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Transverse instability of ion acoustic solitons in a magnetized plasma including q-nonextensive electrons and positrons

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A nonlinear Zakharov–Kuznetsov (ZK) equation for ion acoustic solitary waves (IASWs) is derived using the reductive perturbation method (RPM) for magnetized plasmas in which the inertialess electrons and positrons are nonextensively q-distributed while ions are assumed to be warm and inertial. It is found that both compressive as well as rarefactive solitons coexist in the present model for different regions of non-extensive electron and positron parameters, q_e and q_p. The magnetic field has no effect on the amplitude of the IASW, whereas the obliqueness angle of the wave propagation, the ion-to-electron temperature ratio σ and positron-to-ion density concentration ratio p affect both the amplitude and the width of the solitary wave structures. The transverse instability analysis illustrates that the one soliton solution has a constant growth rate, and it suffers from instability in the transverse direction. The relevance of the present study to astrophysical space plasmas is also discussed.

1. Introduction

Recently, nonlinear wave propagation in plasmas has become one of the most important subjects in plasma physics. The properties of these nonlinear plasma waves can be described by different nonlinear differential equations, e.g. the Korteweg–de Vries (KdV) equation, modified KdV equation and nonlinear Schrödinger equation Dodd *et al.* (1982), in one dimension. The ion acoustic (IA) mode is a fundamental mode in a plasma. This mode has been studied in different plasma systems having different combinations of plasma species. The pseudopotential approach and the reductive perturbation method (RPM) have been employed to study these nonlinear

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waves in arbitrary and small amplitude limits, respectively. Washimi & Taniuti (1966) were the first to derive the KdV equation for IA solitons in a plasma. Later, Zakharov & Kuznetsov (1974) derived a Zakharov–Kuznetsov (ZK) equation for describing nonlinear IA waves in a magnetized plasma composed of cold ions and hot isothermal electrons. The IA solitons were studied theoretically, and later were observed experimentally (Tran 1979; Nakamura 1982; Lonngren 1998; Saitou & Nakamura 2005). Moreover, Samanta, Saha & Chatterjee (2013) described the nonlinear properties of plasma waves in more than one dimension using ZK and Kadomtsev–Petviashili (KP) equations.

The electron–positron (EP) plasmas are found in different astrophysical plasma situations such as active galactic nuclei, quasars, neutron stars and pulsar magnetospheres. (Stenflo, Shukla & Yu 1985; Miller & Wiita 1987; Iwamoto 1993; Zank & Greaves 1995; Reynolds *et al.* 1996; Hirotani *et al.* 1999). The EP plasma, which consists of identical mass but oppositely charged particles, has quite different dynamics than that of the usual electron–ion (EI) plasma in which both fast and slow time scales exist. The symmetry of EP plasma dynamics breaks in the presence of ions, and both fast and slow time scales occur in electron–positron–ion (EPI) plasma. In most astrophysical environments, EP plasma exists in the relativistic regime. The linear and nonlinear electrostatic and electromagnetic waves in EPI plasmas have already been studied by a number of authors (Yu, Shukla & Stenflo 1986; Popel, Vladimirov & Shukla 1995; Mahajan, Berezhiani & Mikilaszewski 1998; Mahmood, Mushtaq & Saleem 2003; Mahmood & Ur-Rehman 2009; Mahmood, Akhtar & Ur-Rehman 2011).

The behaviours of plasma particles are significantly changed by the inclusion of magnetic field effects (Das & Verheest 1989; El-Taibany, El-Bedwehy & El-Shamy 2011). Mace & Hellberg (2001) investigated the effect of an external magnetic field on weakly nonlinear electron acoustic waves and they derived what they termed the KdV–ZK equation for describing such waves. Later, Mahmood & Akhtar (2008) studied the IA solitary wave (IASW) structures in multi-component magnetized plasmas with adiabatically heated ions. They explained the variations of these solitary wave structures in the presence of an external magnetic field and applied their findings to pulsar cusp regions. The nonlinear electrostatic drift waves in dense EPI and multi-component plasmas have also been investigated by (Haque, Mahmood & Mushtaq 2008). Mushtaq, Saeed & Haque (2009) have studied IA waves in a magnetized pair EI plasma and have derived a ZK equation for pair EI plasmas. It is revealed that nonlinear profiles of the IASW are significantly affected by the obliqueness propagation angle, magnetic field and the electron concentration. Also, Kourakis *et al.* (2009) have derived a ZK equation to study the rotational effect on the formation of multi-dimensional solitons in EPI and pair ion plasmas.

The Maxwellian distribution is a velocity distribution describing the plasma particles in a thermal equilibrium but it is not suitable for non-thermal space and laboratory plasmas (Maksimovic, Pierrard & Riley 1997; Leubner 2004; Christon *et al.* 2012). Most plasma studies have been carried out considering the system components are in thermal equilibrium, whereas the ideal thermal equilibrium assumption is no longer valid where some external agents (e.g. external force fields present in natural space plasma environments, wave-particle interactions, turbulence, etc.) disturb this equilibrium state. The deviation of the Maxwellian distribution takes place when the fluid velocity is much greater than the thermal velocity, or the plasma particles move very fast compared to their thermal velocities, particularly in space and astrophysical environments (Asbridge, Bame & Strong 1968; Divine & Garret 1983; Futaana *et al.* 2003).

Over the last few decades, a great deal of attention has been paid to a non-extensive generalization of the Boltzmann–Gibbs–Shannon (BGS) entropy, first recognized by Renyi (1955) and subsequently proposed by Tsallis (1988). In some space regions, distribution functions having non-Maxwellian energy tails, or flat topped structures with pronounced shoulders, are observed. During the last two decades, it has been proven that Boltzmann–Gibbs (BG) statistics are not appropriate to describe correctly systems with long range interaction, long time memory and fractality of the corresponding space-time/phase-space etc. The main reason for this failure is due to the fact that BG statistics are extensive and additive in its formalism. In order to analyse the statistical properties of the system with long range correlations, Tsallis (1988, 1995) extended BG thermodynamics by generalizing the concept of entropy to cover the non-extensive regime. Non-extensivity means that the entropy of the composition $(A+B)$ of two independent systems, A and B, is equal to $S_q(A+B) = S_q(A) + S_q(B) - (1-q)S_q(A)S_q(B)$ where the parameter q defines the non-extensivity property and it is referred to as the entropic index of the non-extensive system under consideration. Statistically, this distribution can be viewed as a simple reparametrization of the student distribution first introduced by Gosset (1908). Among the various physical systems in which connections with Tsallis entropy have been found are plasma physics (Lima, Silva & Santos 2000; Munoz 2006), long-range Hamiltonian systems (Latora, Rapisarda & Tsallis 2001), gravitational systems (Taruya & Sakagami 2003) and many other applications (Abe & Okamoto 2001). Lima *et al.* (2000) have found that the q-non-extensive pattern is very important for systems having long-range interactions (i.e. studying interactions that are comparable to the size of the system under consideration) such as those occurring in astrophysics and plasma physics. Moreover, Liu *et al.* (1994) and Lima *et al.* (2000) have showed that the plasma experimental results lead to a non-Maxwellian velocity distribution. Very recently, the Tsallis q-entropy and the ensuing generalized statistics have been employed with noticeable success to various plasma physics models (Jain, Tiwari & Sharma 1990; Du 2004; Verheest *et al.* 2006; Liyan & Jiulin 2008; Tribeche, Djebarni & Amour 2010; Bains *et al.* 2011; El-Taibany & Tribeche 2012; Ghosh, Chatterjee & Roychoudhury 2012; Akhtar, El-Taibany & Mahmood 2013; Hussain, Akhtar & Mahmood 2013; Shan & Akhtar 2013). For example, Ghosh *et al.* (2012) have investigated the head-on collisions of IASW in the presence of q-distributed electrons. Also, the arbitrary-amplitude IASW is studied in a two component plasma with q-non-extensive electron velocity distribution, (Tribeche *et al.* 2010). Their results show that the plasma model admits both compressive and rarefactive IA solitons whose amplitudes are sensitively dependent on the q-non-extensive parameter. In addition, Bains *et al.* (2011) have discussed a magnetized plasma model, including q-distributed electrons, by deriving a nonlinear ZK equation. The nonlinear propagation of large amplitude IA waves in a magnetized plasma, composed of kappa-distributed electrons and an inertial ion fluid, has been recently investigated by Sultana *et al.* (2010) using the Sagdeev potential analysis. Their results prove that the presence of excess superthermal electrons and a magnetic field influences the nature of the solitary wave profile. Sagdeev potential analysis has been used to address the effect of non-extensive electrons and thermal positrons on the IASW and double layers (Sahu 2011). Saini & Shalini (2013) addressed the problem of IASW in a plasma with two temperature electrons that obey q-non-extensive distribution. The modulation properties and stability regions of electron acoustic wave (EAW) are studied in two-dimensional plasma (Saini & Kholi 2014). Nsengiyumva *et al.* (2014) have studied the existence domains of arbitrary amplitude fast mode IA solitons in a plasma composed of warm adiabatic and cold positive ion species in the presence of Boltzmann electrons. The analysis of linear electrostatic waves in four component electron positron plasmas using the fluid equations has been carried out (Lazarus *et al.* 2012*a*). Later, Lazarus *et al.* (2012*b*) investigated the existence of arbitrary amplitude solitons in a relativistic electron positron, unmagnetized plasmas. The IA soliton amplitude increases with the increase in superthermality of electrons as discussed (Singh *et al.* 2013).

