter is one of these extreme conditions. Electron degenerate pressure will halt the gravitational collapse of a star if its mass is below the Chandrasekhar limit (i.e. 1.44 solar masses; Mazzali et al. 2007). This is the pressure that prevents a white dwarf star from collapsing. A star exceeding this limit and without usable nuclear fuel will continue to collapse to form either a neutron star or black hole because the degenerate pressure provided by the electrons is weaker than the inward pull of gravity. For stellar masses of less than about 1.44 solar masses (Chandrasekhar 1931a, 1931b, 1935), the energy from the gravitational collapse is not sufficient to produce the neutrons of a neutron star, so the collapse is halted by degenerate electron to form white dwarfs. This maximum mass for a white dwarf is called the Chandrasekhar limit (Chandrasekhar 1931a, 1931b, 1935). In case of such a compact object, the degenerate electron number density is so high (in white dwarfs it can be of the order of 10^{28} cm^{-3} or even more; Mamun and Shukla 2010a, 2010b; Roy et al. 2012) that the electron Fermi energy is comparable to the electron mass energy, and as a result the electron speed becomes comparable to the speed of light in vacuum. For such interstellar compact objects, the equation of state for degenerate electrons are mathematically explained by Chandrasekhar (1935) for two limits, named as nonrelativistic and ultrarelativistic limits.

Misra (2009) have derived a Korteweg de-Vries Burger (KdVB) equation to describe the DIA shocks in a quantum dusty plasma taking into account the kinematic viscosity of ion species. While, the temperature effects of the ion species is ignored in that model. It is revealed that the structure of shock wave depends on the viscosity parameters and the ion species density ratio. The necessary condition for the existence of monotonic and oscillatory magentoacoustic shock waves in dispersive degenerate plasma is calculated by Hussain and Mahmood (2011). Zobaer et al. (2013a, 2013b, 2013c) have investigated IA shock waves in a degenerate dense plasma containing nonrelativistic degenerate cold ion fluid with nonrelativistic and ultrarelativistic degenerate electrons. They examined the basic features of the shock structures and pointed out that the shock wave potential increases more rapidly for ultrarelativistic electrons and nonrelativistic degenerate ions in comparison to the other case of where electrons are nonrelativistic degenerate species. The nonlinear propagation of two kinds of the DIA (solitary and shocks) waves in a dusty multi-ion dense plasma (with the constituents being degenerate, either nonrelativistic or ultra-relativistic) has been investigated by Zobaer et al. (2013b) where they derived appropriate Korteweg de-Vries (K-dV) equation and Burger equation. In addition, Zobaer et al. (2013c) have derived a modified Burger equation by using higher stretching coordinates. Very recently, Mamun and Zobaer (2014) have derived a new evolution equation which they call it as "M-Z" equation in their study of DIA shock and DL solutions. This M-Z equation is actually a Gardner equation with an additional dissipative term. They have applied their theoretical results to a non-rotating white dwarf situation.

On the other side, to remove discrepancies between experimentally observed characteristics of IA solitons and theoretical predictions, a number of authors (Tran and Hirt 1974; El-Labany 1993; El-Labany et al. 2005, 2007; Tiwari and Mishra 2006; Tiwari 2009; Chatterjee et al. 2009; Yasmin et al. 2012; Mehdipoor and Esfandyari-Kalejahi 2012; Ghorui et al. 2013) have proved the necessity of including higher-order nonlinearity and dispersion in studying nonlinear plasma waves. The new soliton solutions are called "*dressed soliton*". All of these studies are concerned only with the soliton solutions. Moreover, in a set of research papers (Samanta et al. 2013; Saha and Chatterjee 2014a, 2014b), the bifurcation theory has been employed searching on new wave solutions in dusty plasma models. Various travelling wave solutions; periodic wave, kink and anti-kink waves, have been found for DIA waves (Samanta et al. 2013; Saha and Chatterjee 2014a) and dust acoustic waves (Saha and Chatterjee 2014b). To the best of our knowledge, no investigation has been made to examine the contribution of higher-order nonlinear and dissipation terms through Burger-type equation under extreme condition of matter for both nonrelativistic and ultrarelativistic limits of plasma species that is the main motive of the present study. We consider a degenerate dense plasma model containing nonrelativistic degenerate ion fluid, and degenerate electrons (being nonrelativistic or ultrarelativistic), and stationary negatively charged static dust (as it is possible to be some heavy element in the system). The basic features of the DIA shock waves in such degenerate dense plasma will be studied. Also, the present results will be investigated numerically based on the relevant data of the compact interstellar objects (e.g., white dwarf, neutron star, etc.), Mamun and Shukla (2010b) . The paper is organized as follows. The basic equations governing our plasma system is presented in Sect. 2. A Burger equation is derived, and its stationary shock wave solution is analyzed in Sect. 3. Moreover, a linear inhomogeneous Burger-type equation is derived. The stationary solution of the Burger-type equation is derived in Sect. 4. A brief discussion for the present results and their applications are provided in Sect. 5.

