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Stability of three-dimensional obliquely propagating dust acoustic waves in dusty plasma including the polarization force effect

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Abstract. Propagation of dust acoustic solitary waves (DASWs) in a magnetized dusty plasma consisting of extremely massive, negatively/positively charged dust fluid and Boltzmann distributed electrons and ions is studied. A nonlinear Zakharov-Kuznetsov (ZK) equation adequate for describing the solitary waves is derived by applying a reductive perturbation technique. Moreover, an extended Zakharov Kuznetsov (EZK) equation is derived at the vicinity of the critical phase velocity. The effects of the polarization force are explicitly discussed and the growth rate of the produced waves is calculated. It is found that the physical parameters have strong effects on the instability criterion as well as on the growth rate. It is noted that the phase velocity decreases as the polarization force, the effective-to-ion temperature ratio, and the ion-to-electron temperature ratio increase. Moreover, the nonlinearity coefficient and the critical phase velocity increase by increasing the polarization force. The relevance of these findings to a recent plasma experiment and astrophysical plasma observations is briefly discussed.

1 Introduction

Few decades ago, there has been a great research interest in the study of different types of collective processes in dusty plasmas [1–4] because of their vital roles in laboratory, space, and astrophysical environments. It has been shown both theoretically and experimentally that the presence of extremely massive and highly charged dust grains modifies the existing plasma wave modes, whereas their dynamics introduce new eigenmodes [1,2]. Those of low frequency are named as dust acoustic solitary waves (DASWs) [1,3] where the dust particles provide the inertia and the pressures of inertialess electrons and ions provide the restoring force [4]. Dusty plasmas with two opposite polarity dusts have been found in different regions of space, *e.g.*, Jupiter's magnetosphere [5,6], cometary tails [6], Earth's mesosphere [7], etc. The consideration of negatively charged dust is based on the fact that in low-temperature plasmas, the collections of plasma particles (electrons and ions) are the only important charging process.

Based on the theoretical predictions and satellite observations, Mamun [8], El-Labany *et al.* [9], Chow *et al.* [10], and Mendis [11] have investigated dusty plasma models containing both negatively and positively charged dust particles, positively charged ions, and Boltzmann electrons. These dusty plasmas are particularly relevant to cometary tails, upper mesosphere, Jupiter, Saturn, comets, and in the interplanetary medium.

A limited number of research work regarding three-dimensional DASWs are focused on a magnetized three-component dusty plasma. The first attempt to model a three-dimensional magnetized plasma system was done by Zakharov and Kuznetsov [12]. So, it is among our interest in the present work to study the propagation characteristics and the stability of three-dimensional DASWs in a magnetized dusty plasma.

On the other side, few researchers [13–16] studied the instability of three-dimensional solitary waves using small-k expansion perturbation method in dusty plasmas. For example, Mamun [14] and Mamun *et al.* [15] have investigated the instability of DASW propagating obliquely in a magnetized dusty plasma. They found that the obliqueness angle and the magnitude of the external magnetic field leads to unstable wave structures as well as change drastically wave main characteristics (the amplitude and the width). Three-dimensional DASWs and their instability in a magnetized dusty plasma have been studied by employing the small-k expansion method [17]. It is found that the basic features of the DASWs, the instability criterion and the growth rate are significantly modified by the presence of opposite polarity

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dust particles and the external magnetic field. Recently, El-Labany *et al.* [18] studied the stability of three-dimensional DASWs in a four component dusty plasma with two opposite polarity dust species. They investigated the instability criterion for obliquely propagating DASWs in such plasma.

The dynamics and dust particles wave motion are obviously influenced by the deformation of Debye sheath around the dust grains (*i.e.*, the polarization force) due to the presence of the electric field of other plasma species [19,20]. In the last few years, some researchers considered the effect of the polarization force on DASWs and plasma instabilities [21–26]. For instance, Khrapak *et al.* [21] have investigated theoretically the effect of the polarization force on the propagation of DASWs in a dusty plasma model, and they stated that the DASWs instability is driven by the polarization force. They demonstrated that the phase velocity of DASWs decreases as the polarization interaction increases. Then the theoretical predictions of ref. [21] were proved experimentally by Heinrich *et al.* [22] in a dc-glow-discharge dusty plasma. Recently, Prajapati [25,26] has illustrated the influence of the polarization force on the Jeans instability of interstellar dusty plasma situation. He found that the polarization force results in an increase of the growth rate of Jeans instability.

To the best of our knowledge, the instability of three-dimensional DASWs in a three-component magnetized dusty plasma including the polarization force effect has not been addressed yet. In this paper, we propose a model for describing three-dimensional DASWs in a magnetized dusty plasma containing extremely massive, negatively/positively charged dust fluid and Boltzmann distributed electrons and ions, taking into account the polarization force effect. This paper is organized in the following fashion. In sect. 2, the basic set of equations governing the plasma system under consideration is presented and a nonlinear Zakharov-Kuznetsov (ZK) equation is derived by employing a reductive perturbation technique (RPT). Also, the extended ZK (EZK) equation, adequate for describing the propagation of the DASWs at the vicinity of critical phase velocity is deduced. The stability analysis of the EZK equation is discussed in sect. 3. Section 4 is devoted for the conclusion.

2 The basic equations-Derivation of ZK and EZK equations

We consider a three-component dusty plasma consisting of extremely massive, negatively/positively charged dust fluid and Boltzmann distributed electrons and ions in the presence of an external static magnetic field ($B_o \| \hat{z}$) where $\hat{z}$ is the unit vector along the z-direction). Thus, at equilibrium we have $n_{io} + sZ_d n_{do} = n_{eo}$, where n_{io}, n_{do} and n_{eo} are the unperturbed number densities of ions, dust grains and electrons, respectively. Z_d is the equilibrium number of charges residing on the dust grain surface and $s = -1$ (1) for negative (positive) dust particles.

For the low phase velocity (in comparison with electron and ion thermal velocities) DASWs, the electron and ion number densities obey the Maxwellian distribution, and their dimensionless number densities are, respectively, given by $n_e = \nu e^{\beta\sigma\varphi}$ and $n_i = \mu e^{-\sigma\varphi}$, where n_e and n_i are normalized by $Z_d n_{do}$. $\beta = T_i/T_e$, $\nu = n_{eo}/Z_d n_{do}$, $\mu = n_{io}/Z_d n_{do}$ and $\sigma = T_{\text{eff}}/T_i = (\mu + \nu\beta)^{-1}$. $T_i(T_e)$ is the ion (electron) temperature in energy units. While, T_{eff} is the effective temperature defined by $T_{\text{eff}} = T_e T_i/(T_e + T_i)$.