In the present paper, employing RPM (Washimi & Taniuti 1966; Kourakis *et al.* 2009), we derive a ZK equation adequate for describing IASW in a warm magnetized EPI plasma including q-non-extensive electrons and positrons and inertial ion fluid. The stability of the produced two-dimensional solitons is also addressed. The paper is organized as follows: the basic set of equations governing the dynamics of nonlinear IASW waves in a magnetized EPI plasma including non-extensive electrons and positrons is presented in § 2. Using RPM, a ZK equation is derived and the predicted soliton solutions are derived in § 3. The introduction of the instability of the produced solitons using the small k-expansion method (Moslem *et al.* 2007; Ghosh & Chakrabart 2013) is in § 3 as well. The present results are numerically illustrated and discussed in § 4. The conclusions are summarized in § 5.

2. Basic set of equations

A magnetized EPI plasma is under consideration, which is comprised of q-distributed electrons and positrons, as well as inertial warm ions. An external magnetic field of strength B_0 is applied along the x-axis i.e. $\boldsymbol{B}_0 = B_0\hat{x}$. The charge neutrality condition at equilibrium is $n_{e0} = n_{i0} + n_{p0}$, where n_{j0} is the equilibrium number density of jth species ($j = \mathrm{e}$ for electrons, i for ions and p for positrons).

The nonlinear dynamics of the IA waves in the proposed magnetized plasma can be described by the following equation of ion continuity,

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla} \cdot (n_i \boldsymbol{V}_i) = 0, \tag{2.1}$$

the equation of ion momentum,

$$\left(\frac{\partial}{\partial t} + \boldsymbol{V}_i \cdot \boldsymbol{\nabla}\right) \boldsymbol{V}_i = -\frac{e}{m_i}\boldsymbol{\nabla}\varphi + \omega_{ci}(\boldsymbol{V}_i \times \hat{\boldsymbol{x}}) - \frac{1}{n_i m_i}\boldsymbol{\nabla} P_i \tag{2.2}$$

and the Poisson equation,

$$\nabla^2\varphi = 4\pi e(n_e - n_p - n_i). \tag{2.3}$$

The one-dimensional equilibrium q-distribution function determining the non-extensivity behaviours of electrons and positrons ($j = \mathrm{e, p}$) is given by (Silva, Plastino & Lima 1998; Verheest 2013)

$$f(v_j) = C_{qj}\left[1 - (q_j - 1)\left(\frac{m_j v_j^2}{2k_B T_j} + \frac{q_j\varphi}{k_B T_j}\right)\right]^{1/(q_j-1)}, \tag{2.4}$$

where

$$C_{qj} = n_{j0}\frac{\Gamma\left(\dfrac{1}{1-q_j}\right)}{\Gamma\left(\dfrac{1}{1-q_j} - \dfrac{1}{2}\right)}\sqrt{\frac{m_j(1-q_j)}{2\pi k_B T_j}} \quad \text{for } -1 < q_j < 1 \tag{2.5}$$

and

$$C_{qj} = n_{j0}\left(\frac{1+q_j}{2}\right)\frac{\Gamma\left(\dfrac{1}{q_j-1}+\dfrac{1}{2}\right)}{\Gamma\left(\dfrac{1}{q_j-1}\right)}\sqrt{\frac{m_j(q_j-1)}{2\pi k_B T_j}} \quad \text{for } q_j > 1. \tag{2.6}$$

Here, the parameter q_j stands for the strength of non-extensivity. It is useful to note that for $q_j < -1$, the q-distribution is not normalizable. In the extensive limiting case ($q_j = 1$), the q-distribution reduces to the well-known Maxwell–Boltzmann distribution. Note that for $q_j > 1$, the q-distribution function exhibits a thermal cutoff on the maximum value allowed for the velocity of the jth-particles, which is given by (Pakzad & Tribeche 2011)

$$v_{j\,max} = \sqrt{\frac{2T_j}{m_j}\left(-\frac{q_j\varphi}{T_j}+\frac{1}{q_j-1}\right)}. \tag{2.7}$$

The densities of q-distributed non-thermal electrons and positrons are obtained from the zeroth order moment of the distribution function and integrating over v_j i.e. $-\infty$ to $+\infty$ (for $q_j < 1$) and v_j from $-v_{jmax}$ to $+v_{j\,max}$ (for $q_j > 1$) case, which is described as follows,

$$n_e = n_{e0}\left[1+(q_e-1)\frac{e\varphi}{T_e}\right]^{(q_e+1)/2(q_e-1)}, \tag{2.8}$$

$$n_p = n_{p0}\left[1-(q_p-1)\frac{e\varphi}{T_p}\right]^{(q_p+1)/2(q_p-1)}. \tag{2.9}$$

The parameters q_e and q_p stand for the strengths of electron and positron non-extensivity. Unlike the description of Maxwellian distribution, such that most of the particles are centred around the thermal speed, a power law distribution characterizes the systems containing an ample supply of superthermal particles with the constraint q_e, $q_p < 1$, or by including a large number of low velocity particles with the restriction q_e and $q_p > 1$ (Liyan & Jiulin 2008; El-Taibany & Tribeche 2012).

The energy of the q-distributed particles can be obtained from the second moment of the distribution function i.e. involving the integral of $v_j^2 f(v_j)$ over v_j, which comes out to be

$$E_j = \frac{1}{2}\int v_j^2 f(v_j)\,\mathrm{d}v_j = \frac{1}{(3q_j-1)}k_B T_j \tag{2.10}$$

for both $-1 < q_j < 1$ and $q_j > 1$ cases. The range of the non-extensive parameter q_j (for the case $q_j < 1$) for which the energy of the system is positive should be $1/3 < q_j < 1$. This is in contrast to kappa distribution theory, where the argument that the energy should remain finite has led to the standard requirement that $\kappa > 3/2$ (Vasyliunas 1968; Hellberg *et al.* 2009). There is no maximum limit for $q_j > 1$ case, but its values are not taken to be very large in the literature (Silva *et al.* 1998; Pakzad & Tribeche 2011; Verheest 2013) for a non-thermal plasma system.

Here $\nabla = (\partial/\partial x, \partial/\partial y, 0)$ and $\boldsymbol{V}_i = (V_{ix}, V_{iy}, V_{iz})$ and the quantities n_j and φ are the number density of jth species and the electrostatic potential, respectively. The ion gyro-frequency is denoted by $\omega_{ci} = eB_0/m_i c$, e is the electronic charge, m_i is the mass of ions and c is the speed of light in a vacuum. For the warm ion pressure we have $P_i = \gamma n_i T_i$ (Tiwari, Jain & Chawla 2007), where γ is the adiabatic constant and for a magnetized plasma we have $\gamma = 5/3$ and $T_e(T_p)$ is the electron (positron) temperature.

Using the already mentioned dimensional equations (2.1)–(2.9), the dispersion relation in normalized form is obtained as follows

$$\omega^4 - E\omega^2 + F = 0, \tag{2.11}$$

where

$$E = \Omega^2 + \sigma k^2 + \frac{k^2}{\left[k^2 + \dfrac{\beta p\left(1+q_p\right)}{2} + \dfrac{\mu_e\left(1+q_e\right)}{2}\right]},$$

$$F = \left[\sigma + \frac{1}{\left[k^2 + \dfrac{\beta p\left(1+q_p\right)}{2} + \dfrac{\mu_e\left(1+q_e\right)}{2}\right]}\right]\Omega^2 k_x^2. \tag{2.12}$$

Here k_x and k_y are the wave vectors in the x- and y-directions joined by the relation $k^2 = k_x^2 + k_y^2$. Also, $\lambda_D = \sqrt{T_e/4\pi n_{i0} e^2}$ (Debye length), $\omega_{pi} = \sqrt{4\pi n_{i0} e^2/m_i}$ (ion plasma frequency), $C_s = \omega_{pi}\lambda_D = \sqrt{T_e/m_i}$ (IA speed) and $\Omega = \omega_{ci}/\omega_{pi}$, $\sigma = \gamma T_i/T_e$, $\beta = T_e/T_p$, $\mu_e = n_{e0}/n_{i0}$ and $p = n_{p0}/n_{i0}$. The roots of the quartic equation are given by

$$\omega_{\pm}^2 = \frac{E \pm \sqrt{E^2 - 4F}}{2}. \tag{2.13}$$

The limiting cases for obliquely propagating electrostatic waves in the parallel and perpendicular directions to the magnetic field $\boldsymbol{B}_0$ are discussed below.

