Dust acoustic solitary waves and double layers in a dusty plasma with trapped electrons

S. K. El-Labany and W. F. El-Taibany[a)]
Department of Physics, Faculty of Science-Damietta, Damietta El-Gedida, Egypt

(Received 29 January 2003; accepted 11 September 2003)

The effect of variable dust charge, dust temperature, and trapped electrons on small amplitude dust acoustic waves is investigated. It is found that both compressive and rarefactive solitons as well as double layers exist depending on the nonisothermality parameter. A modified Korteweg–de Vries is derived. At critical density, the Korteweg–de Vries equation is obtained. Employing quasipotential analysis, the Sagdeev potential equation with the inclusion of different new effects has been derived. Because of the presence of free and trapped electrons, the plasma acoustic wave has gained features of various solitary waves. The Sagdeev potential equation, at a small amplitude, shows that the ordering of nonisothermality plays a unique role. In the case of a plasma with first-order nonisothermality, the Sagdeev potential equation shows the compressive solitary wave propagation, while for plasma with higher-order nonisothermality, the solution of this equation reveals the coexistence of both compressive and rarefactive solitary waves. In addition, for certain plasma parameters, the solitary wave disappears and a double layer is expected. Again, with the better approximation in the Sagdeev potential equation, more features of solitary waves, e.g., spiky and explosive, along with the double layers, are also highlighted. The findings of this investigation may be useful in understanding laboratory plasma phenomena and astrophysical situations. [DOI: 10.1063/1.1623764]

I. INTRODUCTION

Recently, there has been much interest in studying the dusty plasmas which are characterized as an ionized gas containing electrons, ions and highly charged massive dust particles. Dusty plasmas differ from the usual multicomponent plasmas,[1–4] since the ratio between the dust charge and the dust mass is nonuniform and it is considered as a new dynamical variable.

The growing interest in physics of dusty plasmas not only because of dust being an omnipresent ingredient of our universe, but also because of its vital role in understanding different collective processes (mode modification, new eigenmodes, coherent structures, etc.) in astrophysical and space environments.[5–9] The consideration of charged dust grains in a plasma does not only modify the existing plasma wave spectra,[10] but also introduces a number of new novel eigenmodes, such as dust acoustic (DA) waves,[11,12] dust ion acoustic (DIA) waves,[13] dust lattice waves,[14] etc.

Highly charged massive dust grains present in a plasma may exhibit charge fluctuations in response to certain types of oscillations incorporated to the plasma. Under this situation, the grain charge becomes a time dependent and self-consistent variable.[15,16] The consequent modifications in the collective properties of a dusty plasma in response to the variation of charge is studied for noncomplicated plasma systems.[17,18] It may be noted that the existence of DA wave on a very slow time scale of dust dynamics was investigated for the first time by Rao *et al.*[11] They also showed the formation of rarefactive type DA soliton solution in a simple dusty plasma system. Similarly, Ma and Liu[19] discussed the existence of rarefactive DA soliton solution in a plasma in presence of dust charge fluctuations. Using a reductive perturbation theory, Xie *et al.*[20] derived small amplitude DA solitons and double layers in dusty plasma with varying dust charges and they have shown that only rarefactive solitary waves exist when the Mach number lies within an appropriate regime depending on the system parameters. Also, the amplitudes of the dust solitary waves become smaller and the regime of Mach number is extended wider for the variable dust charge situation with the case of constant dust charge. The topics of nonlinear grain charge variation and electrostatic ion waves[21] have been reported by regarding dust grains as point charges, where the Debye length is much larger than the intergrain distance.

If streaming particles are injected in plasmas, we often find that they evolve towards a coherent trapped particle state, instead of developing into a turbulent one. This has been confirmed by computer simulations[22,23] and experiments.[24,25] The onset of an electron trapping is also seen in the formation of double layers[26] and computer simulation.[27] It is well known that the presence of trapped particles can significantly modify the wave propagation characteristics in collisionless plasmas.[28] Later, Das and co-workers[29,30] used the tanh method[31–33] to investigate dynamical aspects of various solitary waves and double layers with the inclusion of trapped and free electrons in cold dusty plasma system.

Although, dusty plasma physics has much interests and some papers considered the effect of trapped electrons,[29,30]

[a)]Author to whom correspondence should be addressed. Electronic mail: eltaibany@hotmail.com

self-consistent dust charge fluctuations[20,34,35] and dust temperature,[34] no one has considered the effects of the dust temperature, trapped electrons, and dust charge variation. The motivation of this paper is the study of the dynamics of nonlinear DA solitary waves in a warm dusty plasma system containing both free and trapped electrons. Also, we study both small and large amplitude DA waves and double layers in the presence of self-consistent variational dust charge system.

The manuscript is organized as follows: The basic equations describing the dusty plasma system under consideration is given in Sec. II. In Sec. III, using current balance condition, the dependence of the dust charge on the plasma parameters is obtained. In Sec. IV, using a reductive perturbation technique, the small amplitude DA solitary structures are studied. In Sec. V, critical density for the system is discussed and the Korteweg–de Vries (KdV) and its solution are obtained and the condition under which the double layers can be formed is also found. In Sec. VI, we derive Sagdeev potential and investigate the existence of different solitary waves and double layers and also its tendency to small amplitude solitons. Section VII is devoted to the discussion and conclusion.

II. BASIC EQUATIONS

The dusty plasma we are going to study, consists of three components; extremely massive and highly negatively charged dust grains, ions together with free and trapped electrons. The charge neutrality at equilibrium requires

$$n_{io}=n_{eo}+Z_{do}n_{do}, \tag{1}$$

where n_{eo}, n_{io}, and n_{do} are the unperturbed electron, ion, and dust number densities, respectively, and Z_{do} is the unperturbed number of charges residing on the dust grain measured in unit of the electron charge.