Therefore, the dynamics of low phase velocity (lying between the ion and dust thermal velocities, viz, $v_{td} \ll v_p \ll v_{ti}$) DASWs are governed by

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \boldsymbol{u_d}) = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u_d}}{\partial t} + (\boldsymbol{u_d} \cdot \nabla)\boldsymbol{u_d} + s\left[1 - R_j\left(\frac{n_j}{\mu_j}\right)^{1/2}\right]\nabla\varphi + \frac{\gamma}{n_d}\nabla n_d^{5/3} - s\Omega(\boldsymbol{u_d} \times \hat{\boldsymbol{z}}) = 0, \tag{2}$$

$$\nabla^2\varphi = -sn_d + n_e - n_i, \tag{3}$$

where n_d (normalized by n_{do}) is the dimensionless dust number density, where n_{do} is the dimensional dust number density at equilibrium, u_d is the dust fluid velocity normalized by the dust acoustic speed $C_d = (Z_d T_{\text{eff}}/m_d)^{1/2}$. m_d is the mass of charged dust particles, the electrostatic wave potential φ is normalized by T_{eff}/e and e is the magnitude of the electron charge $\gamma = T_d/T_{\text{eff}}$. The time and space variables are normalized by the dust plasma period $\omega_{pd}^{-1} = (m_d/4\pi n_{do} Z_d^2 e^2)^{1/2}$ and the Debye length $\lambda_{Dd} = (T_{\text{eff}}/4\pi Z_d n_{do} e^2)^{1/2}$, respectively $\Omega = eZ_d B_o/m_d$ is the dust cyclotron frequency normalized by ω_{pd}.

The interaction between thermal ions (electrons) and highly negatively (positively) charged dust grains is termed as the polarization force. The concept of polarization force (acting on a charged dust grain) and its importance in dusty plasma physics have been explained by Hamaguchi and Farouki [19,20]. The polarization force (F_p) acting on a dust grain is mathematically defined as [19–21]

$$F_p = -\frac{q_d^2 \nabla \lambda_D}{2\lambda_D^2},$$

where $\lambda_D = \lambda_{Di(e)} \Big/ \left[1 + \frac{\lambda_{Di(e)}^2}{\lambda_{De(i)}^2}\right]^{\frac{1}{2}}$ and $\lambda_{Di(e)} = [T_{i(e)}/4\pi e^2 n_{i(e)}]^{1/2}$.

Now, using $T_i n_e \ll T_e n_i$ ($T_e n_i \ll T_i n_e$) which is a good approximation [19,20] for any dusty plasma with highly negatively (positively) charged dust particles, *i.e.* for $q_d = sZ_d e$ with $T_i \nabla n_i = -n_i e \boldsymbol{\nabla}\varphi$ ($T_e \nabla n_e = n_e e \boldsymbol{\nabla}\varphi$), one can simplify the polarization force as $F_p = sZ_d e R_j (n_j/n_{jo})^{1/2} \boldsymbol{\nabla}\varphi$, where $j = e(i)$ for electron (ion), n_{jo} is the number density of species j at equilibrium and R_j is a parameter determining the effects of the polarization force. Where, $R_j = R_i = Z_d e^2/4T_i \lambda_{Dio}$ for negatively charged dust grain, while $R_j = R_e = Z_d e^2/4T_e \lambda_{Deo}$ for positively charged dust. It is obvious that i) the polarization force is oppositely directed to the electrostatic force and ii) in a dusty plasma with highly negatively (positively) charged dust, the polarization force arises mainly due to the polarization of plasma ions (electrons) around the dust grain. It is worth to mention that the polarization term that appeared in the momentum equation, (2) takes the form $-R_i\left(\frac{n_i}{\mu}\right)^{1/2}\boldsymbol{\nabla}\varphi$ $\left[R_e\left(\frac{n_e}{\nu}\right)^{1/2}\boldsymbol{\nabla}\varphi\right]$ in the case of negatively [positively] charged dust grain where $s = -1$, $n_j = n_i$ and $\mu_j = \mu[s = +1$, $n_j = n_e$ and $\mu_j = \nu]$, respectively. Thereafter, in the following calculations, we will use Λ_j as $\Lambda_j = -\sigma$ for $s = -1$ but $\Lambda_j = \beta\sigma$ for $s = +1$.

In order to study the nonlinear propagation of DAWs in such magnetized dusty plasma, we employ the standard RPT [27]. According to this method, the independent variables are stretched as

$$X = \epsilon^{1/2} x, \qquad Y = \epsilon^{1/2} y, \qquad Z = \epsilon^{1/2}(z - \lambda t) \qquad \text{and} \qquad T = \epsilon^{3/2} t, \tag{4}$$

where ϵ is a small parameter measuring the amplitude of the wave perturbation, λ is the normalized velocity of the moving frame and it will be determined later self-consistently. We can now expand the physical quantities about their equilibrium values in power series of ϵ as

$$\left.\begin{aligned} n_d &= 1 + \epsilon n_{d1} + \epsilon^2 n_{d2} + \epsilon^3 n_{d3} + \ldots, \\ u_{x,y} &= \epsilon^{3/2} u_{x,y1} + \epsilon^2 u_{x,y2} + \epsilon^{5/2} u_{x,y3} + \ldots, \\ u_z &= \epsilon u_{z1} + \epsilon^2 u_{z2} + \epsilon^3 u_{x3} + \ldots, \\ \varphi &= \epsilon\varphi_1 + \epsilon^2\varphi_2 + \epsilon^3\varphi_3 + \ldots. \end{aligned}\right\} \tag{5}$$

Substituting eqs. (4) and (5) into the set of eqs. (1)–(3) and collecting terms of the same powers of ϵ. For the lowest orders, we get

$$n_{d1} = N\varphi_1, \qquad u_{x1} = \frac{s\lambda N}{\Omega}\frac{\partial\varphi_1}{\partial Y}, \qquad u_{y1} = -\frac{s\lambda N}{\Omega}\frac{\partial\varphi_1}{\partial X} \qquad \text{and} \qquad u_{z1} = \lambda N\varphi_1, \tag{6}$$

where $N = as(1 - R_j)$, $a = (\lambda^2 - \frac{5}{3}\gamma)^{-1}$ and $s = \nu - \mu$.

Moreover, we can derive the linear phase velocity that is given by

$$\lambda = \sqrt{\frac{5\gamma}{3} + \frac{1 - R_j}{\sigma(\nu\beta + \mu)}}\,. \tag{7}$$

The dependences of the phase velocity λ (represented by eq. (7)) on the polarization parameter, R_j, the effective-to-ion temperature ratio, σ and the ion-to-electron temperature ratio, β for negatively (positively) charged dust are demonstrated in fig. 1. Figure 1 shows that λ decreases with an increase of $R_{i,e}$, σ and β in both different cases of dust charge.