In order to study wave propagation in a direction parallel to the magnetic field $\boldsymbol{B}_0$, we put $k_y = 0$ in (2.13) to obtain the following two roots:

$$\omega_+^2 = \Omega^2, \quad \omega_-^2 = \frac{\left[\sigma k_x^2 + \sigma\left(\dfrac{\beta p(1+q_p)}{2} + \dfrac{\mu_e(1+q_e)}{2}\right) + 1\right]}{\left(k_x^2 + \dfrac{\beta p(1+q_p)}{2} + \dfrac{\mu_e(1+q_e)}{2}\right)} k_x^2. \tag{2.14a,b}$$

The positive root is a frequency and can therefore be neglected; the second root is the dispersion relation for an IA wave propagating along the magnetic field and the phase velocity of IA wave depends on the non-extensive parameters of the electrons and positrons, temperature and their density ratios.

On the other side, to find the electrostatic wave propagating in a direction perpendicular to the magnetic field $\boldsymbol{B}_0$, we put $k_x = 0$ in (2.13) and obtain

$$\omega_+^2 = \Omega^2 + \frac{\left[\sigma k_y^2 + \sigma\left(\dfrac{\beta p(1+q_p)}{2} + \dfrac{\mu_e(1+q_e)}{2}\right) + 1\right]}{\left(k_y^2 + \dfrac{\beta p(1+q_p)}{2} + \dfrac{\mu_e(1+q_e)}{2}\right)} k_y^2, \quad \omega_-^2 = 0. \tag{2.15a,b}$$

The positive root gives the dispersion relation for an ion cyclotron wave (ICW), whereas the slow wave vanishes. Here, the phase velocity of the ICW is not independent of k_y and also depends on the non-extensive parameters of the electrons and positrons, temperature and their density ratios.

We can express (2.1)–(2.9) in the form of scalar components as follows,

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}n_iV_{ix}+\frac{\partial}{\partial y}n_iV_{iy}=0, \tag{2.16}$$

$$\left(\frac{\partial}{\partial t}+V_{ix}\frac{\partial}{\partial x}+V_{iy}\frac{\partial}{\partial y}\right)V_{ix}=-\frac{\partial \Phi}{\partial x}-\frac{\sigma}{n_i}\frac{\partial n_i}{\partial x}, \tag{2.17}$$

$$\left(\frac{\partial}{\partial t}+V_{ix}\frac{\partial}{\partial x}+V_{iy}\frac{\partial}{\partial y}\right)V_{iy}=-\frac{\partial \Phi}{\partial y}+\Omega V_{iz}-\frac{\sigma}{n_i}\frac{\partial n_i}{\partial y}, \tag{2.18}$$

$$\left(\frac{\partial}{\partial t}+V_{ix}\frac{\partial}{\partial x}+V_{iy}\frac{\partial}{\partial y}\right)V_{iz}=-\Omega V_{iy}, \tag{2.19}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi=\mu_e n_e-n_i-(\mu_e-1)n_p. \tag{2.20}$$

$$n_e=1+c_1\Phi-c_1c_2\Phi^2+\cdots, \tag{2.21}$$

$$n_p=1-p_1\Phi-p_1p_2\Phi^2+\cdots, \tag{2.22}$$

where we have used the normalizations as $n_{e,i,p}=n_{e,i,p}/n_{e0,i0,p0}$, $\boldsymbol{V}_i=\boldsymbol{V}_i/C_s$, $\boldsymbol{\nabla}=\hat{\boldsymbol{x}}\partial/\partial x+\hat{\boldsymbol{y}}\partial/\partial y\equiv\boldsymbol{\nabla}\lambda_D$, $t=t\omega_{pi}$, $\Omega=\omega_{ci}/\omega_{pi}$, and $\Phi=e\varphi/T_e$. Here $\mu_e=(1+p)$, with, $c_1=(q_e+1)/2$, $c_2=(q_e-3)/4$, $p_1=(q_p+1)\beta/2$, $p_2=(q_p-3)\beta/4$. The non-extensive effects due to the electrons and positrons are represented by the indices q_e and q_p respectively.

3. Nonlinear analysis

In order to derive a ZK equation for the proposed magnetized EPI plasmas, we define the stretching of independent variables, from (Washimi & Taniuti 1966; Kourakis *et al.* 2009)

$$\xi=\epsilon^{1/2}(x-\lambda t),\quad \eta=\epsilon^{1/2}y,\quad \tau=\epsilon^{3/2}t, \tag{3.1a–c}$$

where ϵ is a small $(0<\epsilon\leqslant 1)$ expansion parameter characterizing the strength of the nonlinearity and λ (normalized by C_s) is the phase speed of the wave, which will be determined later. Using the RPM, Kourakis *et al.* (2009), we can expand the perturbed quantities about their equilibrium values as follows:

$$\left.\begin{aligned} n_j&=1+\epsilon n_{j1}+\epsilon^2 n_{j2}+\cdots,\\ V_{ix}&=\epsilon V_{ix1}+\epsilon^2V_{ix2}+\epsilon^3V_{ix3}+\cdots,\\ V_{iy}&=\epsilon^2V_{iy1}+\epsilon^3V_{iy2}+\epsilon^4V_{iy3}+\cdots,\\ V_{iz}&=\epsilon^{3/2}V_{iz1}+\epsilon^{5/2}V_{iz2}+\epsilon^{7/2}V_{iz3}+\cdots,\\ \Phi&=\epsilon\Phi_1+\epsilon^2\Phi_2+\epsilon^3\Phi_3+\cdots.\end{aligned}\right\} \tag{3.2}$$

Substituting (3.1) and (3.2) into (2.16)–(2.22) and collecting terms of lowest order ($\sim \epsilon^{3/2}$) terms of continuity and x-, y-, z-components of the momentum equation for the ions, we obtain

$$n_{i1} = \frac{\Phi_1}{N\lambda^2}, \quad V_{ix1} = \frac{\Phi_1}{N\lambda} \tag{3.3a,b}$$

$$V_{iy1} = \frac{1}{\Omega^2 N}\frac{\partial^2 \Phi_1}{\partial \xi \eta}, \quad V_{iz1} = \frac{1}{\Omega N}\frac{\partial \Phi_1}{\partial \eta} \tag{3.4a,b}$$

where $N = 1 - (\sigma/\lambda^2)$.

The lowest order ($\sim\epsilon$) terms for the non-extensive electrons and positrons from (2.21)–(2.22) give

$$n_{e1} = c_1 \Phi_1, \quad n_{p1} = -p_1 \Phi_1. \tag{3.5a,b}$$

The Poisson equation gives the lowest order ($\sim \epsilon$) terms,

$$0 = \mu_e n_{e1} - n_{i1} + (1-\mu_e) n_{p1}. \tag{3.6}$$

The phase speed λ of the wave is obtained using (3.3), (3.5) and (3.6), i.e.

$$\lambda = (N[\mu_e c_1 - (1-\mu_e) p_1])^{-1/2}, \tag{3.7}$$

which is the normalized phase velocity of the IA wave which can be obtained from (2.14) for a dispersionless medium.

Proceeding through the perturbation theory and using the relations provided in (3.3)–(3.7), we can finally obtain the ZK equation for an electrostatic wave in magnetized EPI plasmas in terms of Φ_1 as follows,

$$\frac{\partial \Phi_1}{\partial \tau} + A\Phi_1 \frac{\partial \Phi_1}{\partial \xi} + B\frac{\partial^3 \Phi_1}{\partial \xi^3} + C\frac{\partial^3 \Phi_1}{\partial \xi \partial \eta^2} = 0, \tag{3.8}$$

where the nonlinear coefficient A and the dispersive coefficients B and C are defined as,

$$A = \frac{1}{2}\left(\frac{3}{N\lambda} - \frac{\sigma}{N\lambda^3} + 2N^2\lambda^3[\mu_e c_1 c_2 + (1-\mu_e) p_1 p_2]\right), \tag{3.9a}$$

$$B = \frac{N^2\lambda^3}{2}, \quad \text{and} \quad C = \frac{N^2\lambda^3}{2}\left(1 + \frac{1}{\Omega^2 N^2 \lambda}\right)\frac{\partial \Phi_1}{\partial \eta}. \tag{3.9b,c}$$

The ZK equation (3.8) has a well-known soliton solution (Kourakis *et al.* 2009), which is given by

$$\Phi_1 = \phi_m \operatorname{sech}^2(\zeta/W), \tag{3.10}$$

where $\phi_m = 3u_0/A$ and $W = \sqrt{4(B+C)/u_0}$ are the amplitude and the width of the soliton, and η is the transformed coordinate in the co-moving frame with speed u_0 i.e. $\zeta = \ell_x \xi + \ell_y \eta - u_0 \tau$. Here $l_x^2 + l_y^2 = 1$. The coefficients of the ZK equation, which depend on the entropic indices i.e. q_e and q_p, directly affect the basic features of the produced soliton.