For one-dimensional low-frequency DA motions, we have the following nondimensional equations for the warm dust fluids:[20]

$$\frac{\partial n_d}{\partial t}+\frac{\partial(n_d u_d)}{\partial x}=0, \tag{2}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}+3\sigma_d n_d\frac{\partial n_d}{\partial x}-Z_d\frac{\partial \phi}{\partial x}=0, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2}=Z_d n_d+n_e-n_i, \tag{4}$$

where n_d and u_d refer to the dust number density and fluid velocity, respectively. Also, $\sigma_d=(T_d/Z_{do}T_{\rm eff})$, $T_{\rm eff}=[Z_{do}n_{do}T_iT_e/(n_{io}T_e+n_{eo}T_i)]$. We normalize all densities by $n_{do}Z_{do}$. The space coordinate x, time t, velocities and electrostatic potential ϕ are normalized by the Debye length $\lambda_{Dd}=(T_{\rm eff}/4\pi Z_{do}n_{do}e^2)^{1/2}$, the inverse dust plasma frequency $\omega_{pd}^{-1}=(m_d/4\pi Z_{do}^2 n_{do}e^2)^{1/2}$, the DA speed $C_d=(Z_{do}T_{\rm eff}/m_d)^{1/2}$, and $T_{\rm eff}/e$, respectively.

In the dynamical system, some of the electrons are attached to the dust grains to form the dust charged grains, while some of the remaining electrons are bounded back and forth in the potential well, lose energy continuously and, as a result, being ultimately trapped electrons. By now the appearance of the trapped electrons has been supported theoretically, and observations of them in the bow shock and in the dynamics of the beam plasma instability are deserving of mention. Moreover, Wong *et al.*[36] experimentally supported the existence of trapped electrons in plasma waves, and there is also the observation of such electrons in the soliton structure in stable plasma waves reported first by Tran *et al.*[37] In this case, the electron density is defined from the Vlasov equation consisting of free and trapped electrons. Following Schamel[28,38] and Das and co-workers,[29,30] the nonisothermality of the plasma is introduced through the electron densities that have the normalized form as

$$n_e(\phi)=\int_{-\infty}^{\infty}f_e(x,v)\,dv=n_{eo}\left[\exp(\Gamma)(1-\mathrm{erf}(\sqrt{\Gamma}))+\frac{1}{\sqrt{\beta_h}}\begin{cases}\exp(\Gamma\beta_h)\mathrm{erf}(\sqrt{\Gamma\beta_h}) & \text{for}\quad \beta_h\geqslant 0,\\ \dfrac{2}{\sqrt{\pi}}\exp(\Gamma\beta_h\displaystyle\int_0^{\sqrt{-\Gamma\beta_h}}\exp(X^2)\,dX) & \text{for}\quad \beta_h<0.\end{cases}\right],$$

where $\Gamma=e\phi/T_e$ and $f_e(x,v)$ and β_h represent the electron distribution function and the ratio of the free to the trapped electron temperatures, T_e/T_t, respectively. Considering the Maxwellian distribution, the Taylor expansion of the last equation, for $\phi\ll 1$, derives the electron density, n_e, as a combination of free and trapped electrons as

$$n_e=n_{eo}[1+\Gamma-\tfrac{4}{3}b_1\Gamma^{3/2}+\tfrac{1}{2}\Gamma^2-\tfrac{8}{15}b_2\Gamma^{5/2}+\tfrac{1}{6}\Gamma^3+\cdots.$$

The cases $\beta_h=1$ and $\beta_h=0$ correspond to the plasma having the Maxwellian and flat topped distributions, respectively. For an isothermal plasma, one can derive the electron density by imposing $b_1=0$ and $b_2=0$, whereas for the nonisothermal plasma, we have $0<b_1,b_2<1/\sqrt{\pi}$. Thus the nonisothermality of the plasma is expressed through the electron density, n_e, by the following modified form:[29,30]

$$n_e=n_{eo}[\exp(\Gamma)-G(\Gamma)],$$

where $G(\Gamma)=\Sigma_{k=1}^{n}[2^{(k+1)}b_k(\Gamma)^{(2k+1)/2}/\Pi(2k+1)]$ with the nonisothermal parameter defined as $b_k=(1-\beta_h^k)/\sqrt{\pi}$.

Now, the dimensionless number densities of ions and electrons are expressed as

$$n_i=\mu\exp(-s\phi), \tag{5}$$

$$n_e=\nu[\exp(s\beta\phi)-G(s\beta\phi)], \tag{6}$$

where

$$\mu=\frac{n_{io}}{Z_{do}n_{do}},\quad \nu=\frac{n_{eo}}{Z_{do}n_{do}},\quad s=\frac{1}{(\mu+\beta\nu)},\quad \text{and}\quad \beta=\frac{T_i}{T_e}.$$

Obviously, Eq. (1) leads to $\mu-\nu=1$.

III. CHARGING OF DUST GRAINS

Dust particles are charged due to a variety of processes including the bombardment of the dust grain surface by background plasma electrons, ions and incident ion beams, photoelectron emission by ultraviolet radiation, ion sputtering, secondary electron production, etc. In low-temperature laboratory plasmas, dust particles are mainly negatively charged when any plasma electrons hitting the surface of the dust grains are attached to it and simply lost from the background plasma.[8] In general, the dust charge variable Q_d is determined by the charge current balance equation[16]

$$\frac{\partial Q_d}{\partial t}+u_d\frac{\partial Q_d}{\partial x}=I_e+I_i, \tag{7}$$

where Q_d is the dust charge variable. We notice that the characteristic time for dust motion is of order of tens of milliseconds for micrometer sized grains,[12] while the dust charging time is typically of order of 10^{-8} s. Therefore, on the hydrodynamic time scale, the dust charge can quickly reach local equilibrium, at which the currents from the electrons and ions to the dust are balanced. The current balance equation reads[9]

$$I_{eo}+I_{io}\approx 0. \tag{8}$$

According to the well-known orbit-motion-limited probe model,[15] we have the following expressions for the electron and ion currents for spherical dust grains with radius r:[20,21,35]

$$I_e=-e\pi r^2(8T_e/\pi m_e)^{1/2}n_e\exp\left(\frac{e\Phi}{T_e}\right),$$

and

$$I_i=e\pi r^2(8T_i/\pi m_i)^{1/2}n_i\left(1-\frac{e\Phi}{T_i}\right),$$

where Φ denotes the dust grain surface potential relative to the plasma potential ϕ. To compare our results with the previous ones, we introduce $\delta=\mu/\nu$ and the current balance equation becomes

$$\begin{aligned}&[\exp(s\beta[\Psi+\phi])-G(s\beta\phi)\exp(s\beta\Psi)]\\ &\quad=\alpha\delta(1-s\Psi)\exp(-s\phi),\end{aligned} \tag{9}$$