For the next higher-order of ϵ, we can calculate the transverse velocities as

$$u_{x2} = \frac{\lambda^2 N}{\Omega^2}\frac{\partial^2\varphi_1}{\partial Z\partial X} \qquad \text{and} \qquad u_{y2} = \frac{\lambda^2 N}{\Omega^2}\frac{\partial^2\varphi_1}{\partial Z\partial Y}\,. \tag{8}$$

For $O(\epsilon^2)$ perturbed quantities, eqs. (1)–(3) lead to the following set of nonlinear partial differential equations:

$$\left.\begin{aligned} &-\lambda\frac{\partial n_{d2}}{\partial Z} + \frac{\partial u_{z2}}{\partial Z} + \frac{\partial u_{x2}}{\partial X} + \frac{\partial u_{y2}}{\partial Y} = -\frac{\partial n_{d1}}{\partial T} - \frac{\partial(n_{d1}u_{z1})}{\partial Z}, \\ &-\lambda\frac{\partial u_{z2}}{\partial Z} + \frac{5\gamma}{3}\frac{\partial n_{d2}}{\partial Z} + s(1 - R_j)\frac{\partial\varphi_2}{\partial Z} = -\frac{\partial u_{z1}}{\partial T} - u_{z1}\frac{\partial u_{z1}}{\partial Z} + \frac{s}{2}R_j\Lambda_j\varphi_1\frac{\partial\varphi_1}{\partial Z} + \frac{5\gamma}{9}n_{d1}\frac{\partial n_{d1}}{\partial Z}, \\ &\frac{\partial^2\varphi_1}{\partial X^2} + \frac{\partial^2\varphi_1}{\partial Y^2} + \frac{\partial^2\varphi_1}{\partial Z^2} = -sn_{d2} + \sigma(\nu\beta + \mu)\varphi_2 + \frac{\sigma^2}{2}(\nu\beta^2 - \mu)\varphi_1^2, \end{aligned}\right\} \tag{9}$$

solving eqs. (9) by eliminating the second-order perturbed quantities and with the aid of eqs. (6)–(8), we can readily obtain the following ZK equation:

$$\frac{\partial\varphi_1}{\partial T} + A\varphi_1\frac{\partial\varphi_1}{\partial Z} + B\frac{\partial^3\varphi_1}{\partial Z^3} + C\left(\frac{\partial^3\varphi_1}{\partial Z\partial X^2} + \frac{\partial^2\varphi_1}{\partial Z\partial Y^2}\right) = 0, \tag{10}$$

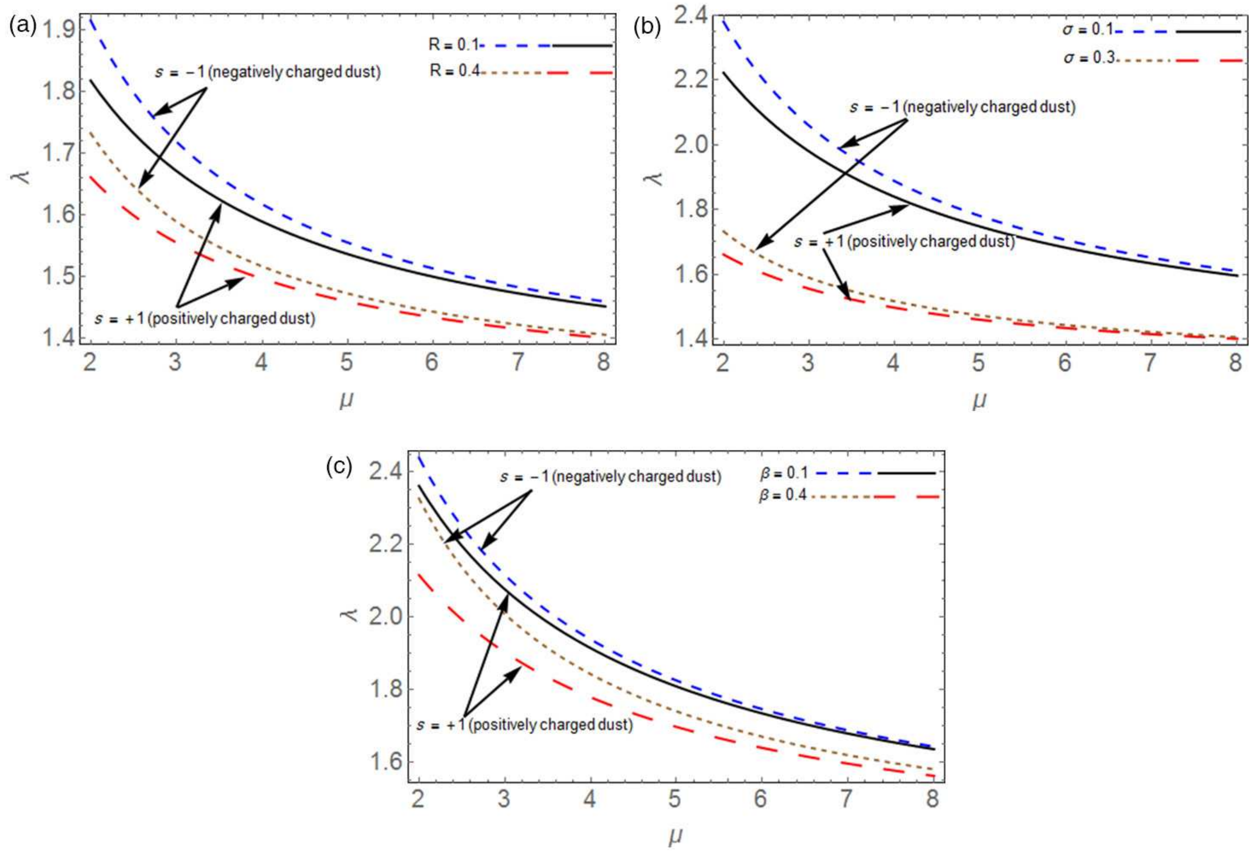

Fig. 1. The variation of λ against μ for different values of $R(\equiv R_{i,e})$ with $\beta = 0.25$, $\gamma = 1$ and $\sigma = 0.2$ in (a) and for different values of σ in (b) with $\beta = 0.25$, $\gamma = 1$ and $R = 0.1$. In (c). It is plotted for different values of β with $R = 0.1$, $\gamma = 1$.

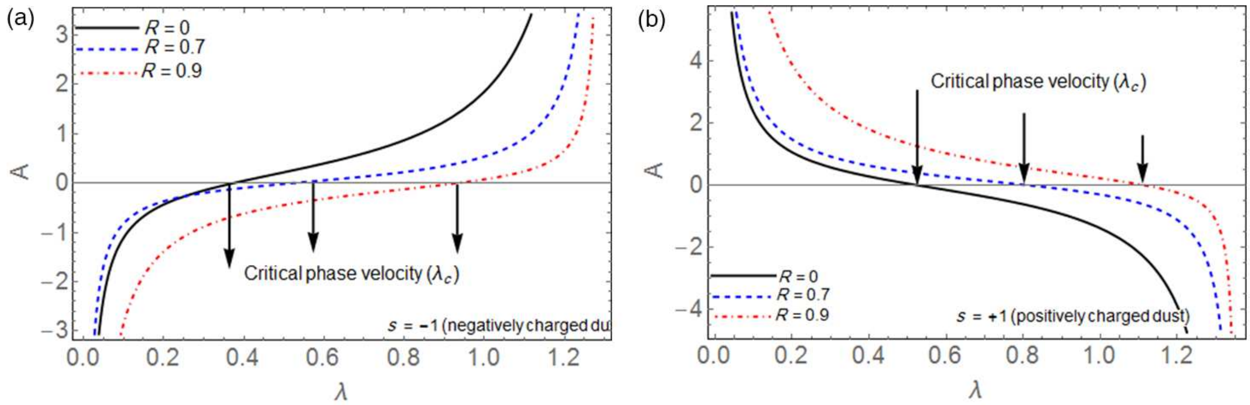

Fig. 2. Variation of the nonlinear coefficient, A, against phase velocity, λ, for different values of $R(\equiv R_{i,e})$, in (a) for negatively charged dust grain where $\gamma = 1$, $\sigma = 0.2$, $\nu = 2$, $\beta = 0.25$ and $\mu = 3$ and in (b) for positively charged dust.