From (3.10) and the relation $\boldsymbol{E}_1 = -\nabla \Phi_1$, the normalized electric field of the obliquely propagating two-dimensional IAW becomes

$$E_1 = \frac{3u_0^{2/3}}{A\sqrt{(B+C)}} \tanh(\zeta/W)\mathrm{sech}^2(\zeta/W). \tag{3.11}$$

Stability analysis. To study the stability of the two-dimensional solitons satisfying (3.8), we apply a small-k expansion perturbation method (Moslem *et al.* 2007; Ghosh & Chakrabart 2013). Using the following transformations

$$X = \sqrt{\frac{A}{B}}\xi, \quad T = \sqrt{\frac{A^3}{B}}\tau, \quad Y = \sqrt{\frac{A}{C}}\eta \quad \text{and} \quad \Phi_1 \equiv \Psi, \tag{3.12a–d}$$

we can rewrite (3.8) as

$$\frac{\partial \Psi}{\partial T} + \Psi\frac{\partial \Psi}{\partial X} + \frac{\partial^3 \Psi}{\partial X^3} + \frac{\partial^3 \Psi}{\partial X \partial Y^2} = 0. \tag{3.13}$$

Supposing $\chi = X - \nu_0 T$, (3.13) becomes

$$-\nu_0\frac{\partial \varphi_0}{\partial \chi} + \varphi_0\frac{\partial \varphi_0}{\partial \chi} + \frac{\partial^3 \varphi_0}{\partial \chi^3} = 0, \tag{3.14}$$

where we assume that (3.13) has a planar solution φ_0, that is given by

$$\varphi_0(\chi) = \varphi_{0m}\mathrm{sech}^2(\chi/\Delta), \tag{3.15}$$

with $\varphi_{0m} = 3\nu_0$ and $\Delta = 2/\sqrt{\nu_0}$. Now, let us write the full solution of (3.13) as

$$\Psi(\chi, Y, T) = \varphi_0(\chi) + \psi(\chi)\exp(\mathrm{i}kY - \mathrm{i}\Upsilon T). \tag{3.16}$$

Substituting (3.16) into (3.13) and linearizing with respect to ψ, we obtain

$$\frac{\mathrm{d}^3\psi}{\mathrm{d}\chi^3} + \frac{\mathrm{d}(\varphi_0\psi)}{\mathrm{d}\chi} - (\nu_0 + k^2)\frac{\mathrm{d}\psi}{\mathrm{d}\chi} = \mathrm{i}\Upsilon\psi. \tag{3.17}$$

Now expanding ψ and Υ as a power series of k

$$\left.\begin{aligned} \psi &= \psi_0 + k\psi_1 + k^2\psi_2 + \cdots \\ \Upsilon &= k\Upsilon_1 + k^2\Upsilon_2 + \cdots, \end{aligned}\right\} \tag{3.18}$$

then substituting into (3.17) and equating the coefficients of equal powers of k, the zeroth order yields

$$\frac{\mathrm{d}^3\psi_0}{\mathrm{d}\chi^3} + \frac{\mathrm{d}(\varphi_0\psi_0)}{\mathrm{d}\chi} - \nu_0\frac{\mathrm{d}\psi_0}{\mathrm{d}\chi} = 0, \tag{3.19}$$

which can be written as

$$\frac{\mathrm{d}}{\mathrm{d}\chi}\hat{L}(\psi_0) = 0, \tag{3.20}$$

where $\hat{L} = ((\mathrm{d}^2/\mathrm{d}\chi^2) + \varphi_0 - \nu_0)$. Though, (3.14) can be given in terms of the operator $\hat{L}$ as

$$\hat{L}\left(\frac{\mathrm{d}\varphi_0}{\mathrm{d}\chi}\right) = 0. \tag{3.21}$$

Differentiating with respect to χ, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}\chi}\hat{L}\left(\frac{\mathrm{d}\varphi_0}{\mathrm{d}\chi}\right)=0. \tag{3.22}$$

Comparing (3.20) and (3.22), we can write

$$\psi_0=\frac{\mathrm{d}\varphi_0}{\mathrm{d}\chi}. \tag{3.23}$$

So,

$$\psi_0(\chi)=\psi_{0m}\mathrm{sech}^2(\chi/\varDelta)\tanh(\chi/\varDelta), \tag{3.24}$$

where $\psi_{0m}=-3\nu_0^{3/2}$. Considering the next order of k, one obtains

$$\frac{\mathrm{d}^3\psi_1}{\mathrm{d}\chi^3}+\frac{\mathrm{d}\,(\varphi_0\psi_1)}{\mathrm{d}\chi}-\nu_0\frac{\mathrm{d}\psi_1}{\mathrm{d}\chi}=\mathrm{i}\varUpsilon_1\psi_0. \tag{3.25}$$

Integrating with respect to χ, we obtain

$$\hat{L}(\psi_1)=\mathrm{i}\varUpsilon_1\varphi_0+S_1, \tag{3.26}$$

where S_1 is the integration constant. To solve (3.26), it is useful to write $\psi_1=V(\chi)\psi_0(\chi)$, where $V(\chi)$ is an arbitrary function, to be determined. Substituting into (3.26) and looking for the localized solution under the condition $\psi_1\rightarrow 0$ as $|\chi|\rightarrow\infty$, we obtain

$$V(\chi)=\frac{\mathrm{i}}{8}\varUpsilon_1\varDelta^2\left[\chi-\varDelta\coth\left(\frac{\chi}{\varDelta}\right)\right]. \tag{3.27}$$

So,

$$\psi_1=\psi_{1m}\mathrm{sech}^2\left(\frac{\chi}{\varDelta}\right)\left[\chi\tanh\left(\frac{\chi}{\varDelta}\right)-\varDelta\right], \tag{3.28}$$

where $\psi_{1m}=-(3/4)\mathrm{i}\varUpsilon_1\nu_0\varDelta$. Considering the next order of k, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}\chi}\hat{L}(\psi_2)=\mathrm{i}\varUpsilon_2\psi_0+\mathrm{i}\varUpsilon_1\psi_1+\frac{\mathrm{d}\psi_0}{\mathrm{d}\chi}. \tag{3.29}$$

To have a solution for (3.29), its left-hand side must be orthogonal to the kernal φ_0, i.e.

$$\int_{-\infty}^{\infty}\varphi_0\frac{\mathrm{d}}{\mathrm{d}\chi}\hat{L}(\psi_2)\,\mathrm{d}\chi=0. \tag{3.30}$$

This orthogonality condition yields the following consistency condition

$$\mathrm{i}\varUpsilon_2\int_{-\infty}^{\infty}\varphi_0\psi_0\,\mathrm{d}\chi+\mathrm{i}\varUpsilon_1\int_{-\infty}^{\infty}\varphi_0\psi_1\,\mathrm{d}\chi+\int_{-\infty}^{\infty}\varphi_0\frac{\mathrm{d}\psi_0}{\mathrm{d}\chi}\,\mathrm{d}\chi=0, \tag{3.31}$$

from which we can determine $\varUpsilon_1$. After performing these integrals, we obtain

$$\varUpsilon_1=\pm\mathrm{i}\frac{2\nu_0}{\sqrt{15}}. \tag{3.32}$$

For the present problem, ν_0 is always positive, so it is clear that the positive sign in the above expression leads to instability of the one-dimensional soliton solution in the transverse direction. Also, the instability growth rate is directly proportional to ν_0. The above result, (3.32), agrees exactly with that obtained in Moslem *et al.* (2007) and Ghosh & Chakrabart (2013) when $\nu_o=4$; $|\varUpsilon_1|=8/\sqrt{15}$.