where $\Psi=e\Phi/T_{\mathrm{eff}}$, $\alpha=(\beta/\mu_i)^{1/2}$, and $\mu_i=m_i/m_e\approx 1840$. Equation (9) is important for determining the dust charges $Q_d=C\Phi$; C is the capacitance of dust grain. We have the normalized dust charge $Z_d=\Psi/\Psi_o$, where Ψ_o is the dust surface floating potential with respect to the unperturbed plasma potential at infinite place. Ψ_o can be determined from Eq. (9) with $\phi=0$ and it has the form of

$$\Psi_o=\frac{1}{s\beta}\ln[\alpha\delta(1-s\Psi_o)]. \tag{10}$$

As can be seen, the dust charge is very sensitive to the small disturbance of ϕ. This point is very important to explain how the variable dust charge influences on the shape of solitons and solitary waves. Obviously, Eq. (9) includes strongly nonlinear terms. Thus, its solution can be obtained numerically.

IV. DUST ACOUSTIC SOLITONS

In order to study the dynamics of small amplitude DA solitary waves in the presence of variation of dust charges, we derive an evolution equation from the system of Eqs. (2)–(6), employing a reductive perturbation technique[39] by introducing the stretched coordinates[38] $\xi=\varepsilon^{1/4}(x-\lambda t)$, and $\tau=\varepsilon^{3/4}t$, where ε is a small parameter and λ is the solitary wave velocity normalized by C_d. The variables n_d, u_d, Z_d, and ϕ are expanded as

$$n_d=1+\varepsilon n_{d1}+\varepsilon^{3/2}n_{d2}+\varepsilon^2 n_{d3}+\varepsilon^{5/2}n_{d4}+\cdots,$$

$$u_d=\varepsilon u_{d1}+\varepsilon^{3/2}u_{d2}+\varepsilon^2 u_{d3}+\varepsilon^{5/2}u_{d4}+\cdots,$$

$$Z_d=1+\varepsilon Z_{d1}+\varepsilon^{3/2}Z_{d2}+\varepsilon^2 Z_{d3}+\varepsilon^{5/2}Z_{d4}+\cdots,$$

$$\phi=\varepsilon\phi_1+\varepsilon^{3/2}\phi_2+\varepsilon^2\phi_3+\varepsilon^{5/2}\phi_4+\cdots.$$

Substituting these expansions into Eqs. (2)–(6), and using $\Psi=\Psi_o Z_d$ in Eq. (9), then collecting terms of different powers of ε, in the lowest order we obtain

$$n_{d1}=-\phi_1 R,\quad u_{d1}=-\lambda\phi_1 R,\quad Z_{d1}=\gamma_1\phi_1, \tag{11}$$

where

$$R=(\lambda^2-3\sigma_d)^{-1}\quad\text{and}\quad \gamma_1=\frac{-(1+\beta)(1-s\Psi_o)}{\Psi_o(1+\beta(1-s\Psi_o))}.$$

The linear dispersion relation is given by

$$\gamma_1+1=R, \tag{12}$$

from which one can get the wave velocity λ as

$$\lambda=[3\sigma_d+[\gamma_1+1]^{-1}]^{1/2}, \tag{13}$$

putting $\sigma_d=0$, this equation agrees exactly with Xie *et al.*[20] It is obvious that the inclusion of the dust temperature increases the velocity and the contrary with the effect of dust charge variation where $\gamma_1>0$.

The next order in $\varepsilon, O(\varepsilon^{3/2})$ yields a system of equations that leads to the MKdV equation

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=0, \tag{14}$$

where

$$B^{-1}=2\lambda R^2,\quad A=B\left[\frac{2(s\beta)^{3/2}b_1}{(\delta-1)}-\frac{3\gamma_2}{2}\right]$$

and

$$\gamma_2=\frac{4\sqrt{s\beta^3}(1-s\Psi_o)b_1}{3\Psi_o(1+\beta(1-s\Psi_o))}.$$

Using the boundary conditions

$$\phi_1(\eta)\to 0,\frac{d\phi_1(\eta)}{d\eta}\to 0,\frac{d^2\phi_1(\eta)}{d\eta^2}\to 0\quad\text{as}\quad|\eta|\to\infty;$$

$$\eta=\xi-u_o\tau, \tag{15}$$

the stationary solution of Eq. (14) is given by

$$\phi_1=\phi_{1m}\,\mathrm{sech}^4[\eta/w_1], \tag{16}$$

where the amplitude ϕ_{1m} and the width w_1 are given by $(15u_o/8A)^2$ and $4\sqrt{B/u_o}$, respectively. As $u_o>0$, there exist solitary waves with positive potential only. As u_o increases, the amplitude increases and the width decreases. The inclusion of the dust temperature and dust charge variation is observed as increment of the width but decrement of the amplitude. These effects will be studied later numerically.

V. DERIVATION OF THE KdV EQUATION

The propagation of compressive solitons (that admitted only) depends on the sign of the nonlinear coefficient of the MKdV equation, A. We can ensure that the dispersion coefficient of the MKdV equation, B is always positive and thus the DA waves are compressive if $A>0$. If $A=0$, corresponds to the critical density ratio δ_c, the MKdV equation breaks down. The condition $A=0$ can be satisfied if $\beta_h=1$ (all electrons are isothermal) or at

$$\delta_c=\frac{\pm\sqrt{\beta}\Psi_o\sqrt{4(1-\Psi_o)+\beta(\Psi_o-2)^2}+\beta(2+\Psi_o(\Psi_o-2))}{2(\Psi_o-1)}.$$

Now, one has to seek for another equation suitable for describing the evolution of the system. Using the stretching coordinates[40] $\xi=\varepsilon^{1/2}(x-\lambda t)$, and $\tau=\varepsilon^{3/2}t$, and following the procedure used before, we obtain the same relations as Eq. (11) for the lowest order of ε [coefficient of $O(\varepsilon)$], and to the order $\varepsilon^{3/2}$, we get

$$n_{d2}=-R\phi_2,u_{d2}=-\lambda R\phi_2,Z_{d2}=\gamma_1\phi_2+\gamma_2\phi_1^{3/2}, \tag{17}$$

$$[\gamma_1-R+1)]\phi_2=\left[\frac{4(s\beta)^{3/2}b_1}{3(\delta-1)}-\gamma_2\right]\phi_1^{3/2}. \tag{18}$$