where

$$A = \frac{1}{2\lambda N}\left\{\left(3\lambda^2 - \frac{5\gamma}{9}\right)N^2 - \frac{s}{2}\left[R_j \Lambda_j + \sigma^2(\mu - \nu\beta^2)\right]\right\}, \qquad B = 1/(2Nas)$$

and

$$C = B\left[1 + (1 - R_j)\left(\frac{a\lambda^2}{\Omega}\right)^2\right].$$

The dependence of the nonlinearity coefficient, A, on λ and R_j for negatively (positively) charged dust is illustrated in fig. 2. It is noted that the critical phase velocity λ_c (at which $A = 0$) is strongly influenced by the variation of the polarization term. For compressive soliton, the nonlinear coefficient A increases (decreases) by increasing λ for plasma models containing negative (positive) dust grains. However vice versa is occurred when we study the rarefactive soliton ($A < 0$). Moreover, the critical phase λ_c (λ at $A = 0$) velocity increases as the polarization force increases for both cases of positive/negative dust grains.

At the critical phase velocity ($\lambda = \lambda_c$), $A = 0$ which results as eq. (10) fails to describe the nonlinear propagation of the DAWs in the suggested model. Therefore, we must replace eq. (4) by new modified stretched variables which are defined as

$$X = \epsilon x, \qquad Y = \epsilon y, \qquad Z = \epsilon(z - \lambda t), \qquad \text{and} \qquad T = \epsilon^3 t. \tag{11}$$

Consequently, the plasma physical dependent parameters are expanded as

$$\left.\begin{aligned} n_d &= 1 + \epsilon n_{d1} + \epsilon^2 n_{d2} + \epsilon^3 n_{d3} + \dots, \\ u_{x,y} &= \epsilon^2 u_{x,y1} + \epsilon^3 u_{x,y2} + \epsilon^4 u_{x,y3} + \dots, \\ u_z &= \epsilon u_{z1} + \epsilon^2 u_{z2} + \epsilon^3 u_{z3} + \dots, \\ \varphi &= \epsilon \varphi_1 + \epsilon^2 \varphi_2 + \epsilon^3 \varphi_3 + \dots. \end{aligned}\right\}. \tag{12}$$

Using eqs. (11) and (12) in the basic equations (1)–(3), we get for the lowest orders of ϵ the same relations as presented in eq. (6). The next higher order of ϵ leads to a system of equations, from which we can evaluate the second order-perturbed quantities that are given as

$$\left.\begin{aligned} n_{d2} &= N\varphi_2 + \frac{1}{2}a\left[\left(3\lambda^2 - \frac{5\gamma}{9}\right)N^2 - \frac{s}{2}R_j\Lambda_j\right]\varphi_1^2, \\ u_{x2} &= \frac{\lambda^2}{\Omega}\left[sN\frac{\partial \varphi_2}{\partial Y} + \frac{\lambda N}{\Omega}\frac{\partial^2 \varphi_1}{\partial Z \partial X} + a\left(\frac{40}{9}\gamma s N^2 - \frac{s}{2}R_j\Lambda_j\right)\varphi_1\frac{\partial \varphi_1}{\partial Y}\right], \\ u_{y2} &= \frac{\lambda^2}{\Omega}\left[-sN\frac{\partial \varphi_2}{\partial X} + \frac{\lambda N}{\Omega}\frac{\partial^2 \varphi_1}{\partial Z \partial Y} - a\left(\frac{40}{9}\gamma s N^2 - \frac{s}{2}R_j\Lambda_j\right)\varphi_1\frac{\partial \varphi_1}{\partial X}\right], \\ u_{z2} &= \lambda\left\{N\varphi_2 + \frac{1}{2}a\left[\left(\lambda^2 + \frac{25\gamma}{9}\right)N^2 - \frac{s}{2}R_j\Lambda_j\right]\varphi_1^2\right\}, \end{aligned}\right\} \tag{13}$$

while Poisson's equation, eq. (3), gives

$$\varphi_2[sN - \sigma(\nu\beta + \mu)] = \left\{\sigma^2(\nu\beta^2 - \mu) - \frac{sa}{2}\left[\left(3\lambda^2 - \frac{5\gamma}{9}\right)N^2 - \frac{s}{2}R_j\Lambda_j\right]\right\}\varphi_1^2, \tag{14}$$

that can be rewritten as $\varphi_2[sN - \sigma(\nu\beta + \mu)] = Q\varphi_1^2$. It is obvious from eq. (14) that the coefficient of φ_2 is identically zero, because of the linear dispersion relation (7). As $\varphi_1 \neq 0$, it is supposed that Q could be at least of order ϵ and this term should be included in the next order of the equation of motion.

If we consider the next order in the RPT, we obtain the following set of equations:

$$\left.\begin{aligned} &-\lambda\frac{\partial n_{d3}}{\partial Z} + \frac{\partial u_{z3}}{\partial Z} + \frac{\partial u_{x2}}{\partial X} + \frac{\partial u_{y2}}{\partial Y} + \frac{\partial n_{d1}u_{x1}}{\partial X} + \frac{\partial n_{d1}u_{y1}}{\partial Y} = -\frac{\partial(n_{d1}u_{z2} + n_{d2}u_{z1})}{\partial Z} - \frac{\partial n_{d1}}{\partial T}, \\ &-\lambda\frac{\partial u_{z3}}{\partial Z} + \frac{5\gamma}{3}\frac{\partial n_{d3}}{\partial Z} + s(1 - R_j)\frac{\partial \varphi_3}{\partial Z} = -\frac{\partial u_{z1}}{\partial T} + \frac{s}{2}R_j\Lambda_j\left(\frac{\partial \varphi_1\varphi_2}{\partial Z} + \frac{1}{4}\Lambda_j\varphi_1^2\frac{\partial \varphi_1}{\partial Z}\right) \\ &-\frac{\partial u_{z1}u_{z2}}{\partial Z} + \frac{5\gamma}{9}\left(\frac{\partial n_{d1}n_{d2}}{\partial Z} + \frac{2}{3}n_{d1}^2\frac{\partial n_{d1}}{\partial Z}\right) - 2Qs(1 - R_j)\varphi_1\frac{\partial \varphi_1}{\partial Z}, \\ &\frac{\partial^2 \varphi_1}{\partial X^2} + \frac{\partial^2 \varphi_1}{\partial Y^2} + \frac{\partial^2 \varphi_1}{\partial Z^2} = -sn_{d3} + \sigma(\nu\beta + \mu)\varphi_3 + \sigma^2(\nu\beta^2 - \mu)\varphi_1\varphi_2 + \frac{1}{6}\sigma^3(\nu\beta^3 + \mu)\varphi_1^3, \end{aligned}\right\} \tag{15}$$