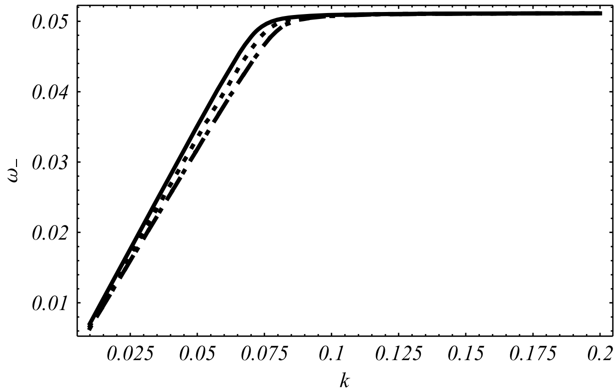

FIGURE 1. The low frequency mode of the electrostatic wave i.e. ω_- versus k is plotted for different values of entropic indices i.e. q_e, q_p; $q_e = q_p = 0.35$ (solid curve), $q_e = q_p = 0.9$ (dotted curve), $q_e = q_p = 2$ (dashed dotted curve). For the plasma $\mu = 5$, $p = 4$, $T_e = T_p = 10^3$ K, $T_i = 0.2T_e$, $k_y = 0.1k_x$ and $B_0 \sim 10^2$ G.

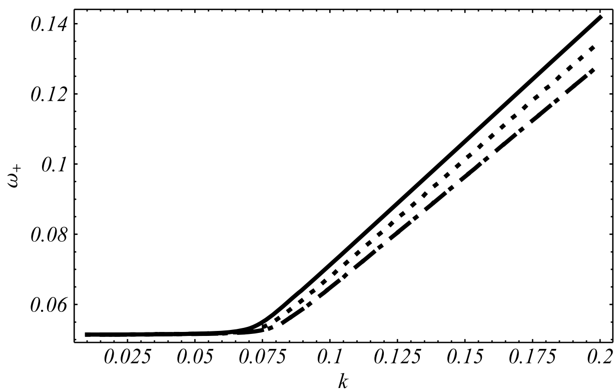

FIGURE 2. The high frequency mode of the electrostatic wave i.e. ω_+ versus k is plotted for different values of entropic indices i.e. q_e, q_p; $q_e = q_p = 0.35$ (solid curve), $q_e = q_p = 0.9$ (dotted curve), $q_e = q_p = 2$ (dashed dotted curve). For the plasma $\mu = 5$, $p = 4$, $T_e = T_p = 10^3$ K, $T_i = 0.2T_e$, $k_y = 0.1k_x$ and $B_0 \sim 10^2$ G.

4. Numerical illustrations and discussion

In this section we illustrate the characteristics and instability of two-dimensional IASW in magnetized EPI plasmas based on the results presented in §§ 3 and 4.

The high and low frequency modes of the dispersion relation $\omega_{+,-}$ versus k are plotted with variations of non-extensive parameter for electrons q_e (same value for q_p) and the magnetic field B_0. In figures 1 and 2 the low (IA wave) and high (ion cyclotron wave) frequency modes of the dispersion relation are shown at different values of the non-extensive parameter i.e. q_e. It can be seen from figure 1 that as we increase the values of q_e, the saturation of the curve for the low frequency mode occurs at larger values of k, while for the high frequency mode, saturation of the curves are created at small wavelengths or at large k values as shown in figure 2. Magnetic field variation on the low and high frequency modes of the dispersion relation are presented in figures 3 and 4, respectively. The phase velocity of the low

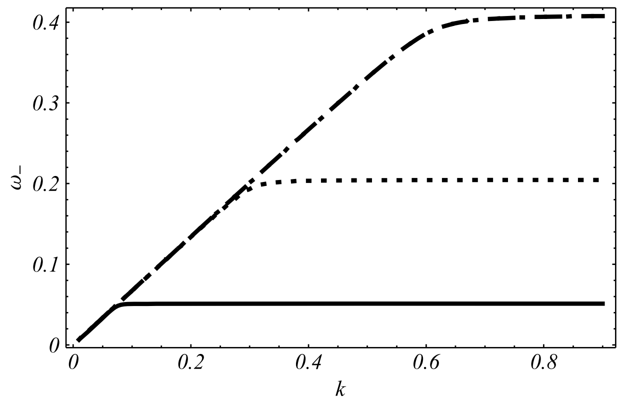

FIGURE 3. The low frequency mode of the electrostatic wave i.e. ω_- versus k is plotted for different values of magnetic field. The plasma parameters are $\mu = 5$, $p = 4$, $T_e = T_p = 10^3$ K, $T_i = 0.2T_e$, $k_y = 0.1k_x$, $q_e = q_p = 0.9$ for $B_0 = 10^2$ G (solid curve), $B_0 = 4 \times 10^2$ G (dotted curve), $B_0 = 8 \times 10^2$ G (dashed dotted curve).

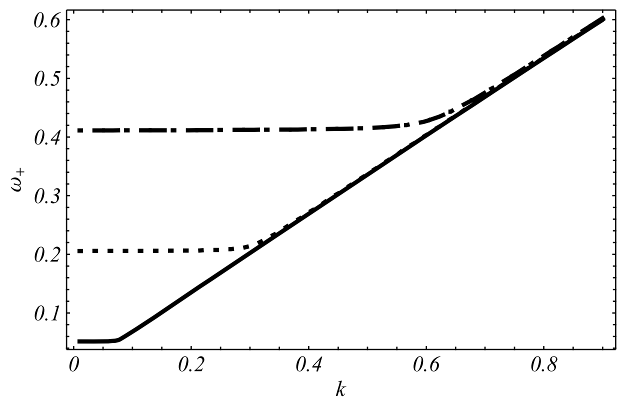

FIGURE 4. The high frequency mode of the electrostatic wave i.e. ω_+ vs k is plotted for different values of magnetic field. The plasma parameters are $\mu = 5$, $p = 4$, $T_e = T_p = 10^3$ K, $T_i = 0.2T_e$, $k_y = 0.1k_x$, $q_e = q_p = 0.9$ for $B_0 = 10^2$ G (solid curve), $B_0 = 4 \times 10^2$ G (dotted curve), $B_0 = 8 \times 10^2$ G (dashed dotted curve).

wave increases by enhancing the magnetic field strength while the saturation of the curve occurs at large values of k shown in figure 3. The phase velocity of the high wave also increases with the increase in the magnetic field strength and the wave dispersion occurs at large k values described in figure 4. The phase velocity of low or IAW (λ) defined in (3.7) is plotted in figure 5 in EPI plasmas for different cases of ion temperature i.e. σ (ratio of ion-to-electron temperature) and variations of entropic indices q_e, q_p for the non-thermal electrons and positrons. It is found that the phase velocity of IA wave is decreased with an increase of entropic indices i.e. q_e, q_p for the electrons and positrons. The phase velocity of IA wave increases in the presence of an ion temperature in comparison with the cold ions case (i.e. $\sigma = 0$) for q_e, $q_p > 0$. In figure 6 phase velocity, λ, is plotted for the different positron concentrations versus entropic indices i.e. q_e, q_p for the electrons and positrons, respectively. The phase velocity, λ, is larger in the presence of positrons in comparison with the absence of

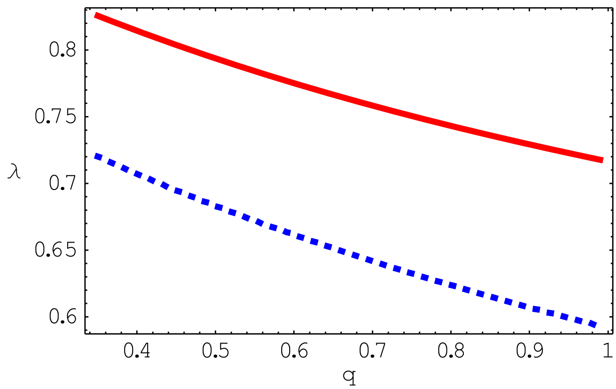

FIGURE 5. The phase velocity λ is plotted for IA wave in EPI plasmas at different ion temperature (T_i) against entropic indices q_e, q_p for the non-thermal electrons and positrons. The plasma parameters are $\mu = 1.2$, $p = 0.2$, $T_e = T_p = 10^3$ K for $T_i = 0.2T_e$ (red solid curve), $T_i = 0$ (blue dotted curve).