If we consider the next-order in ε; $O(\varepsilon^2)$, we obtain a system of equations, with the aid of Eqs. (10), (11), and (17), yields

$$\frac{\partial\phi_1}{\partial\tau}+A\frac{\partial\phi_1^{1/2}\phi_2}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}+C\phi_1\frac{\partial\phi_1}{\partial\xi}=0, \tag{19}$$

where

$$C=B\left[\frac{(\delta-1)(\delta-\beta^2)}{(\delta+\beta)^2}+3(\gamma_1-[\lambda^2+\sigma_d]R^2)R-2\gamma_3\right],$$

$$\gamma_3=\frac{-s(1+\beta)^2(1-s\Psi_o)}{2\Psi_o(1+\beta(1-s\Psi_o))^3}.$$

The solution of this equation with $A=0$ is given by

$$\phi_1=\phi_{2m}\,\mathrm{sech}^2[\eta/w_2], \tag{20}$$

where the amplitude ϕ_{2m} and the width w_2 are given here by $3u_o/C$ and $2\sqrt{B/u_o}=0.5\,w_1$, respectively. Also, as u_o increases the amplitude increases and the width decreases. Since $\gamma_1\geqslant 0$, $\gamma_3\geqslant 0$ and $u_o>0$, Eq. (20) clearly indicates that only rarefactive soliton waves exist. The form of γ_3 agrees exactly with that obtained by Xie *et al.*[20] When one compares between the expression of the amplitude and the width of the MKdV and KdV solutions, he may deduce that the case of MKdV has a larger amplitude and width than those of KdV but we will prove later, using numerical analysis, that the nonlinear terms have the major role in the calculation of the amplitude and the width precisely.

On the other hand, when $A\to 0$ but $\neq 0$, Eq. (20) would reduce to

$$\frac{\partial\phi_1}{\partial\tau}+D\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}+C\phi_1\frac{\partial\phi_1}{\partial\xi}=0, \tag{21}$$

where we have used[40] $A\phi_2\to 2D\phi_1/3$. This equation has a double layer solution of the form[26]

$$\phi_1=\frac{\phi_m}{4}(1-\tanh[(\xi-u_o\tau)/w])^2, \tag{22}$$

with

$$\phi_m=\left(\frac{4D}{5C}\right)^2,\quad u_o=\frac{-16D^2}{75C},\quad\text{and}\quad w=\frac{5}{D}\sqrt{-3BC}. \tag{23}$$

However, Eq. (22) has the same form obtained by Schamel[26] and Das *et al.*,[29] its amplitude and width strongly depend on the introduced effects. Obviously, the formation of the two types of DA double layers (compressive and rarefactive), depends on the sign of D, while C should take negative values.

VI. QUASIPOTENTIAL ANALYSIS

Now we turn our attention to investigate the properties of large amplitude DA solitary waves. We assume that all variables in Eqs. (2)–(6) depend only on a single variable $\zeta=x-Mt$, where ζ is normalized by λ_{Dd} and M is the Mach number (solitary wave velocity divided by C_d). In this stationary frame and using $\Psi=\Psi_oZ_d$, Eqs. (2) and (3) can be integrated to give the following expression for dust number density:

$$n_d=\frac{1}{\sqrt{1+2(V_d(\phi)/(M^2-3\sigma_d))}}, \tag{24}$$

where we have imposed the appropriate boundary conditions for localized disturbances, viz., $\phi\to 0,n_d\to 1,u_d\to 0,n_{eo}\to\nu,n_i\to\mu$ as $\zeta\to\pm\infty$, and

$$V_d(\phi)=\int_0^\phi Z_d\,d\phi=\frac{1}{\Psi_o}\int_0^\phi\Psi(\phi)\,d\phi. \tag{25}$$

Although, we can evaluate the value of Ψ_o [Eq. (10)] the value of Ψ has a complicated form and is given by

$$\Psi=\frac{1}{s\beta}\ln\left[\frac{\alpha\delta(1-s\Psi)\exp(-s\phi)}{\exp(s\beta\phi)-G(s\beta\phi)}\right]. \tag{26}$$

The introduction of the nonisothermal term in this equation prevents us to put a general recurrence relation for the dust surface potential. For isothermal plasma, Eq. (26) agrees exactly with that obtained by Xie *et al.*[20]

Substituting for the normalized number densities of ion and electrons, the dust charge number and the dust density number into Poisson equation (4) and integrating it, imposing the boundary conditions for localized solutions; namely $\phi\to 0$ and $d\phi/d\zeta\to 0$ as $\zeta\to\pm\infty$, we get

$$\frac{1}{2}\left(\frac{d\phi}{d\zeta}\right)^2+V(\phi)=0, \tag{27}$$

where the Sagdeev quasipotential reads

$$V(\phi)=\frac{\nu}{\beta s}\left[1-\exp(s\beta\phi)+\int_0^\phi G(s\beta\phi)d\phi\right]+\frac{\mu}{s}(1-\exp(-s\phi))+\left\{1-\sqrt{1+\frac{2V_d(\phi)}{(M^2-3\sigma_d)}}\right\}(M^2-3\sigma_d). \tag{28}$$

The overall Sagdeev potential analysis requires the condition for the existence of soliton structure qualitatively as $V(\phi)$ to be negative. Otherwise double layers might be possible. This condition is strongly sensitive at the edges $\phi=0$ and $\phi=\phi_m$, around which the classical potential $V(\phi)<0$ is expected. The condition yields the requirement $V(0)=V(\phi_m)=0$ and $V'(0)=V'(\phi_m)$ and in between $V(\phi)$ is always negative. From Eq. (28), these conditions lead to

$$M>M_l=\sqrt{\frac{1}{\gamma_1+1}+3\sigma_d}. \tag{29}$$

This fixes the lower limit of M, which is equivalent to the value of λ obtained in Sec. IV. We also prove later by numerical investigation that the lower Mach number should be one but not zero as in Ref. 19. However, our result agrees exactly with Xie *et al.*[20] when $\sigma_d=0$. The upper limit of M for which negative solitary waves exist can be found from the condition $V(\phi_{\min})=0$, where $\phi_{\min}$ is the minimum value of ϕ for which the dust density n_d is real. This minimum value variation due to the inclusion of the dust charge fluctuation, trapped electrons and dust temperature effects is investigated in the next section.