2 Basic equations

We consider the propagation of electrostatic perturbation in a degenerate dense plasma containing nonrelativistic degenerate ion fluid, and electrons (being either nonrelativistic or ultrarelativistic degenerate), and negatively charged static dust to study the basic features of the DIA shock waves in such degenerate dense plasma. The nonlinear dynamics of these electrostatic waves propagating in such a degenerate plasma are governed by (Zobear et al. 2013b, 2013c; Mamun and Zobear 2014)

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_i}\frac{\partial n_i^{\alpha}}{\partial x} + \eta \frac{\partial^2 u_i}{\partial x^2} = 0, \tag{2}$$

$$n_e \frac{\partial \phi}{\partial x} - K_2 \frac{\partial n_e^{\gamma}}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1-\mu) n_e - n_i + \mu, \tag{4}$$

where n_i (n_e) is the ion (electron) number density normalized by its equilibrium value $n_{i0}(n_{e_0})$, u_i is the ion fluid speed normalized by $C_i = (m_e c^2/m_i)^{1/2}$ with m_e (m_i) being the electron (ion) rest mass and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $(m_e c^2/e)$ with e being the magnitude of the electron charge, the time variable t is normalized by $\omega_{pi}^{-1} = (4\pi n_{i0} e^2/m_i)^{-1/2}$, and the space variable x is normalized by $\lambda_s = (m_e c^2/4\pi n_{i0} e^2)^{1/2}$. The kinematic viscosity coefficient η is normalized by $\omega_{pi}\lambda_s^2 m_i n_{i0}$ and $\mu(=Z n_d/n_{i0})$ is the the dust charge number multiplied by the ratio of the dust-to-ion number densities. At equilibrium, we have $n_{e0} + Z n_d = n_{i0}$ where n_d is the equilibrium dust number density. The constants $K_1 = n_{i0}^{\alpha-1} K_i/m_e c^2$ and $K_2 = n_{e0}^{\gamma-1} K_e/m_e c^2$. The ion equation of state (Chandrasekhar 1931a, 1931b, 1935; El-Taibany et al. 2012) is given by

$$P_i = K_i n_i^{\alpha}, \tag{5}$$

where $\alpha = 5/3$ and $K_i = [3\pi\hbar^2 \sqrt[3]{(\pi/3)}]/(5m_i) \simeq 3\Lambda_c \hbar c/5$ for the nonrelativistic limit (where $\Lambda_c = \pi\hbar/m_i c = 6.6 \times 10^{-14}$ cm, and $\hbar$ is the Planck's constant divided by 2π). While for the electron fluid, we set

$$P_e = K_e n_e^{\gamma}. \tag{6}$$

For nonrelativistic limit, $\gamma = \alpha$, however for the ultrarelativistic limit (Chandrasekhar 1931a, 1931b, 1935; El-Taibany and Mamun 2012), we have $\gamma = 4/3$ with $K_e = 3\hbar c(\sqrt[3]{\pi^2/9})/4$. It is noted here that the ion pressure effect is ignored in the study presented by Zobaer et al. (2013c).

3 Burger equations

In this section, we will study the propagation of the electrostatic DIA perturbations in such degenerate dense plasma including the effect of dissipation. At first, let us introduce the stretched coordinates (Zobaer et al. 2013a, 2013b, 2013c; Mamun and Zobaer 2014)

$$\zeta = \epsilon(x - V_p t) \quad \text{and} \quad \tau = \epsilon^2 t, \tag{7}$$

where V_p is the phase velocity of DIA solitary structures. The physical quantities are expanded as power series of ϵ [ϵ is a smallness parameter $(0<\epsilon<1)$]

$$\begin{aligned} n_i &= 1+\epsilon n_i^{(1)}+\epsilon^2 n_i^{(2)}+\epsilon^3 n_i^{(3)}+\cdots, \\ n_e &= 1+\epsilon n_e^{(1)}+\epsilon^2 n_e^{(2)}+\epsilon^3 n_e^{(3)}+\cdots, \\ u_i &= \epsilon u_i^{(1)}+\epsilon^2 u_i^{(2)}+\epsilon^3 u_i^{(3)}+\cdots, \\ \phi &= \epsilon \phi^{(1)}+\epsilon^2 \phi^{(2)}+\epsilon^3 \phi^{(3)}+\cdots \end{aligned} \tag{8}$$

Substituting Eqs. (7) and (8) into the basic set of equations; Eqs. (1)–(4), for the lowest order of $\in$, we have

$$n_i^{(1)} = \phi^{(1)}/\big(V_p^2-\alpha K_1\big), \qquad u_i^{(1)} = V_p\phi^{(1)}/\big(V_p^2-\alpha K_1\big) \quad \text{and} \quad n_e^{(1)} = \phi^{(1)}/\gamma K_2. \tag{9}$$