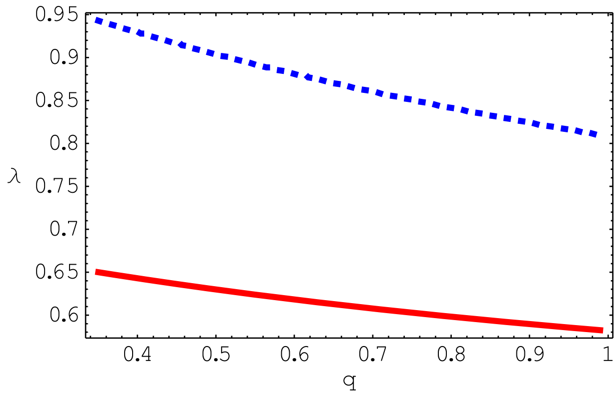

FIGURE 6. The phase velocity λ is plotted for IA wave in EPI plasmas at different positrons density against entropic indices q_e, q_p for the non-thermal electrons and positrons. The plasma parameters are $T_i = 0.2T_e$, $T_e = T_p = 10^3$ K, $p = 0.9$ (red solid curve), $p = 0$ (blue dotted curve).

a positron concentration as shown in the figure. It is noticed that the phase velocity of the IA wave decreases with the increase in the value of entropic indices i.e. q_e, q_p for the electrons and positrons keeping positron concentration fixed. The amplitude of the soliton $\phi_m = (3u_0/A)$ is plotted in figure 7 for different values of positron concentration against the entropic indices of electrons and positrons. The amplitude of the IA soliton increases as we decrease the positron concentration, keeping all others parameters fixed. The width of the soliton $W = \sqrt{4(B+C)/u_0}$ is plotted in figure 8 for all the same parameters used in figure 7, and we found a similar trend. The width of the soliton decreases at larger values of positron concentration and non-extensive parameter. The amplitude of the soliton is plotted in figure 9 for different values of ion temperature and entropic indices q_e, q_p. The amplitude of the soliton decreases for the larger values of σ and entropic indices q_e, q_p. The soliton width W is plotted for the temperature coefficient σ in figure 10. The width of the

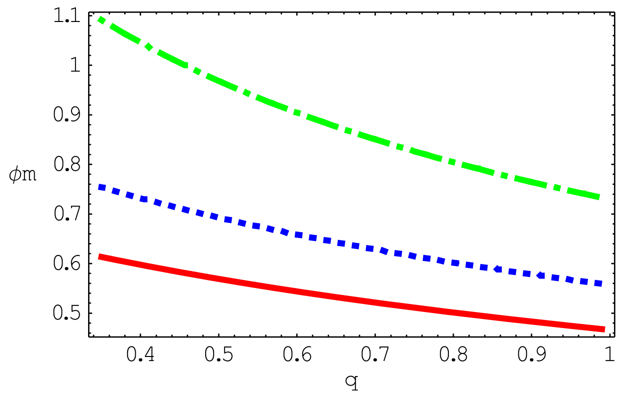

FIGURE 7. The IA soliton amplitude $\phi_m = (3u_0/A)$ is plotted for different values of positron density concentration and entropic indices q_e, q_p for the non-thermal electrons and positrons. The plasma parameters are $u_0 = 0.3$, $T_i = 0.2T_e$, $T_e = T_p = 10^3$ K, $B_0 \sim 10$ G, $p = 0.4$ (red solid curve), $p = 0.2$ (blue dotted curve) and $p = 0$ (green dashed dotted curve).

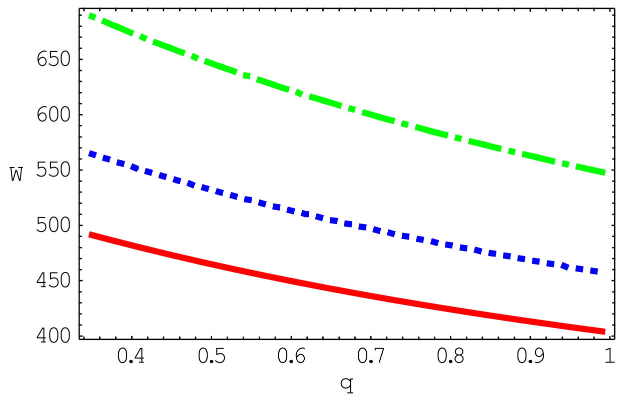

FIGURE 8. The width of the soliton $W = \sqrt{4(B+C)/u_0}$ is plotted for different values of positron density concentration and entropic indices q_e, q_p for the non-thermal electrons and positrons. The plasma parameters are $u_0 = 0.3$, $T_i = 0.2T_e$, $T_e = T_p = 10^3$ K, $B_0 \sim 10$ G, $p = 0.4$ (red solid curve), $p = 0.2$ (blue dotted curve) and $p = 0$ (green dashed dotted curve).

soliton increases for large values of σ and *vice versa* for the non-extensive parameters. The width for the ion acoustic wave in EPI is plotted for the magnetic field intensity. It can be seen that there is no change in the amplitude, whereas the width of the soliton reduces significantly with the increase in magnetic field strength i.e. B_0 (0.1 G, 20 G, 100 G) as shown in figure 11. It is also evident from (3.10) that the intensity of the external magnetic field B_0 has no direct effect on the amplitude of the solitary waves. The contribution of the ion gyro-frequency Ω comes into play on the width of the soliton through the coefficient C in (3.9). The amplitude of the soliton remains the same while the width decreases with increasing Ω i.e. a stronger magnetic field leads to steeper and thus narrower soliton profiles, which agrees with the results of Sultana *et al.* (2010). Figure 12 shows the variation of the planar solution, φ_0, that

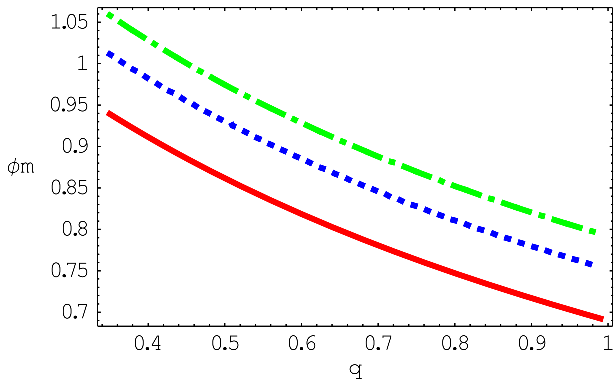

FIGURE 9. The amplitude of the soliton $\phi_m = (3u_0/A)$ is plotted against entropic indices q_e, q_p for the electrons and positrons for different values of σ. The plasma parameters are $u_0 = 0.3$, $\mu = 1.2$, $p = 0.2$, $T_e = T_p = 10^3$ K, $B_0 \sim 10$ G, $T_i = 0.28T_e$ (red solid curve), $T_i = 0.1T_e$ (blue dotted curve) and $T_i = 0$ (green dashed dotted curve).

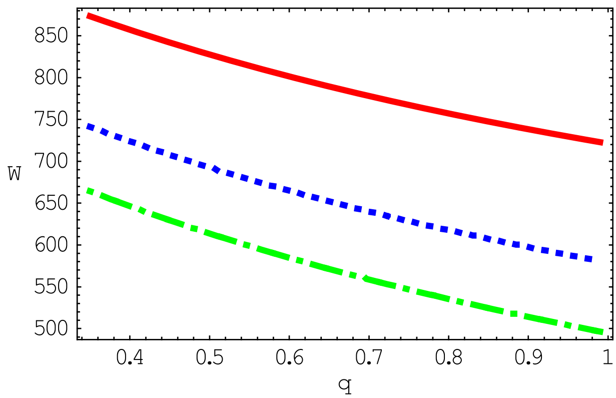

FIGURE 10. The soliton width $W = \sqrt{4(B+C)/u_0}$ is plotted against entropic indices q_e, q_p for the electrons and positrons for different values of σ. The plasma parameters are $u_0 = 0.3$, $\mu = 1.2$, $p = 0.2$, $T_e = T_p = 10^3$ K, $B_0 \sim 10$ G, $T_i = 0.28T_e$ (red solid curve), $T_i = 0.1T_e$ (blue dotted curve) and $T_i = 0$ (green dashed dotted curve).

is given by (3.15), and the function $\psi = \psi_0 + k\psi_1 + \cdots$ appears in the perturbed solution; (3.18) against χ and different values of ν_0. It is clear that φ_0 (ψ) decreases (increases) as χ increases (decreases). For $\chi < 1$, φ_0 increases as ν_0 increases. The opposite effect is observed where $\chi > 1$. Since, φ_0 changes its behaviour with ν_0 at $\chi = 1$, this ψ-corresponding characteristic point occurs at $\chi \simeq 1.75$. Moreover, the variation curves of ψ versus χ shows the existence of minimum points appearing at smaller χ by increasing ν_0. Figure 12 proves that there are some points at which $|\psi| > |\varphi_0|$ that may lead to instability of the produced solitons in the transverse direction (comparing the dash dotted curves in the two panels that correspond to higher values of ν_0).