The critical upper limit of Mach number $M_{\max}$ for which positive plasma solitary waves exist can be found from the condition $V(\phi_{\max})=0$, where $\phi_{\max}$ is the maximum value of ϕ, meanwhile $V(\phi)$ is tangent to the ϕ-axis for which $dV(\phi_{\max})/d\phi=0$. On the other hand, for the formation of double layer the following equations must be satisfied:

$$\frac{\nu}{\beta s}\left[1-\exp(\beta s\phi)+\int_0^\phi G(s\beta\phi)d\phi\right]+\frac{\mu}{s}(1-\exp(-s\phi))+\left\{1-\sqrt{1+\frac{2V_d(\phi)}{(M^2-3\sigma_d)}}\right\}(M^2-3\sigma_d)=0,$$

and

$$\mu\exp(-s\phi)-\nu[\exp(\beta s\phi)-G(\beta s\phi)]+\left(Z_d/\sqrt{1+\frac{2V_d(\phi)}{(M^2-3\sigma_d)}}\right)=0.$$

These coupled transcendental equations can be solved numerically for the largest positive potential amplitude and upper Mach number of the associated double layer.

Now, we use the tanh method[31–33] to investigate the effect of introducing new effects into our system. If we expand the expression for quasipotential around $\phi=0$, for small amplitude we will recover all the results of the small amplitude DA soliton obtained by the reduced perturbation technique in Sec. IV and Sec. V. For example, for small ϕ, we can write to the order of $O(\phi^{5/2})$

$$V_d(\phi)=\phi+\frac{\gamma_1}{2}\phi^2+\frac{2\gamma_2}{5}\phi^{5/2}.$$

Substituting for $V_d(\phi)$ and the dimensionless number density of dusts, electrons and ions into Eq. (28) we have

$$V(\phi)=\frac{1}{2}\{(M^2-3\sigma_d)^{-1}-1-\gamma_1\}\phi^2+\frac{2}{5}\left[\frac{4(s\beta)^{3/2}b_1}{3(\delta-1)}-\gamma_2\right]\phi^{5/2}.$$

Equation (27) can be rewritten as

$$\left(\frac{d\phi}{d\zeta}\right)^2=a_1\phi^2-a_2\phi^{5/2}, \tag{30}$$

where $a_1=M/B$ and $a_2=8A/15B$. Equation (30) has the soliton solution as Eq. (16). For a slightly larger values of ϕ, we can get, following the same procedure, the solution[30]

$$\phi(\zeta)=\left[\frac{a_1}{4a_3}\pm\sqrt{\frac{8a_1a_3+3a_2^2}{8a_3^2}}\operatorname{sech}\left(\frac{\zeta}{\sqrt{\frac{32a_3}{8a_1a_3+3a_2^2}}}\right)\right]. \tag{31}$$

Equation (31) can be transformed to a shock-like wave solution as Eq. (22) or a soliton solution like Eq. (20) for adjusted constant values. However, this form agrees exactly with that obtained by Das *et al.*[30] However, the constants enclosed are dependent on the charge fluctuation and dust temperature. Other types of solitons, viz., spiky type solitary waves, collapsible waves, etc., can be obtained by taking higher order terms. Since the expression for $V(\phi)$ derived in Eq. (28) is exact, one can expand it up to any desirable power of ϕ and then can obtain all different types of solitary waves depending on the nonisothermal parameter, dust temperature and the charge variation effect obtained by perturbation theory. For higher power of ϕ, we have Z_d as

$$Z_d = 1 + \gamma_1 \phi + \gamma_2 \phi^{3/2} + \gamma_3 \phi^2 + \gamma_4 \phi^{5/2},$$

where

$$\gamma_4 = \frac{\gamma_{a1} + \gamma_{a2} + \gamma_{a3} + \gamma_{a4}}{-15\Psi_o(1+\beta(1-s\Psi_o))^3},$$

and

$$\gamma_{a1} = -8\beta b_2 (s\beta)^{3/2}(1-s\Psi_o),\ \gamma_{a2} = -15 s\gamma_2 \Psi_o(1 - \beta^2(1-s\Psi_o)),$$

$$\gamma_{a3} = \frac{-15}{8\sqrt{\beta s}}\gamma_1\gamma_2\gamma_{a1}\Psi_o^2,\ \gamma_{a4} = \frac{5b_1}{2b_2}\gamma_1\gamma_{a1}\Psi_o.$$

Thus, Sagdeev potential can be given as

$$\begin{aligned} V(\phi) = {} & \frac{1}{2}\{(M^2-3\sigma_d)^{-1} - 1 - \gamma_1\}\phi^2 \\ & + \frac{2}{5}\left[\frac{4(s\beta)^{3/2}b_1}{3(\delta-1)} - \gamma_2\right]\phi^{5/2} + \phi^3\left(\frac{(\delta-1)(\delta-\beta^2)}{6(\delta+\beta)^2}\right. \\ & \left. + \frac{1}{2}\gamma_1(M^2-3\sigma_d)^{-1} - \frac{1}{3}\gamma_3 - \frac{1}{2}(M^2-3\sigma_d)^{-2}\right) \\ & + \phi^{7/2}\left[\frac{2}{5}\gamma_2(M^2-3\sigma_d)^{-1} - \frac{2}{7}\gamma_4 + \frac{16(s\beta)^{5/2}b_2}{105(\delta-1)}\right]. \end{aligned}$$