On the other hand, the dispersion relation of the DIA solitary wave structures in the present degenerate plasma can be derived as,

$$V_p = \sqrt{\frac{\gamma K_2}{1-\mu}+\alpha K_1}, \tag{10}$$

which agrees exactly with Zobaer et al. (2013a, 2013b) and Mamun and Zobaer (2014). It is clear that the presence of heavy dust grains leads to the increment of the phase velocity of the DIA shocks as stated in Luo et al. (1999), Nakamura et al. (1999), Nakamura (2002). From the next order in the perturbation theory, we get

$$\begin{aligned} &\frac{\partial n_i^{(1)}}{\partial \tau} - V_p\frac{\partial n_i^{(2)}}{\partial \zeta} + \frac{\partial u_i^{(2)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\big(n_i^{(1)}u_i^{(1)}\big) = 0, \\ &\frac{\partial u_i^{(1)}}{\partial \tau} - V_p\frac{\partial u_i^{(2)}}{\partial \zeta} + u_i^{(1)}\frac{\partial u_i^{(1)}}{\partial \zeta} + \frac{\partial \phi^{(2)}}{\partial \zeta} + \alpha K_1\frac{\partial n_i^{(2)}}{\partial \zeta} \\ &\quad + \alpha(\alpha-2)K_1 n_i^{(1)}\frac{\partial n_i^{(1)}}{\partial \zeta} + \eta\frac{\partial^2 u_i^{(1)}}{\partial \zeta^2} = 0, \\ &\frac{\partial \phi^{(2)}}{\partial \zeta} + n_e^{(1)}\frac{\partial \phi^{(1)}}{\partial \zeta} - \gamma K_2\frac{\partial n_e^{(2)}}{\partial \zeta} \\ &\quad - \gamma K_2(\gamma-1)n_e^{(1)}\frac{\partial n_e^{(1)}}{\partial \zeta} = 0, \\ &n_e^{(2)} - \frac{n_i^{(2)}}{1-\mu} = 0. \end{aligned} \tag{11}$$

Eliminating the second-order perturbed quantities from Eqs. (11) with the help of Eqs. (9) and (10), one can derive a Burger's equation,

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \zeta} + B\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = 0, \tag{12}$$

where

$$A = \frac{V_p^2-\alpha K_1}{2V_p}\left[\frac{\gamma-2}{\gamma K_2} + \frac{3V_p^2+\alpha(\alpha-2)K_1}{(V_p^2-\alpha K_1)^2}\right] \quad \text{and}$$

$$B = \frac{\eta}{2}.$$

Equation (12) has a DIA shock wave solution as

$$\phi^{(1)} = \frac{2B}{A}(1+\tanh\beta), \tag{13}$$

whose amplitude equals $2B/A$ with $\beta=\zeta-2B\tau$. Furthermore, the second-order perturbed quantities; $n_i^{(2)}$, $n_e^{(2)}$ and $u_i^{(2)}$, can be calculated in terms of $[\phi^{(1)}]^2$ and $\phi^{(2)}$, and given by

$$\begin{aligned} n_i^{(2)} &= \frac{D}{2}\big[\phi^{(1)}\big]^2 + E\phi^{(2)}, \\ n_e^{(2)} &= \frac{1}{\gamma K_2}\left\{\frac{DE}{2}\big[\phi^{(1)}\big]^2 + \phi^{(2)}\right\}, \\ u_i^{(2)} &= \frac{F}{2}\big[\phi^{(1)}\big]^2 + EV_p\phi^{(2)} + \frac{\eta E}{2}\frac{\partial \phi^{(1)}}{\partial \zeta}, \end{aligned} \tag{14}$$

where the coefficients D, E and F are given in the Appendix. Going further to the higher-order perturbation theory, leads to a system of nonlinear differential equations in the third-order perturbed quantities as follows,

$$\begin{aligned} &\frac{\partial n_i^{(2)}}{\partial \tau} - V_p\frac{\partial n_i^{(3)}}{\partial \zeta} + \frac{\partial u_i^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\big(n_i^{(1)}u_i^{(2)}\big) \\ &\quad + \frac{\partial}{\partial \zeta}\big(n_i^{(2)}u_i^{(1)}\big) = 0, \\ &\frac{\partial u_i^{(2)}}{\partial \tau} - V_p\frac{\partial u_i^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\big(u_i^{(1)}u_i^{(2)}\big) + \frac{\partial \phi^{(3)}}{\partial \zeta} + \eta\frac{\partial^2 u_i^{(2)}}{\partial \zeta^2} \\ &\quad + \alpha K_1\bigg[\frac{\partial n_i^{(3)}}{\partial \zeta} + (\alpha-2)\frac{\partial}{\partial \zeta}\big(n_i^{(1)}n_i^{(2)}\big) \\ &\quad + \frac{1}{2}(\alpha-3)(\alpha-2)n_i^{(1)^2}\frac{\partial n_i^{(1)}}{\partial \zeta}\bigg] = 0, \\ &\frac{\partial \phi^{(3)}}{\partial \zeta} + n_e^{(1)}\frac{\partial \phi^{(2)}}{\partial \zeta} + n_e^{(2)}\frac{\partial \phi^{(1)}}{\partial \zeta} \\ &\quad - \gamma K_2\bigg[\frac{\partial n_e^{(3)}}{\partial \zeta} + (\gamma-1)\frac{\partial (n_e^{(1)}n_e^{(2)})}{\partial \zeta} \\ &\quad + \frac{1}{2}(\gamma-2)(\gamma-1)\big(n_e^{(1)}\big)^2\frac{\partial n_e^{(1)}}{\partial \zeta}\bigg] = 0, \\ &\frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = (1-\mu)\frac{\partial n_e^{(3)}}{\partial \zeta} - \frac{\partial n_i^{(3)}}{\partial \zeta}. \end{aligned} \tag{15}$$