solving this set of equations, eq. (15), by eliminating the third-order perturbed quantities, we finally obtain the extended ZK (EZK) equation for the first-order perturbed potential $\varphi^{(1)}$ as

$$\frac{\partial \varphi_1}{\partial T} + Q\varphi_1\frac{\partial}{\partial Z}\varphi_1 + D\varphi_1^2\frac{\partial \varphi_1}{\partial Z} + B\frac{\partial^3 \varphi_1}{\partial Z^3} + C\left(\frac{\partial^3 \varphi_1}{\partial Z \partial X^2} + \frac{\partial^2 \varphi_1}{\partial Z \partial Y^2}\right) = 0, \tag{16}$$

where

$$D = \frac{1}{2\lambda N}\left\{\left(\frac{15}{2}\lambda^4 + \frac{15}{2}\lambda^4 - \frac{5\gamma}{9}\right)N^2 - \frac{s}{2}R_j\Lambda_j - \frac{s}{2}\sigma^2(\mu - \nu\beta^2)\right\}.$$

It is remarked here that eq. (16) consists of the nonlinear term $\varphi_1\frac{\partial \varphi_1}{\partial Z}$ of the ZK equation besides the higher-order nonlinearity term $\varphi_1^2\frac{\partial \varphi_1}{\partial Z}$. The presence of lower-order nonlinearity in ZK equation describes the solitary waves, while the presence of both nonlinear terms in EZK equation allows the propagation of either solitary or shock waves.

3 The instabilities growth rate of solitary-wave solution of EZK equation

The EZK equation (16) can be transformed into the following form, using the variables $\tilde{X} = \sqrt{\frac{B}{C}}X$ and $\tilde{Y} = \sqrt{\frac{B}{C}}Y$:

$$\varphi_{1t} + (Q\varphi_1 + D\varphi_1^2)\varphi_{1Z} + B\nabla^2\varphi_{1Z} = 0, \tag{17}$$

where the subscripts denote partial differentiation. In order to get the solitary pulse solution, following Hongsit *et al.* [28] and introducing

$$\varphi_1' = \frac{\varphi_1}{|Q|}, \qquad t' = \frac{|Q|^3 t}{\sqrt{|B|}}, \qquad \text{and} \qquad (X', Y', Z') = \frac{|Q|}{\sqrt{|B|}}(\tilde{X}, \tilde{Y}, Z),$$

we obtain

$$\varphi_{1t} + (\sigma_Q\varphi_1 + D\varphi_1^2)\varphi_{1Z} + \sigma_B\nabla^2\varphi_{1Z} = 0, \tag{18}$$

where $\sigma_Q \equiv \text{sgn(Q)}$ and $\sigma_B \equiv \text{sgn(B)}$. This equation has plane solitary pulse solution when $\sigma_B\text{D} > 0$ or $D = 0$, $\sigma_B = 1$. The solution can be written as [28,29]

$$\varphi_1(Z,t) = \frac{\sigma_Q\sigma_B\beta_1}{\alpha + \cosh[\eta(Z - Vt)]}, \tag{19}$$

where η is a free real parameter, $V = \sigma_B\eta^2$, and $\beta_1 = 6\alpha|V|$, $\alpha = \frac{1}{\sqrt{1+6DV}}$. The dependence of the electrostatic potential $\varphi_1(Z,t)$ $[\equiv \Phi_1]$ (given by equation (19)) on μ, ν, β and R_j is depicted in fig. 3. It is obvious that increasing ν, β, μ and R_j leads to an increase in $\varphi_1(Z,t)$ in the presence of negatively charged dust. While, in the presence of positively charged dust, $\varphi_1(Z,t)$ decreases with the increase of ν and β but it increases with an increase in either μ or R_j.

Before performing the stability analysis, we can simplify our system further by transforming eq. (18) to a frame moving at speed V along the Z-axis and then making the change of variables $\varphi_1' = \eta^{-2}\varphi_1$, $(t', X', Y', Z') = \eta(\eta^2 t, \tilde{X}, \tilde{Y}, Z)$, $B' = \eta^2 D$. Dropping the primes once again and multiplying by σ_B leaves the EZK equation in the form

$$\sigma_B\varphi_{1t} + (\sigma_Q\sigma_B\varphi_1 + \sigma_B D\varphi_1^2 - 1)\varphi_{1Z} + \nabla^2\varphi_{1Z} = 0. \tag{20}$$

After replacing V by σ_B in the definitions of α and β_1, the plane solitary pulse solution reduces to

$$\varphi_o(Z) = \frac{\sigma_Q\sigma_B\beta_1}{\alpha + \cosh Z}. \tag{21}$$

To determine the growth rate of long-wavelength transverse instabilities of wave number k, we use the small-k expansion perturbation method [30,31]. We proceed by writing

$$\varphi_1(X,Y,Z,t) = \varphi_o(Z) + \varepsilon\phi(Z)e^{ikX+g_r t}, \tag{22}$$

where g_r is the growth rate of the perturbation $\varepsilon\phi(Z)e^{ikX}$, and $\varepsilon \ll k^2$. Without loss of generality, we have taken the transverse perturbation to be in the X-direction. Substituting eq. (22) into (20) and linearizing with respect to ε, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}Z}L\phi = -\sigma_B g_r\phi + k^2\frac{\mathrm{d}\phi}{\mathrm{d}Z}, \tag{23}$$

where $L = \frac{\mathrm{d}^2}{\mathrm{d}Z^2} + \sigma_Q\sigma_B\varphi_o + \sigma_B D\varphi_o^2 - 1$.

We expand ϕ and g_r as

$$\phi = \phi_o + k\phi_1 + k^2\phi_2 + \ldots, \tag{24}$$

$$g_r = kg_{r1} + k^2 g_{r2} + \ldots, \tag{25}$$

and substitute for them in eq. (23). Equating ascending powers of k in the resulting equation leads to a series of equations that is used to find the ϕ_i and then the g_{ri}. At lowest order, we have $\frac{\mathrm{d}}{\mathrm{d}Z}L\phi_o = 0$, which has the solution $\phi_o = \varphi_{oZ}$ by virtue of the Z-direction translational invariance of the unperturbed solution. Integrating this equation, we get