In this case, Eq. (27) becomes

$$\left(\frac{d\phi}{d\zeta}\right)^2 = a_1\phi^2 - a_2\phi^{5/2} - a_3\phi^3 - a_4\phi^{7/2}, \tag{32}$$

where $a_3 = C/3B$, $a_4 = 8F/35B$, and $F = (B/6)[21\gamma_2(M^2 - 3\sigma_d)^{-1} - 15\gamma_4 + 8(s\beta)^{5/2}\nu b_2]$. If $a_2 = a_3 = 0$, we have a soliton solution like

$$\phi = \phi_{3m}\,\mathrm{sech}^{4/3}[\zeta/w_3],$$

where the amplitude ϕ_{3m} and the width w_3 are given by $(35M/8F)^{2/3}$ and $\frac{4}{3}\sqrt{B/M} = w_1/3$, respectively. The variation of the amplitude and the width of the soliton is considered in Sec. VII. We can reform Eq. (32), using $\phi = \theta^2$, as

$$\left(\frac{d\theta}{d\zeta}\right)^2 = a\theta^2(\mathcal{F} - \theta)^3, \tag{33}$$

where $\mathcal{F} = -a_3/3a_4$, $a = a_4/4$, and $a_3^2 = 3a_2a_4$. Equation (33) can be solved for the soliton profile and the solution can be obtained as an implicit function of ζ in the following form:

$$\phi(\zeta) = \mathcal{F}^2\left(1 - \tanh^2\left[\left(\frac{\mathcal{F}}{\mathcal{F} - \sqrt{\phi(\zeta)}}\right)^{1/2} - \sqrt{a\mathcal{F}^3}\frac{\zeta}{2}\right]\right)^2. \tag{34}$$

This solution gives a profile of spiky solitary wave defined in the region $0 < \phi(\zeta) < \sqrt{\mathcal{F}}$ and is affected by a_1 and a_3, which are functionally dependent on the dust charged grains, as well as by the trapped electrons through a_2 and a_4 and all of them are functions of the dust temperature. On the other hand, for the region defined as $\phi < 0$, the soliton solution can be obtained in a similar manner and it has an explosive solitary wave profile in the plasma acoustic dynamics. Also, if we proceed and take higher nonlinearities terms, we have Z_d as

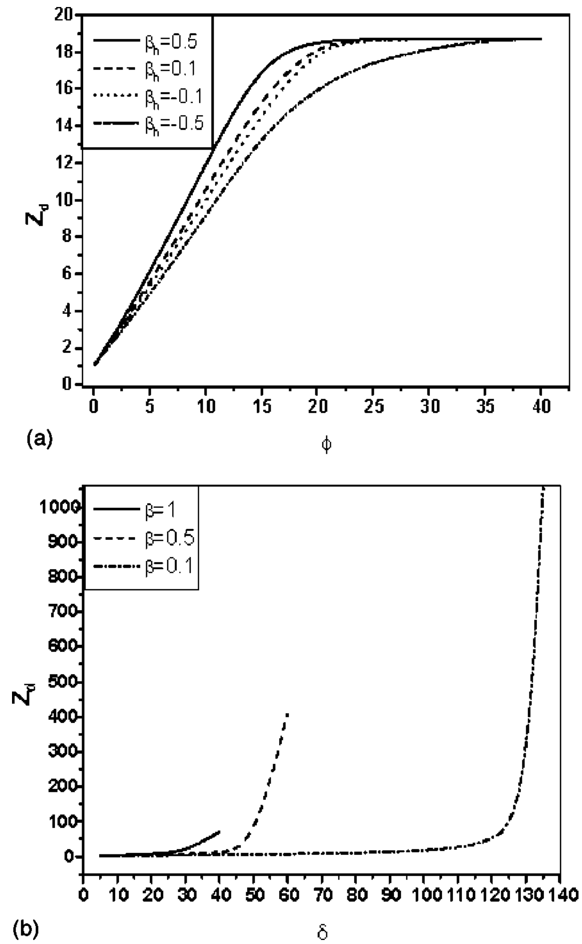

FIG. 1. Z_d is plotted (a) against plasma potential disturbance ϕ for different β_h values with $\delta = 10$ and $\beta = 0.5$, and (b) against δ for different β values with $\phi = 2$ and $\beta_h = 0.9$.

$$Z_d = 1 + \gamma_1\phi + \gamma_2\phi^{3/2} + \gamma_3\phi^2 + \gamma_4\phi^{5/2} + \gamma_5\phi^3,$$

where

$$\gamma_5 = \frac{\Sigma_{i=1}^{6}\gamma_{bi}}{-18\Psi_o(1+\beta(1-s\Psi_o))^5},$$

and

$$\gamma_{b1} = -s^2(1+\beta)(1-s\Psi_o)[-2 + \beta(3 - s\Psi_o + \beta(3 - 2\beta + 2s\Psi_o(\beta-1)))],$$

$$\gamma_{b2} = 3s^2\Psi_o\gamma_1[1 - \beta(1-s\Psi_o) \times (1 + \beta(5 + \beta - s\Psi_o - 2\beta^2(1-s\Psi_o)))],$$

$$\gamma_{b3} = -3(s\beta\Psi_o\gamma_1)^2(1-s\Psi_o)[3 + \beta - 2\beta^2(1-s\Psi_o)],$$

$$\gamma_{b4} = -s^2(\beta\Psi_o\gamma_1)^3(1-s\Psi_o)[1 - 2\beta(1-s\Psi_o)],$$

$$\gamma_{b5} = 8b_1\beta(s\beta)^{3/2}\Psi_o\gamma_2(1-s\Psi_o)[1 + \beta(1-s\Psi_o)],$$

$$\gamma_{b6} = -3s(\beta\Psi_o\gamma_2)^2(1-s\Psi_o)[1 + \beta(1-s\Psi_o)].$$

Thus, Sagdeev potential can be given as

$$V(\phi)=\frac{1}{2}\{(M^2-3\sigma_d)^{-1}-1-\gamma_1\}\phi^2+\frac{2}{5}\left[\frac{4(s\beta)^{3/2}b_1}{3(\delta-1)}-\gamma_2\right]\phi^{5/2}$$
$$+\phi^3\left(\frac{(\delta-1)(\delta-\beta^2)}{6(\delta+\beta)^2}+\frac{1}{2}\gamma_1(M^2-3\sigma_d)^{-1}-\frac{1}{3}\gamma_3-\frac{1}{2}(M^2-3\sigma_d)^{-2}\right)$$
$$+\phi^{7/2}\left[\frac{2}{5}\gamma_2(M^2-3\sigma_d)^{-1}-\frac{2}{7}\gamma_4+\frac{16(s\beta)^{5/2}b_2}{105(\delta-1)}\right]$$
$$+\phi^4\left[\frac{s^3(\delta+\beta^3)}{24(\delta-1)}-\frac{1}{4}\gamma_5+\frac{5}{8}(M^2-3\sigma_d)^{-3}-\frac{3}{4}\gamma_1(M^2-3\sigma_d)^{-2}+\frac{1}{8}\gamma_1^2(M^2-3\sigma_d)^{-1}+\frac{1}{3}\gamma_3(M^2-3\sigma_d)^{-1}\right].$$