Solving this system of equations; Eqs. (15), by eliminating the third-order perturbed quantities, we finally obtain a

Burger-type equation,

$$\frac{\partial\phi^{(2)}}{\partial\tau}+A\frac{\partial}{\partial\zeta}\big(\phi^{(1)}\phi^{(2)}\big)+B\frac{\partial^2\phi^{(2)}}{\partial\zeta^2}$$
$$=G\frac{\partial^3\phi^{(1)}}{\partial\zeta^3}+H{\phi^{(1)}}^2\frac{\partial\phi^{(1)}}{\partial\zeta}+I\left(\frac{\partial\phi^{(1)}}{\partial\zeta}\right)^2$$
$$+J\phi^{(1)}\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} \qquad (16)$$

where the explicit expressions of G, H, I and J are given in the Appendix.

4 The stationary solutions

Using the solution $\phi^{(1)}$ introduced in (13), Eq. (16) can be transformed into the form

$$\frac{d^2\phi^{(2)}}{d\beta^2}+\frac{d}{d\beta}\big(2\phi^{(2)}\tanh\beta\big)=K(\beta), \qquad (17)$$

where

$$K(\beta)=\frac{8}{A^3}\big(GA^2+2B^2H-BJA\big)\operatorname{sech}^2\beta$$
$$+\frac{4}{A^3}\big(BIA-3GA^2-2B^2H+2BJA\big)$$
$$\times\operatorname{sech}^4\beta+\frac{8}{A^3}\big(2B^2H-BJA\big)\operatorname{sech}^2\beta\tanh\beta.$$

The corresponding homogeneous equation of Eq. (17) has two independent solutions, one of them is, $\phi^{(2)}_{c1}=\operatorname{sech}^2\beta$, and the other, which can be derived using reduction of order method along with Abel's theorem, is given by $\phi^{(2)}_{c2}=[\frac{\beta}{2}+\frac{1}{4}\sinh(2\beta)]\operatorname{sech}^2\beta$.

Using the variation of parameters method (El-Labany 1993; El-Taibany and Moslem 2005; El-Labany et al. 2005, 2007), the particular solution of Eq. (17) can be written as

$$\phi^{(2)}=T_1(\beta)\phi^{(2)}_{c1}+T_2(\beta)\phi^{(2)}_{c2},$$

where $T_1(\beta)=\int\frac{K(\beta)\phi^{(2)}_{c2}}{W(\phi^{(2)}_{c1},\phi^{(2)}_{c2})}$ and $T_2(\beta)=\int\frac{K(\beta)\phi^{(2)}_{c1}}{W(\phi^{(2)}_{c1},\phi^{(2)}_{c2})}$, and the Wronskian $W(\phi^{(2)}_{c1},\phi^{(2)}_{c2})=\phi^{(2)}_{c1}\frac{d\phi^{(2)}_{c2}}{d\beta}+\phi^{(2)}_{c2}\frac{d\phi^{(2)}_{c1}}{d\beta}=\operatorname{sech}^2\beta$.

Finally, the full stationary solution of the higher-order nonlinear DIA shock waves can be expressed as

$$\phi=\frac{2B}{A}(1+\tanh\beta)+\frac{4B}{3A^3}(4HB+IA-JA)$$
$$+\frac{2}{3A^3}\big(4JAB-3GA^2-10HB^2-IAB\big)\operatorname{sech}^2\beta$$
$$+\frac{4}{3A^3}\big(IAB-3GA^2-2HB^2+2JAB\big)$$
$$\operatorname{sech}^2\beta\ln[\cosh\beta]-\frac{4B}{A^3}(2HB-JA)\beta\operatorname{sech}^2\beta. \qquad (18)$$