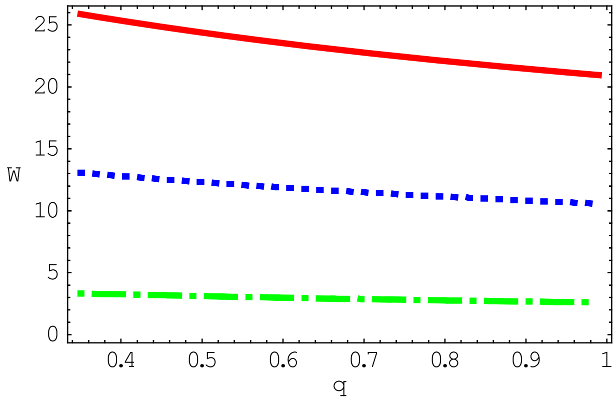

FIGURE 11. The width $W=\sqrt{4(B+C)/u_0}$ of the soliton is plotted for different values of magnetic field intensity B_0 against entropic indices q_e, q_p for the electrons and positrons. The plasma parameters are $u_0=0.3$, $\mu=1.2$, $p=0.2$, $T_e=T_p=10^3$ K, $T_i=0.2T_e$, $B_0=0.1$ G (red solid curve), $B_0=20$ G (blue dotted curve) and $B_0=10^2$ G (green dashed dotted curve).

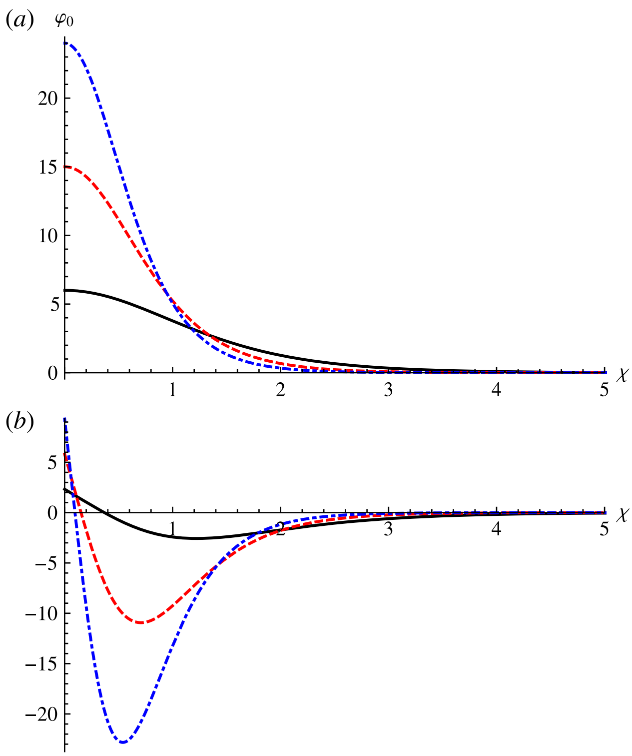

FIGURE 12. The variations of the planar solution, φ_0, given by (3.15), in (*a*) and $\psi=\psi_0+k\psi_1+\cdots$, given by (3.18), in (*b*) versus χ where $\nu_0=2$ (black solid curve), $\nu_0=5$ (red dashed curve) and $\nu_0=8$ (blue dashed dotted curve). Here, $k=0.75$.

5. Conclusion

To conclude, we have studied the nonlinear IASW in magnetized EPI plasmas. The electrons and positrons follow the q-non-extensive distribution while the ions are taken to be warm and inertial. By using the well-known RPM, we drive a ZK equation. The solution to the ZK equation is analysed for different parameters i.e. q_e, q_p, B_0, σ, p. It is evident that the non-extensive parameters q_e, q_p, positron concentration ratio p and the ion-to-electron temperature ratio σ effect on both the amplitude and the width of the soliton, whereas the magnetic field B_0 affects only the soliton width. The instability analysis reveals that the one-dimensional soliton solution suffers from an instability in the transverse direction. The instability growth rate of the IASW is a constant, cf. (3.32). Our theoretical results may be applied to systems with long-range interactions, such as space and astrophysical magnetized plasmas containing non-thermal electrons, positrons and warm ions.

REFERENCES

ABE, S. & OKAMOTO, Y. 2001 *Statistical Mechanics and its Applications*. Springer.

AKHTAR, N., EL-TAIBANY, W. F. & MAHMOOD, S. 2013 Electrostatic double layers in a warm negative ion plasma with nonextensive electrons. *Phys. Lett.* A **377**, 1282–1289.

ASBRIDGE, J. R., BAME, S. J. & STRONG, I. B. 1968 Outward flow of protons from the Earth's bow shock. *J. Geophys. Res.* **73** (17), 5777–5782.

BAINS, A. S., TRIBECHE, M., SAINI, N. S. & GILL, T. S. 2011 A nonlinear Zakharov–Kuznetsov equation in magnetized plasma with q-nonextensive electrons velocity distribution. *Phys. Plasmas* **18**, 104503.1–104503.4.

CHRISTON, S. P. *et al.* 2012 Energy spectra of plasma sheet ions and electrons from $\sim$50 eV e^{-1} to $\sim$1 MeV during plasma temperature transitions. *J. Geophys. Res.* **93**, 2562–2572.

DAS, K. P. & VERHEEST, F. 1989 Ion-acoustic solitons in magnetized multi-component plasmas including negative ions. *J. Plasma Phys.* **41**, 139–155.

DIVINE, N. & GARRET, H. B. 1983 Charged particle distributions in Jupiter's magnetosphere. *J. Geophys. Res.* **88**, 6889–6903.

DODD, R. K. *et al.* 1982 *Solitons and Nonlinear Waves Equations*. Academic.

DU, J. 2004 Nonextensivity in nonequilibrium plasma systems with Coulombian long-range interactions. *Phys. Lett.* A **329**, 262–267.

EL-TAIBANY, W. F., EL-BEDWEHY, N. A. & EL-SHAMY, E. F. 2011 Three-dimensional stability of dust-ion acoustic solitary waves in a magnetized multicomponent dusty plasma with negative ions. *Phys. Plasmas* **18**, 033703.1–033703.8.

EL-TAIBANY, W. F. & TRIBECHE, M. 2012 Nonlinear ion-acoustic solitary waves in electronegative plasmas with electrons featuring Tsallis distribution. *Phys. Plasmas* **19**, 024507.1–024507.4.

FUTAANA, Y. *et al.* 2003 Moon-related nonthermal ions observed by Nozomi: species, sources, and generation mechanisms. *J. Geophys. Res.* **108**, 15.1–15.10.

GHOSH, S. & CHAKRABART, N. 2013 Magnetic electron drift solitons in electron magnetohydrodynamic plasmas. *Plasma Phys. Control. Fusion* **55**, 035008.1–035008.4.

GHOSH, U. N., CHATTERJEE, P. & ROYCHOUDHURY, R. 2012 The effect of q-distributed electrons on the head-on collision of ion acoustic solitary waves. *Phys. Plasmas* **19**, 012113.1–012113.6.

GOSSET, W. 1908 The probable error of a mean. *Biometrika* **6**, 1–25.

HAQUE, Q., MAHMOOD, S. & MUSHTAQ, A. 2008 Nonlinear electrostatic drift waves in dense electron–positron–ion plasmas. *Phys. Plasmas* **15**, 082315.1–082315.5.

HELLBERG, M. A. *et al.* 2009 Comment on 'Mathematical and physical aspects of Kappa velocity distribution' [*Phys. Plasmas* 14, 110702 2007]. *Phys. Plasmas* **16**, 094701.1–094701.5.

HIROTANI, K. *et al.* 1999 Pair plasma dominance in the 3c 279 jet on parsec scales. *Publ. Astron. Soc. Japan* **51** (2), 263–267.

HUSSAIN, S., AKHTAR, N. & MAHMOOD, S. 2013 Propagation of ion acoustic shock waves in negative ion plasmas with nonextensive electrons. *Phys. Plasmas* **20**, 092303.1–092303.7.

IWAMOTO, N. 1993 Collective modes in nonrelativistic electron–positron plasmas. *Phys. Rev.* E **47**, 604–611.

JAIN, S. L., TIWARI, R. S. & SHARMA, S. R. 1990 Large-amplitude ion-acoustic double layers in multispecies plasma. *Can. J. Phys.* **68**, 474–478.

KOURAKIS, I. *et al.* 2009 Nonlinear dynamics of rotating multi-component pair plasmas and epi plasmas. *Plasma Fusion Res.* **4**, 018.1–018.11.

LATORA, V., RAPISARDA, A. & TSALLIS, C. 2001 Non-Gaussian equilibrium in a long-range Hamiltonian system. *Phys. Rev.* E **64**, 056134.1–056134.5.

LAZARUS, I. J. *et al.* 2012*a* Arbitrary amplitude Langmuir solitons in a relativistic electron–positron plasma. *J. Plasma Phys.* **78** (2), 175–180.

LAZARUS, I. J. *et al.* 2012*b* Linear electrostatic waves in two-temperature electron–positron plasmas. *J. Plasma Phys.* **78** (6), 621–628.