$$L\phi_1 = -\sigma_B g_{r1}\varphi_o. \tag{26}$$

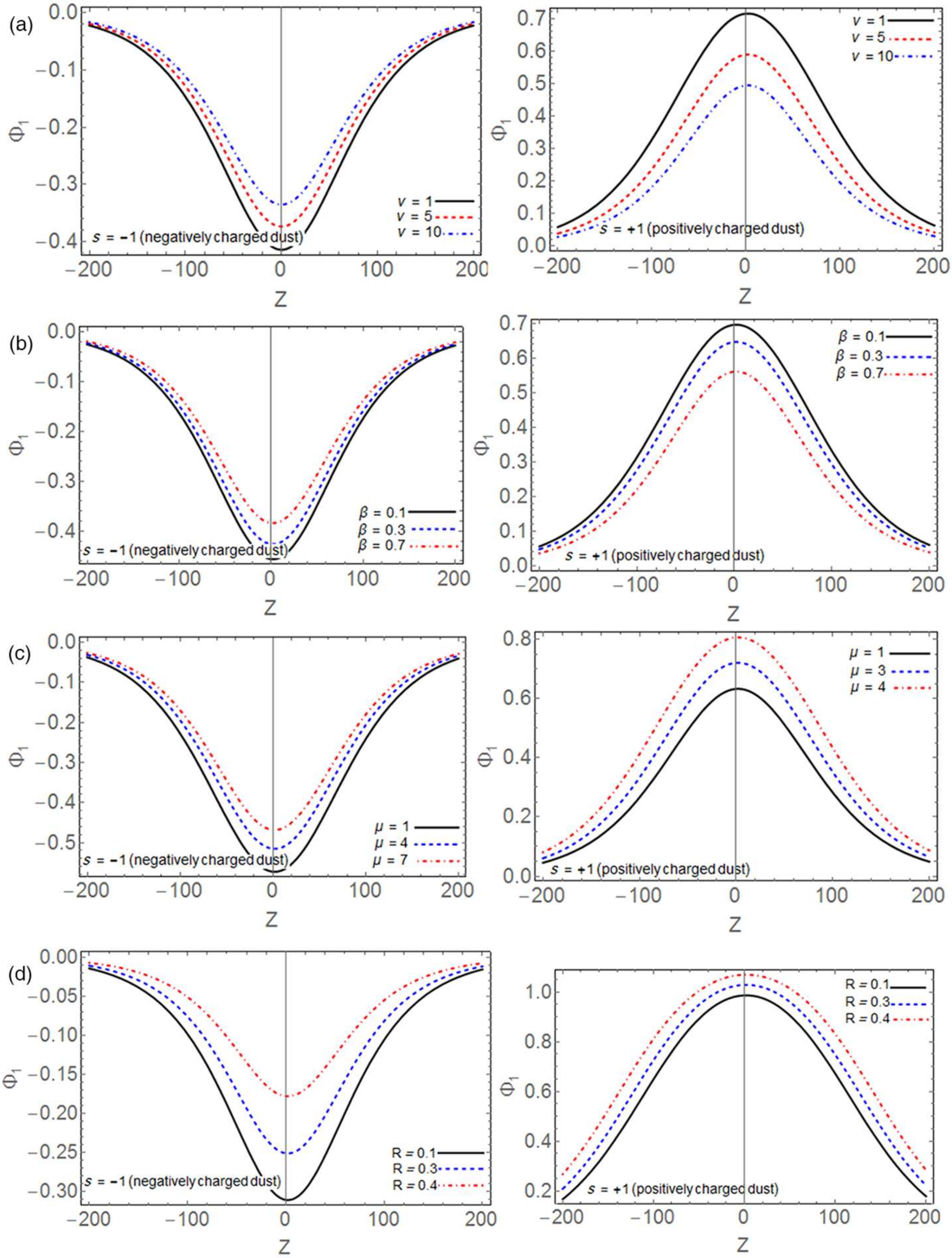

Fig. 3. Variation of Φ_1 $[= \varphi_1(z,t)]$ *vs.* Z for different values of ν where $\beta = 0.25$, $R = 0.1$, $\sigma = 0.4$, $\lambda = 0.5$ and $\gamma = 1.7$ for negatively (positively) charged dust in (a), for different values of β for both polarities in (b) where $\nu = 1$, $R = 0.1$, $\gamma = 0.7$, $\mu = 2$ and $\sigma = 0.6$, in (c) for different values of μ for negatively (positively) charged dust, respectively, and in (d) for different values of $R(\equiv R_{i,e})$.