5 Discussion and conclusion

Based on the numerical physical parameters of the white dwarfs presented in Mamun and Shukla (2010a, 2010b), Zobaer et al. (2013a, 2013b, 2013c), Mamun and Zobaer (2014), we present in this section a number of numerical illustrations corresponding to the two studied cases; the first one is corresponding to including ultrarelativistic electrons and the other is for nonrelativistic electrons. Figure 1 (2) is devoted to the first (second) case, respectively. We have plotted the variation of both of $\phi^{(1)}$ and $\phi^{(2)}$ and ϕ against β in Figs. 1 and 2 with keeping panel (a) in both to present reference curves. The transition from panel (a) to any other panel occurred by changing only one physical parameter stated in the figure caption. It is remarked here that due to the reductive perturbation principals, the condition $\phi^{(2)}\le\phi^{(1)}$ should be satisfied. This condition has been discussed in details in El-Taibany and Moslem (2005), El-Labany et al. (2005, 2007). However, Chatterjee et al. (2009) and Mehdipoor and Esfandyari-Kalejahi (2012), Ghorui et al. (2013) in their studies for higher-order solitons in degenerate plasma models, have employed the condition of $|\phi^{(2)}|\le|\phi^{(1)}|$ at what corresponding here $\beta=0$ only, without concerning the remainder domain. Therefore, in Figs. 1 and 2, the shaded light blue regions refer to where the full analytical solution, ϕ presented by Eq. (18), is allowed for the full β-domain. It is remarked that from Eq. (10), V_p is proportional to K_1 and K_2. In case of including ultrarelativistic (nonrelativistic) electrons, we have (smaller) larger values of K_2. This means the DIA shock have larger velocity, V_p, when ultrarelativistic electrons are included. In addition, from the solution presented in (13), the DIA shock wave amplitude increases (decreases) by increasing the kinematic viscosity (nonlinearity) effects. However, the kinematic viscosity affects implicitly on the DIA shock width through the parameter β. These observations are for the lowest order potential wave solution, but due to the interaction of higher-order nonlinear and dispersion effects, new wave solution features will be detected as shown below.

For ultrarelativistic electrons, Fig. 1 shows that increasing the ion number density results in increasing both of the DIA shock amplitude and its width. Moreover, for a more dense plasma (through increasing the ion number density, n_{i0}), new solitary wave structures are created, which we would like to call as "*humped DIA shock*", c.f. Figs. 1b, 1c, and 1d. According to the neutrality condition of the present