LEUBNER, M. P. 2004 Fundamental issues on kappa-distributions in space plasmas and interplanetary proton distributions. *Phys. Plasmas* **11**, 1308–1316.

LIMA, J. A. S., SILVA, J. R. & SANTOS, J. 2000 Plasma oscillations and nonextensive statistics. *Phys. Rev.* E **61**, 3260–3263.

LIU, J. M. *et al.* 1994 Measurements of inverse bremsstrahlung absorption and non-Maxwellian electron velocity distributions. *Phys. Rev. Lett.* **72** (17), 2717–2720.

LIYAN, L. & JIULIN, D. 2008 Ion acoustic waves in the plasma with the power-law q-distribution in nonextensive statistics. *Physica* A **387**, 4821–4827.

LONNGREN, K. E. 1998 Ion acoustic soliton experiments in a plasma. *Opt. Quant. Electron.* **30**, 615–630.

MACE, R. L. & HELLBERG, M. A. 2001 The Korteweg–de Vries–Zakharov–Kuznetsov equation for electron-acoustic waves. *Phys. Plasmas* **8**, 2649–2656.

MAHAJAN, S. M., BEREZHIANI, V. I. & MIKILASZEWSKI, R. 1998 On the robustness of the localized spatiotemporal structures in electron–positron–ion plasmas. *Phys. Plasmas* **5**, 3264–3269.

MAHMOOD, S. & AKHTAR, N. 2008 Ion acoustic solitary waves with adiabatic ions in magnetized electron–positron–ion plasmas. *Eur. Phys. J.* D **49**, 217–222.

MAHMOOD, S., AKHTAR, N. & UR-REHMAN, H. 2011 Acoustic solitons in magnetized quantum electron–positron plasmas. *Phys. Scr.* **83**, 035505–035510.

MAHMOOD, S., MUSHTAQ, A. & SALEEM, H. 2003 Ion acoustic solitary wave in homogeneous magnetized electron–positron–ion plasmas. *New J. Phys.* **5**, 28.1–28.10.

MAHMOOD, S. & UR-REHMAN, H. 2009 Electrostatic solitons in unmagnetized hot electron–positron–ion plasmas. *Phys. Lett.* A **373**, 2255–2259.

MAKSIMOVIC, M., PIERRARD, V. & RILEY, P. 1997 Ulysses electron distributions fitted with kappa functions. *Geophys. Res. Lett.* **24**, 1511–1519.

MILLER, H. R. & WIITA, P. J. 1987 *Active Galactic Nuclei*. Springer.

MOSLEM, W. M. *et al.* 2007 Solitary, explosive, and periodic solutions of the quantum Zakharov–Kuznetsov equation and its transverse instability. *Phys. Plasmas* **14**, 082308.1–082308.5.

MUNOZ, V. 2006 A nonextensive statistics approach for Langmuir waves in relativistic plasmas. *Nonlinear Process. Geophys.* **13**, 237–241.

MUSHTAQ, A., SAEED, R. & HAQUE, Q. 2009 Ion acoustic solitary waves in magnetized pair-ion electron plasmas. *Phys. Plasmas* **16**, 084501.1–084501.4.

NAKAMURA, Y. 1982 Experiments on ion-acoustic solitons in plasmas invited review article. *IEEE Trans. Plasma Sci.* **10**, 180–195.

NSENGIYUMVA, F. *et al.* 2014 Stopbands in the existence domains of acoustic solitons. *Phys. Plasmas* **21** (10), 102301.1–102301.8.

PAKZAD, H. R. & TRIBECHE, M. 2011 Electron acoustic double layers in a plasma with a q-nonextensive electron velocity distribution. *Astrophys. Space Sci.* **334**, 45–53.

POPEL, S. I., VLADIMIROV, S. V. & SHUKLA, P. K. 1995 Ion-acoustic solitons in electron–positron–ion plasmas. *Phys. Plasmas* **2** (3), 716–719.

RENYI, A. 1955 On a new axiomatic theory of probability. *Acta Math. Acad. Sci. Hung.* **6**, 285–335.

REYNOLDS, C. S. *et al.* 1996 The matter content of the jet in M87: evidence for an electron–positron jet. *Mon. Not. R. Astron. Soc.* **283**, 873–880.

SAHU, B. 2011 Ion acoustic solitary waves and double layers with nonextensive electrons and thermal positrons. *Phys. Plasmas* **18**, 082302.1–082302.6.

SAINI, N. S. & KHOLI, R. 2014 Electrostatic envelope excitations under transverse perturbations in a plasma with nonextensive hot electrons. *Astrophys. Space Sci.* **349**, 293–303.

SAINI, N. S. & SHALINI 2013 Ion acoustic solitons in a nonextensive plasma with multi-temperature electrons. *Astrophys. Space Sci.* **346**, 155–163.

SAITOU, Y. & NAKAMURA, Y. 2005 Ion-acoustic soliton-like waves undergoing Landau damping. *Phys. Lett.* A **343**, 397–402.

SAMANTA, U. K., SAHA, A. & CHATTERJEE, P. 2013 Bifurcations of nonlinear ion acoustic travelling waves in the frame of a Zakharov–Kuznetsov equation in magnetized plasma with a kappa distributed electron. *Phys. Plasmas* **20**, 052111.1–052111.5.

SHAN, S. A. & AKHTAR, N. 2013 Korteweg–de Vries equation for ion acoustic soliton with negative ions in the presence of nonextensive electrons. *Astrophys. Space Sci.* **346**, 367–374.

SILVA, R. JR., PLASTINO, A. R. & LIMA, J. A. S. 1998 A Maxwellian path to the q-nonextensive velocity distribution function. *Phys. Lett.* A **249** (5), 401–408.

SINGH, S. V. *et al.* 2013 Effect of ion temperature on ion-acoustic solitary waves in a magnetized plasma in presence of superthermal electrons. *Phys. Plasmas* **20**, 012306.1–012306.6.

STENFLO, L., SHUKLA, P. K. & YU, M. Y. 1985 Nonlinear propagation of electromagnetic waves in magnetized electron–positron plasmas. *Astrophys. Space Sci.* **117** (2), 303–308.

SULTANA, S. *et al.* 2010 Oblique electrostatic excitations in a magnetized plasma in the presence of excess superthermal electrons. *Phys. Plasmas* **17**, 032310.1–032310.10.

TARUYA, A. & SAKAGAMI, M. A. 2003 Long-term evolution of stellar self-gravitating systems away from thermal equilibrium: connection with nonextensive statistics. *Phys. Rev. Lett.* **90**, 181101.1–181101.4.

TIWARI, R. S., JAIN, S. L. & CHAWLA, J. K. 2007 Ion acoustic cnoidal waves and associated nonlinear ion flux in a warm ion plasma. *Phys. Plasmas* **14**, 022106.1–022106.9.

TRAN, M. Q. 1979 Ion acoustic solitons in a plasma: a review of their experimental properties and related theories. *Phys. Scr.* **20**, 317–327.

TRIBECHE, M., DJEBARNI, L. & AMOUR, R. 2010 Ion-acoustic solitary waves in a plasma with a q-nonextensive electron velocity distribution. *Phys. Plasmas* **17**, 042114.1–042114.6.

TSALLIS, C. 1988 Possible generalization of Boltzmann–Gibbs statistics. *J. Stat. Phys.* **52**, 479–487.

TSALLIS, C. 1995 Non-extensive thermostatistics: brief review and comments. *Physica* A **221**, 277–290.

VASYLIUNAS, V. M. 1968 A survey of low energy electrons in the evening sector of the magnetosphere with OGO-1 and OGO-3. *J. Geophys. Res.* **73**, 2839–2884.

VERHEEST, F. *et al.* 2006 On the existence of ion-acoustic double layers in two-electron temperature plasmas. *Phys. Plasmas* **13**, 042301.1–042301.9.

VERHEEST, F. 2013 Ambiguities in the Tsallis description of non-thermal plasma species. *J. Plasma Phys.* **79** (6), 1031–1034.

WASHIMI, H. & TANIUTI, T. 1966 Propagation of ion-acoustic solitary waves of small amplitude. *Phys. Rev. Lett.* **17**, 996–997.

YU, M. Y., SHUKLA, P. K. & STENFLO, L. 1986 Alfven vortices in a strongly magnetized electron-position plasma. *Astrophys. J.* **309**, L63–L66.

ZAKHAROV, V. E. & KUZNETSOV, E. A. 1974 Three-dimensional solitons. *Sov. Phys. JETP* **39**, 285–286.

ZANK, G. P. & GREAVES, R. G. 1995 Linear and nonlinear modes in nonrelativistic electron–positron plasmas. *Phys. Rev.* E **51**, 6079–6090.