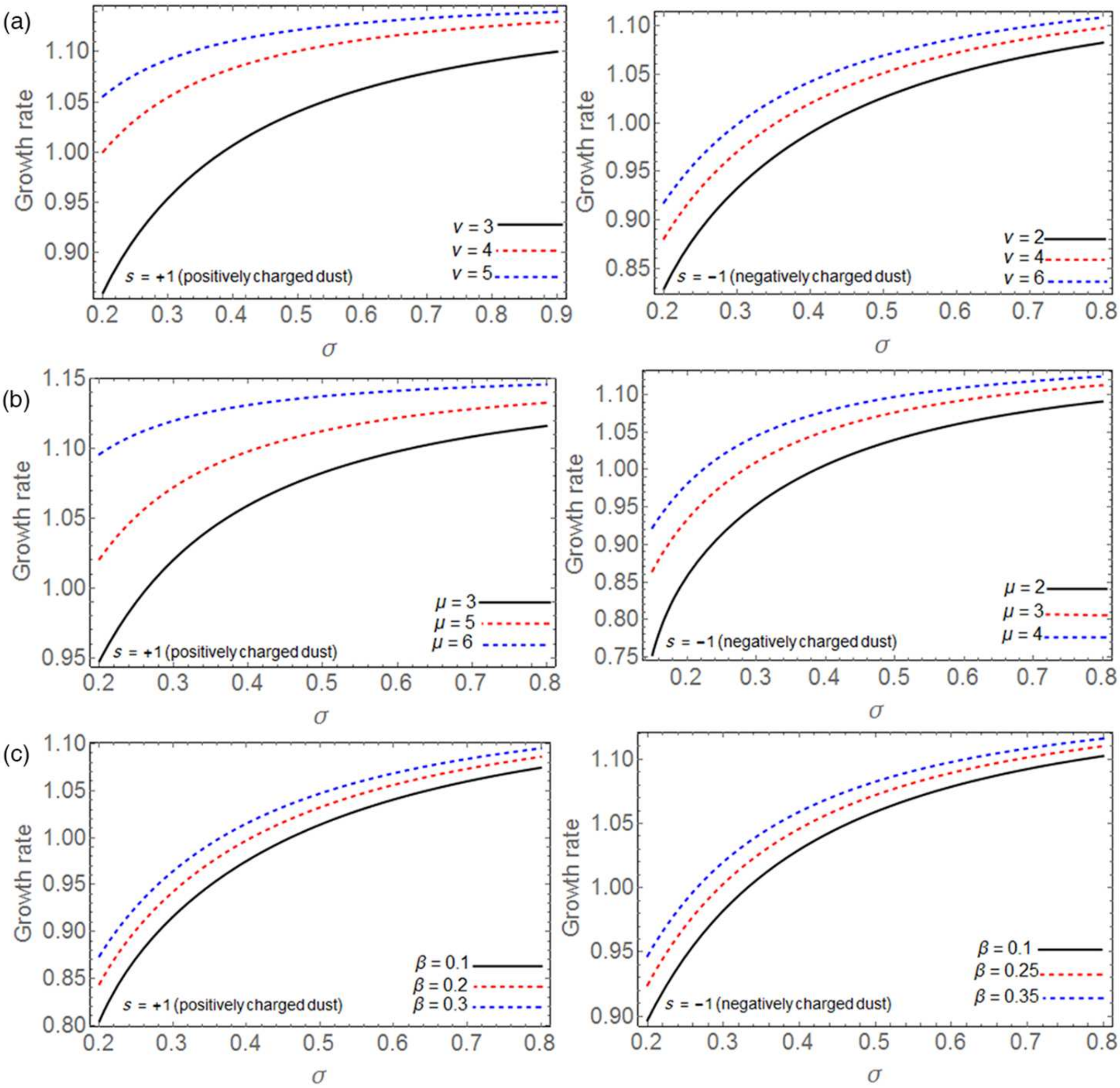

Fig. 4. Variation of the growth rate (given by eq. (27)) *vs.* σ for different values of ν for negatively (positively) charged dust in (a) where $\beta = 0.25$, $R = 0.4$, $\gamma = 1$ and $\mu = 2$. In panels (b)-(d) the growth rate is plotted *vs.* σ for different values of μ, β and γ for both polarities is shown. In panel 3(e), the growth rate *vs.* γ is displayed for different values of polarization term $R(\equiv R_{i,e})$ where $\sigma = 0.3$.

In previous studies involving ZK-type equations, ϕ_1 is determined by applying the L^{-1} operator [32]. This procedure is somewhat involved as it requires the evaluation of some nontrivial integrals. Here, given the solutions for ϕ_1 obtained for specific ZK-type equations elsewhere, we instead assume that ϕ_1 can be written as a linear combination of $Z\varphi_{oZ}$, φ_o, and φ_o^m, where m is to be determined. Then after some algebraic manipulation, we finally obtain (see the appendix for more details)

$$g_{r1}^2 = \frac{4\langle\varphi_{oZ}^2\rangle_s}{(2(1+\alpha^2)-1)\langle\varphi_o^2\rangle_s + 2\sigma_Q\alpha^2 D\langle\varphi_o^3\rangle_s}, \tag{27}$$

where for later convenience we have scaled the integrals, writing $\langle\cdot\rangle_s = \langle\cdot\rangle/(2\beta_1)^2$.

The dependence of the growth rate, g_{r1} (at the vicinity of the critical dust density) on the system parameter variations is displayed in fig. 4. For positively (negatively) charged dust grains, it is noted that g_{r1} increases with the increase of σ, ν, μ, β and γ. Moreover, increasing the polarization parameter yields to an increase of g_{r1} for both dust polarities.

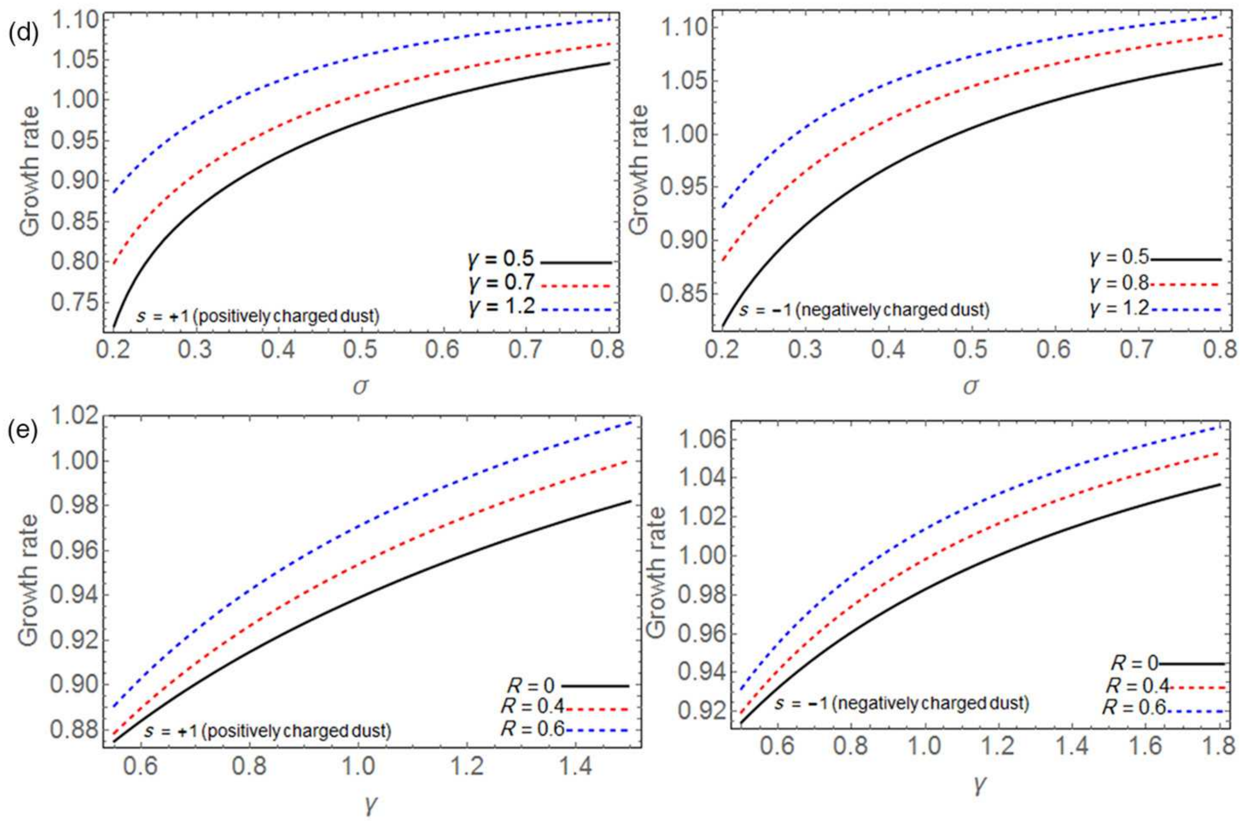

Fig. 4. Continued.