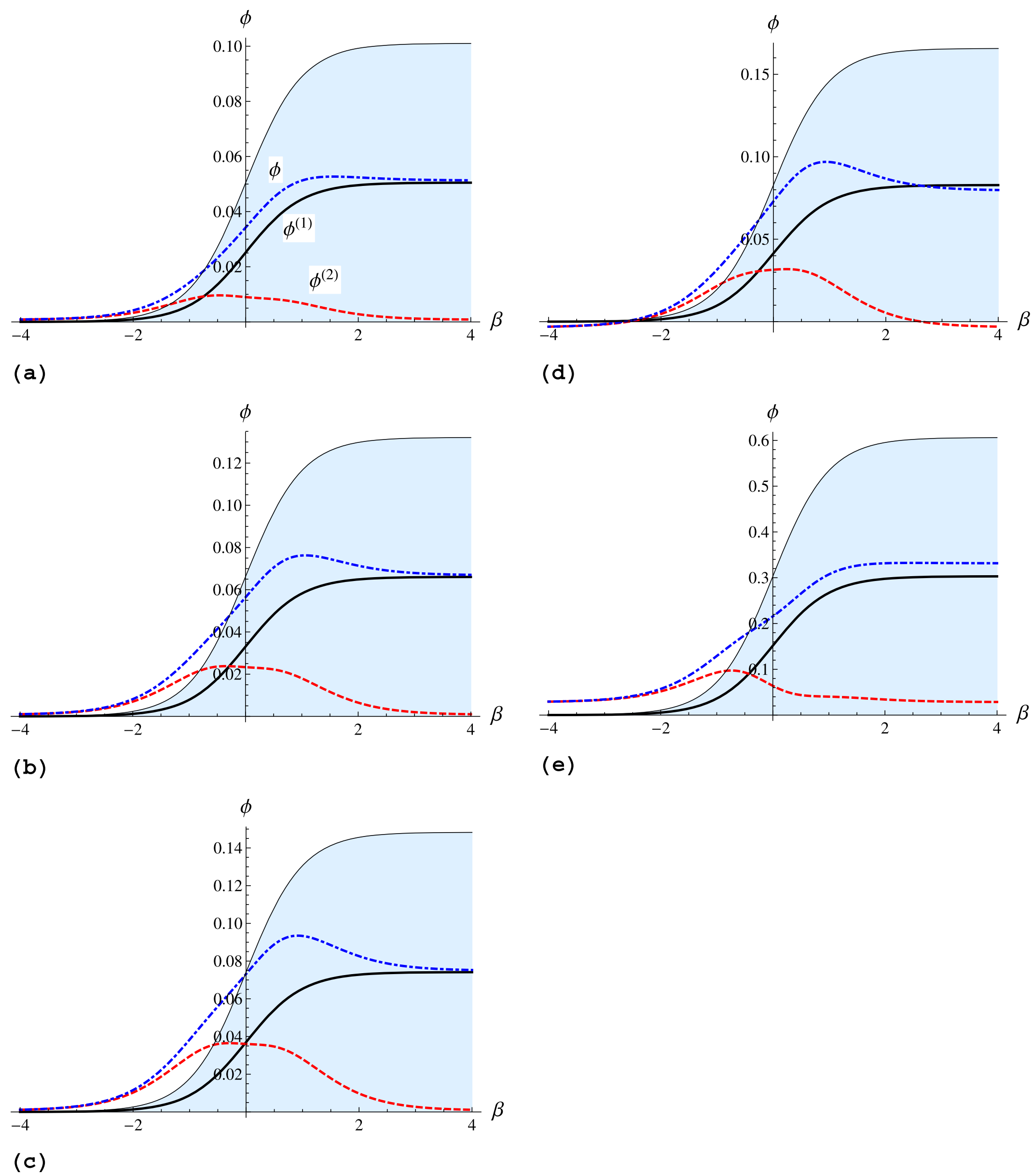

Fig. 1 A degenerate dense space plasma model contains nonrelativistic ions, ultrarelativistic electrons with stationary heavy dust grains. $\phi^{(1)}$ (*solid black curve*), $\phi^{(2)}$ (*dashed red curve*) and ϕ (*dot dashed blue curve*) are plotted against β. In (**a**), the curves appeared in this panel are considered as reference curves to compare among this panel and the other panels against variation in physical parameters. Here $n_{i0} = 10^{28}$, $\mu = 0.1$, $\eta = 0.1$, $\alpha = 5/3$ and $\gamma = 4/3$. In the other panels, only one parameter is changed as follows, in (**b**), $n_{i0} = 5 \times 10^{28}$, in (**c**) $n_{i0} = 10^{29}$, in (**d**) $\mu = 0.5$ and in (**e**) $\eta = 0.6$, respectively. The *shaded light blue area* is the region where the analytical solution, ϕ, presented in (18) is allowed

model, for a certain fixed value of electron number density, the increase of ion number density corresponds to an increase in the dust number density. Therefore, one can recognize that similar responses occurred in the DIA features between panel (d) and those presented in panels (a–c) obtained by increasing the ion number densities. On the other side, increasing either of the dust number density, μ, or the kinematic viscosity parameter, η, leads to a larger DIA shock with wider width, i.e. the shock waves become more dissipative with higher amplitude. Since the kinematic ion viscosity is responsible on the dissipation occurred in the system, the humped DIA shock is destroyed by increasing this dissipation effect and the resultant DIA shocks have larger widths. On contrary, this new humped DIA shock behaviours are obtained by an increment of either the equilibrium ion density, n_{i0}, or the dust species density, μ. In Figs. 1a, 1b, 1c, the variation occurred only in the ion number density, n_{i0}, and μ and η are kept fixed. We observed that the humped DIA