In this case, Eq. (27) becomes

$$\left(\frac{d\phi}{d\zeta}\right)^2=a_1\phi^2-a_2\phi^{5/2}-a_3\phi^3-a_4\phi^{7/2}-a_5\phi^4, \tag{35}$$

where

$$a_5=-\left[\frac{s^3(\delta+\beta^3)}{12(\delta-1)}-\frac{1}{2}\gamma_5+\frac{5}{4}(M^2-3\sigma_d)^{-3}-\frac{3}{2}\gamma_1(M^2-3\sigma_d)^{-2}+\frac{1}{4}\gamma_1^2(M^2-3\sigma_d)^{-1}+\frac{2}{3}\gamma_3(M^2-3\sigma_d)^{-1}\right].$$

Based on the similar mathematical manipulation used before, Eq. (35) is written as

$$\left(\frac{d\theta}{d\zeta}\right)^2=a'\theta^2(F'-\theta)^4,$$

with the following relation: $a'=a_5/4$, $a_2a_4=16a_1a_5$, $a_3a_4=-6a_2a_5$, and $F'=a_4/4a_5$. The wave solution

$$\phi(\zeta)=\frac{F^2}{4}\left(1-\frac{1}{\tanh\left[\sqrt{a'}F'^2(\sqrt{\phi(\zeta)}-F')\frac{\zeta}{2}-\left(\frac{F'}{2(\sqrt{\phi(\zeta)}-F')}\right)\right]}\right)^2. \tag{36}$$

This solution represents a profile of a spiky solitary wave in the region $0<\phi(\zeta)<\sqrt{F'}$ while in the region defined as $\phi<0$, the soliton solution has an explosive solitary wave profile.

VII. DISCUSSION AND CONCLUSION

In this paper, by employing the reductive perturbation technique, we have studied the effects of adiabatic variation of charges, dust temperature and trapped electrons on DA solitary waves in an unmagnetized dusty plasma. The current neutrality from ions and electrons on the dust grains causes an adiabatic variation of dust charges which modifies the shape of DA solitary waves.

Now, we investigate the numerical analysis of the nonlinear equations obtained in this paper. First, we study the dependence of the dust charging effect on the electrostatic potential [Fig. 1(a)]. The dust charge number increases with a large slope increment from its unperturbed value to a higher one corresponding to electrostatic potential variation. If $\beta_h>0$ (particles have kinetic energies larger than the thermal energy) and the amount of trapped electrons decreases, the charge residing on the dust surface will increase. Also, the dust particle will have a stable charge more quickly for small percentage of trapped electrons, that can be explained as, for quickly stable charge, there are much free electrons contribute to charge the dust surface. The same behavior is the same for $\beta_h<0$. Figure 1(b) shows the dependence of Z_d on the ion–electron density and temperature ratios. As the temperature or the density of ions increases the dust charge will increase but with different manner. There exists a maximum admitted density value which can be called $\delta_{\max}$ nearest which the changes have significant and rapidly violence changes. Also, $\delta_{\max}$ increases and admits a larger region for charging dusts, as β decreases.

However, λ equals to the lower limit of the Mach number. These changes strongly affected the admitted Mach number regime for waves propagation. The dependence of the velocity of the small amplitude DA waves on the physical system parameters are shown in Fig. 2. Although, the inclusion of the dust temperature increases the velocity of the wave, the increase of the ion to electron density or temperature causes a decrease of λ. Also, the effect of the dust charge fluctuation is observed as a decrease in λ with respect to the constant dust charge dusty plasma.

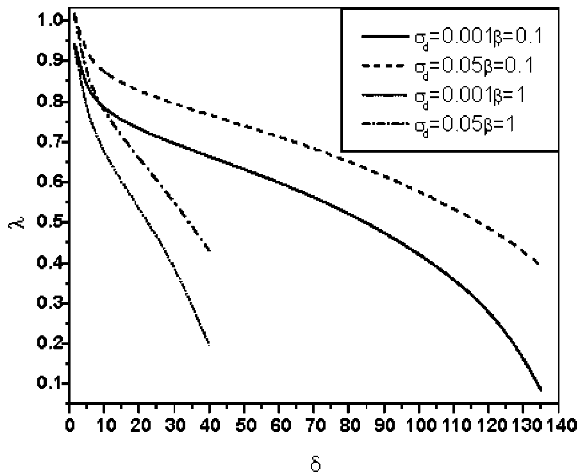

FIG. 2. The variation of phase velocity λ against system parameter variations.

The inclusion of both the free and trapped particles forces the system to be governed by MKdV equation (14). The solution of this equation admits compressive DA solitary waves. At δ_c or $\beta_h=1$ the MKdV equation fails to describe-the system. This forced us to apply a new stretching and we get the KdV equation, Eq. (19), that admits a rarefactive soliton only. The dependence of the amplitude on system parameter variations in both cases is investigated in the Fig. 3. We can conclude that:

(1) The amplitude ϕ_{1m} increases as the trapped electrons amount or the ion temperature increases. For $\delta<3$, ϕ_{1m} increases very rapidly then it goes in a decremental behavior. The effect of σ_d, that evidence from its dependence on constant B is to decrease the amplitude.

(2) The amplitude ϕ_{2m} slows down with a larger slope for $\delta<3$ after which the slope will change to a smaller one until δ goes closest to $\delta_{\max}$ the amplitude behaves the opposite behavior with a large slope increment. These behaviors agree with that obtained by Xie *et al.*[20] and that stated by Shukla and Mamun.[34] The β increment gives the amplitude ϕ_{2m} a chance to increase for $\delta<\delta_{\max}$. Here, the inclusion of σ_d causes an increment of ϕ_{2m} that contrast the ϕ_{1m} behavior.