Using eq. (21), the scaled integrals can be obtained in closed form with the help of *Mathematica*. After making the substitution $w = \cosh Z$, we obtain

$$\langle\varphi_o^2\rangle_s = \frac{\Gamma^2(1)H_1(1,1) - 2\alpha\Gamma^2\left(\frac{3}{2}\right)H_3\left(\frac{3}{2},\frac{3}{2}\right)}{2\Gamma(2)}, \tag{28}$$

$$\langle\varphi_o^3\rangle_s = \frac{\sigma_Q\sigma_B\beta_1}{\Gamma(3)}\left[\Gamma^2\left(\frac{3}{2}\right)H_1\left(\frac{3}{2},\frac{3}{2}\right) - 2\alpha\Gamma^2(2)H_3(2,2)\right], \tag{29}$$

$$\langle\varphi_{oZ}^2\rangle_s = \frac{1}{\Gamma(4)}\left[\Gamma(2)\Gamma(1)H_1(2,1) - 2\alpha\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{5}{2}\right)H_3\left(\frac{3}{2},\frac{5}{2}\right)\right], \tag{30}$$

In which $H_n(a,b) \equiv {}_2F_1\left(a,b;\frac{n}{2};\alpha^2\right)$ where ${}_2F_1$ is the hypergeometric function. When $\alpha^2 = 1$, which corresponds to $D = 0$ and hence the generalization of the ZK equation with a single nonlinear term, eq. (27) reduces to

$$g_{r1}^2 = \frac{4\langle\varphi_{oZ}^2\rangle_s}{3\langle\varphi_o^2\rangle_s}, \tag{31}$$

and eqs. (28) and (30) can be simplified to give

$$\langle\varphi_o^2\rangle_s = \frac{\sqrt{\pi}\Gamma\left(-\frac{3}{2}\right)}{2\Gamma(2)}\left[\frac{\Gamma^2(1)}{\Gamma^2\left(-\frac{1}{2}\right)} - \frac{\Gamma^2\left(\frac{3}{2}\right)}{\Gamma^2(0)}\right], \tag{32}$$

$$\langle\varphi_{oZ}^2\rangle_s = \frac{\sqrt{\pi}\Gamma\left(-\frac{5}{2}\right)}{\Gamma(4)}\left[\frac{\Gamma(1)\Gamma(2)}{\Gamma\left(-\frac{1}{2}\right)\Gamma\left(-\frac{3}{2}\right)} - \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{5}{2}\right)}{\Gamma(-1)\Gamma(0)}\right]. \tag{33}$$

Back substitution of eq. (32) and eq. (33) into eq. (31) gives $g_{r1}^2 = 4/15$.

4 Conclusion

The stability of obliquely propagating three-dimensional DAWs with the effect of the polarization force in a magnetized dusty plasma consisting of extremely massive, negatively/positively charged dust fluid and Boltzmann distributed electrons and ions in the presence of an external static magnetic field is studied. The properties of these solitary waves is discussed through the Zakharov-Kuznetsov (ZK) equation. A new set of expansion physical parameters and stretched coordinates is used to derive the EZK equation valid in the vicinity of the critical phase velocity. The expression for the growth rate of the wave instability is estimated. The findings from the attached numerical investigations can be summarized as follows:

- It is noted that the phase velocity λ decreases by increasing $R_{i,e}$, σ and β whereas it increases with the increase in γ.
- Investigation of the dependence of the nonlinear coefficient, A, on the phase velocity, λ and the polarization term, $R_{i,e}$, for negatively (positively) charged dust grains shows that the critical phase velocity λ_c (λ at which $A = 0$) is strongly influenced by the variation of the polarization term.
- In the vicinity of the critical dust density, it is noted that, for positively (negatively) charged dust, g_{r1} increases with an increase in σ, ν, μ, β or γ. Moreover, increasing the polarization parameter, yields to an increase of g_{r1} for both different dust grains polarities.

Since the used numerical physical parameters are inspired from recent experimental data, Bandyopadhyay *et al.* [34], we believe that the present results are helpful to explain existing and forthcoming experimental and space observations [23, 24], where strongly coupled dusty plasma is present including positive/negative charged dust grains

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Appendix A. Derivation of eq. (27)

From the definition of L and using the fact that $L\varphi_{oZ} = 0$, we have

$$L(z\varphi_{oZ}) = 2\varphi_{oZZ'}, \tag{A.1}$$

$$L(\varphi_o) = \varphi_{oZZ} + \sigma_Q\sigma_B\varphi_o^2 + \sigma_B\varphi_o^3 D - \varphi_o, \tag{A.2}$$

$$L(\varphi_o^m) = m\varphi_o^{m-1}\varphi_{oZZ} + m(m-1)\varphi_o^{m-2}\varphi_{oZ}^2 + \sigma_Q\sigma_B\varphi_o^{m+1} + \sigma_B\varphi_o^{m+2}D - \varphi_o^m. \tag{A.3}$$

Since φ_o is a solution of (20), we have

$$\varphi_{oZZZ} = (1 - \sigma_Q\sigma_B\varphi_o - \sigma_B\varphi_o^2 D)\varphi_{oZ},$$

that leads to (after integration)

$$\varphi_{oZZ} = \varphi_o - \frac{\sigma_Q\sigma_B}{2}\varphi_o^2 - \frac{\sigma_B D}{3}\varphi_o^3. \tag{A.4}$$

Multiplying (A.4) by $2\varphi_{oZ}$ and integrating again yields

$$\varphi_{oZ}^2 = \varphi_o^2 - \frac{\sigma_Q\sigma_B}{3}\varphi_o^3 - \frac{\sigma_B D}{6}\varphi_o^4.$$

Hence we see that the right-hand sides of eqs. (A.1)–(A.3) can be written in terms of powers of φ_o, and to minimize the number of different powers we must choose $m = 2$, then we find that

$$L(\varphi_o^2) = 3\varphi_o^2 - \frac{2\sigma_Q\sigma_B}{3}\varphi_o^3.$$

Matching coefficients of the linear sum operated on by L with the right-hand side of (26), it is then straightforward to show that

$$\phi_1 = -\frac{\sigma_B g_{r1}}{2}[z\varphi_{oZ} + \varphi_o(1+\alpha^2) + \alpha^2\sigma_Q\varphi_o^2 D]. \tag{A.5}$$

To find the first-order growth rate g_{r1}, we must consider the equation obtained at second order in k namely;

$$\frac{\mathrm{d}}{\mathrm{d}Z}L\phi_2 = -\sigma_B g_{r2}\phi_o - \sigma_B g_{r1}\phi_1 + \phi_{oZ}, \tag{A.6}$$

However, we do not need to find ϕ_2. As in ref. [33], we instead first multiply eq. (A.6) by φ_o and integrate over all Z. The left-hand side can be seen to equal zero after integrating by parts and using the self-adjoint property of L and the fact that $L\phi_0 = 0$. The first term on the right-hand side also vanishes as the integrand is odd and we are left with $\sigma_B g_{r1}\langle\phi_1\varphi_o\rangle = \langle\phi_{oZ}\varphi_o\rangle$, where $\langle.\rangle$ is the integral over all Z. Using eq. (A.5) and the results $\langle\varphi_o\varphi_{oZZ}\rangle = -\langle\varphi_{oZ}^2\rangle$ and $\langle Z\varphi_o\varphi_{oZ}\rangle = -\frac{1}{2}\langle\varphi_o^2\rangle$, we finally obtain

$$g_{r1}^2 = \frac{4\langle\varphi_{oZ}^2\rangle_s}{(2(1+\alpha^2)-1)\langle\varphi_o^2\rangle_s + 2\sigma_Q\alpha^2\langle\varphi_o^3\rangle_s D}\,.$$

Notice that since $\mathrm{sgn}(\varphi_o^3) = \sigma_Q\sigma_B$ and if $D \neq 0$ and $\sigma_B D > 0$, the second term in the denominator is never negative.

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