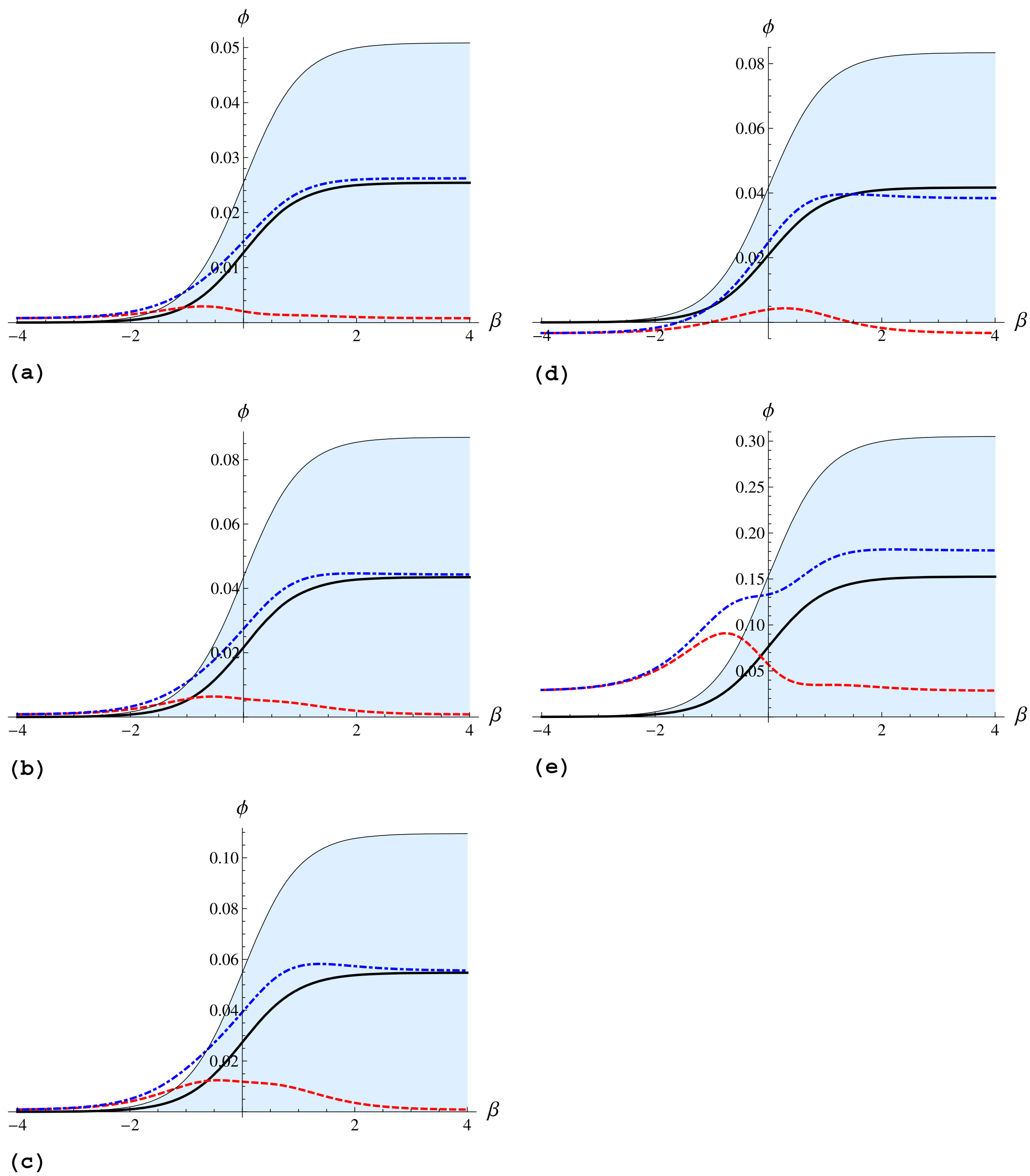

Fig. 2 A degenerate dense space plasma model contains both nonrelativistic ions and electrons with stationary heavy dust grains. $\phi^{(1)}$ (*solid black curve*), $\phi^{(2)}$ (*dashed red curve*) and ϕ (*dot dashed blue curve*) are plotted against β. The same parameter values are selected as in Fig. 1 with $\alpha = \gamma = 4/3$. The *shaded light blue area* is the region where the analytical solution, ϕ, presented in (18) is allowed

shock is created in these panels at small values of β due to changes in nonlinear effects. In Fig. 1(d), increasing η results as shift of the potential wave, ϕ, to higher values for positive/negative β values compared to $\phi^{(1)}$ wave solution. As we have stated in the introduction that the shock wave is created due to the balance between nonlinear and dissipation effects, the DIA shock width is proportional to the dissipation effect which is related to the viscosity term. However its amplitude is reduced by increasing the nonlinear effects.

From the other side, in the second case where the ions and the electrons are both nonrelativistic, including the higher-order nonlinear and dissipation corrections results as small increment of both of the amplitude and the width of the DIA shock waves. The kinematic viscosity effect plays an important role in this second case; increasing its strength (by increasing the parameter η) permits to create two-humped DIA shock waves with a wider width comparing to that of $\phi^{(1)}$. Since the nonlinear effects are functions in the pressures of electrons and ions species obeying the same equation state. Moreover, by carrying out a comparison between Figs. 1 and 2, we notice that the DIA shock waves in case of ultrarelativistic electrons have larger amplitude than that in

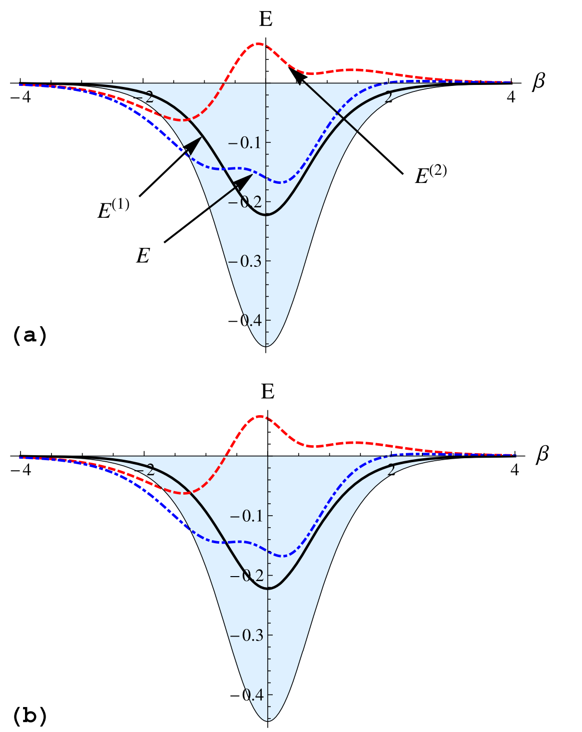

Fig. 3 The variations of the total electric field, E (*dotdashed blue curve*) corresponding to Eq. (18), and the first (second)-order electric field $E_1(E_2)$ shown as *solid black curve* (*dashed red curve*) respectively, against β. Panel (**a**) [(**b**)] corresponds to the case of ultrarelativistic [nonrelativistic] electrons. Here $n_{i0} = 10^{29}$, $\mu = 0.1$, $\eta = 0.6$, in (**a**) [**b**], we set $\alpha = 5/3$ and $\gamma = 4/3$ [$\alpha = \gamma = 4/3$]

nonrelativistic electrons case which agrees exactly with the results of Zobaer et al. (2013b, 2013c). In other words, the nonlinear effects is weaker when the electrons are ultrarelativistic. It is also noted that the model reveals only compressive DIA shocks. Up to the lowest order perturbed potential; $\phi^{(1)}$, the present results agree exactly with those presented in Zobaer et al. (2013b, 2013c), Mamun and Zobaer (2014).

Figure 3 shows the higher-order corrections of the total electric field, E, corresponding to the analytical solution, (18), and the first- (second)-order electric field, $E_1(E_2)$, respectively against β variations. It illustrates that including ultrarelativistic electrons with nonrelativistic ions, shown in panel (a), leads to a decrement of the associated electric field by 28 % at $\beta = 0$. However, in the case of nonrelativistic electrons and ions, shown in panel (b), the total electric field is reduced by 38 % of the first-order electric field, i.e. the reduction of the associated electric field is larger in case of the nonrelativistic model. Also, in both cases the associated total electric field is deformed to including these corrections and the reduction of E values is larger for negative β values and larger viscosity effects.

On the other hand, the observation of successive shock waves in ordinary plasma has been reported in the review article of Block (1978). Also, the transition from oscillatory DIA shock waves to monotonic ones due to increasing dust grain densities has been discussed by Nakamura et al. (1999), Nakamura (2002) and Sarma and Nakamura (2009) though they have considered the source of the dissipation is the ion collisions.

We have considered in the present model, the pressure of all the constituent particles (electrons and ions), as the whole system is degenerate and all the particles should follow the equation of state whatever its limit is (nonrelativistic or ultrarelativistic), again the effect of static charged dust grains is included. Comparing between our investigation and other previous related investigations (Roy et al. 2012; Zobaer et al. 2013a, 2013b, 2013c; Mamun and Zobaer 2014) leads to: the modifications due to introducing the higher-order nonlinear and dispersive effects to the DIA shocks and their associated electric field can't be observed in Zobaer et al. (2013c), Mamun and Zobaer (2014) where the authors used higher-order stretching coordinates. From this point of view, our present investigation is more acceptable and the system constituents have made the validity of our investigations greater than other previous works (Roy et al. 2012; Zobaer et al. 2013a, 2013b, 2013c; Mamun and Zobaer 2014). Therefore, we conclude that it is important to include such higher-order contribution of nonlinearity and dissipation effects in studying DIA shocks. This study presents a first attempt to introduce analytical/numerical studies to include these higher-order corrections for the DIA shocks in dense degenerate space plasma considering the two limits of nonrelativistic and ultrarelativistic states.

In addition, these findings would also be useful to explain the effects of degenerate pressure in interstellar and space plasmas, In our numerical analysis, we have tried to cover the degenerate plasma parameter ranges which are relevant to cosmic environments and compact astrophysical objects (e.g., white dwarfs, neutron stars, black hole, etc.), Michel (1982); Tandberg-Hansen and Emslie (1988); Hoyos et al. (2008), the effects of heavy elements in interstellar and space plasmas (Ferro et al. 2004).

Appendix: The coefficients D, E, F, G, H, I and J appeared in Eqs. (14) and (16)

The explicit expressions of the D, E and F coefficients presented in Eq. (14) are given by

$$D = \frac{1}{(V_p^2 - \alpha K_1)^3}\{[3V_p^2 + \alpha(\alpha - 2)K_1] - 2AV_p(V_p^2 - \alpha K_1)\}, \tag{19}$$

$$E = \frac{1}{V_p^2 - \alpha K_1}, \tag{20}$$

$$F = \frac{1}{(V_p^2 - \alpha K_1)^3}\{V_p[\alpha(\alpha - 2)K_1 + 3V_p^2]$$

$$- 2V_p\big(V_p^2 - \alpha K_1\big) - A\big(V_p^4 - \alpha^2 K_1^2\big)\big\}, \qquad (21)$$

and those coefficients; G, H, I and J, introduced in Eq. (16), are given by

$$G = -\frac{1}{8E^2 V_p}\big(4 + \eta^2 E^2\big), \qquad (22)$$

$$\begin{aligned} H = \bigg\{ & \frac{DE^2}{2\gamma K_2}\bigg[\frac{1}{2E^2 V_p} - 3(\gamma - 1)\bigg] \\ & - \frac{1}{2(\gamma K_2)^2}(\gamma - 2)(\gamma - 1)E - 3FE^2 V_p \\ & - \frac{3D}{2}E^2\big[V_p^2 + \alpha K_1(\alpha - 2)\big] \\ & - \frac{1}{2}E^4 \alpha(\alpha - 3)(\alpha - 2)K_1 + A(DEV_p + EF)\bigg\}, \end{aligned} \qquad (23)$$

$$I = -\frac{\eta}{4EV_p}\big(2E^2 V_p + 2F - AE\big), \qquad (24)$$

$$J = -\frac{\eta}{4EV_p}\big(2E^2 V_p + F - AE - DV_p\big). \qquad (25)$$

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