On the other hand, the amplitude of MKdV solution has a smaller magnitude than of KdV solution and the width in the case of MKdV has a larger one than of KdV, that disagrees with the well known before and stated later by Mamun.[41]

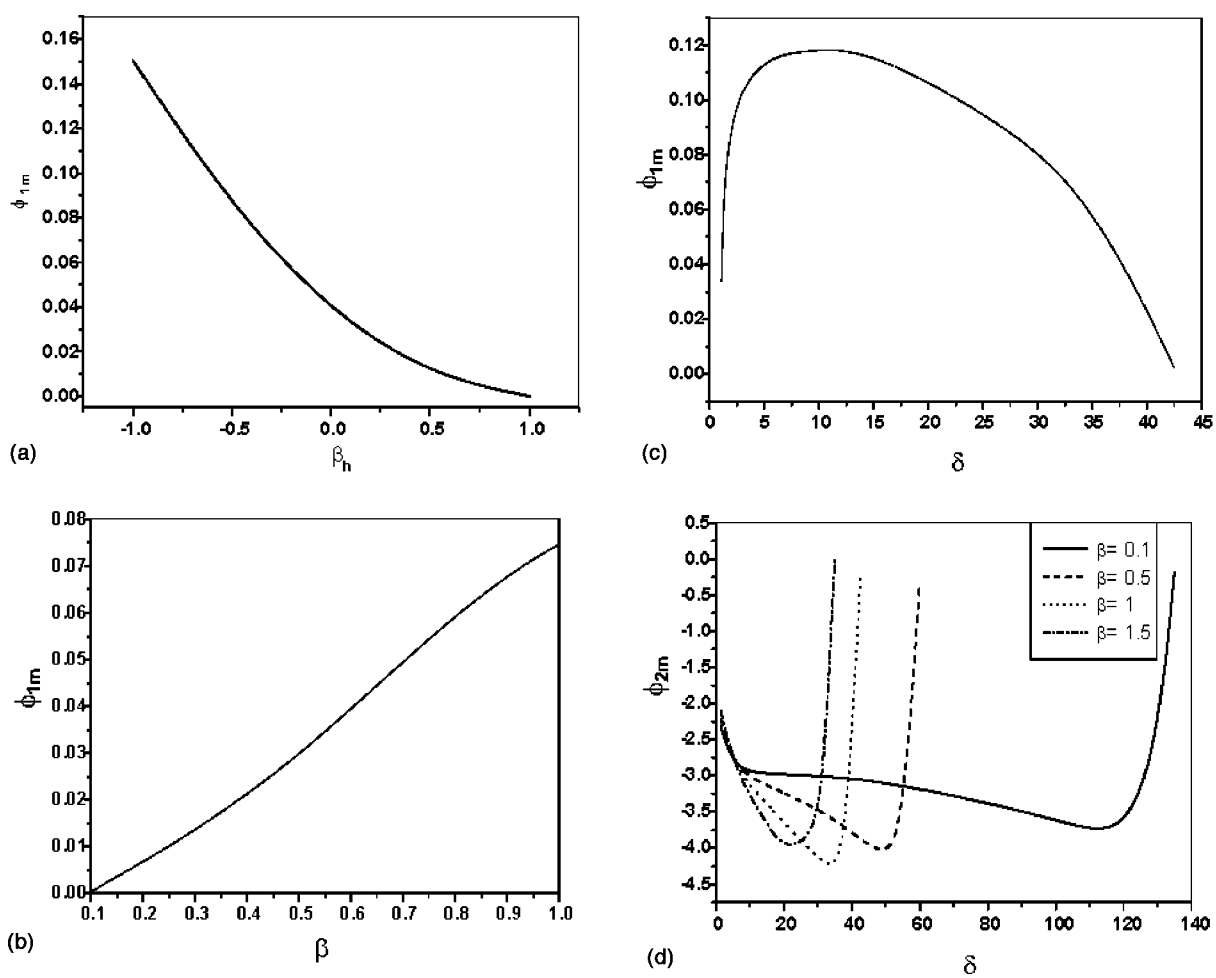

FIG. 3. ϕ_{1m} is plotted (a) against β_h for $\delta=20$, $\beta=1$, $\sigma_d=0.005$, (b) against β for $\delta=30$, $\beta_h=-0.7$, $\sigma_d=0.001$ and (c) against δ for $\sigma_d=0.001$, $\beta=1$, and $\beta_h=-0.7$ and (d) ϕ_{2m} is plotted against δ for $\sigma_d=0.001$ and different β values.

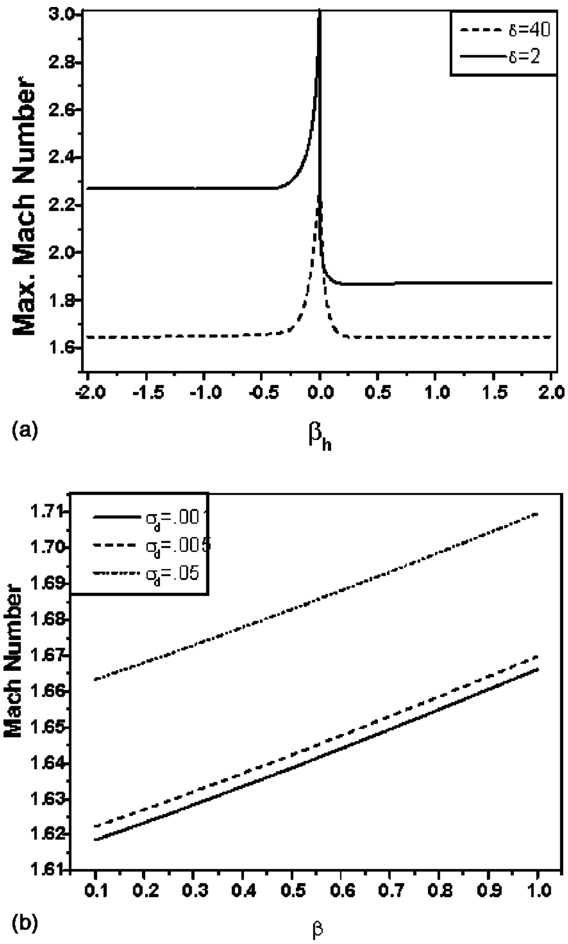

FIG. 4. (a) The variation of the maximum Mach number with β_h for two different values of δ with $\beta=0.5$ and $\sigma_d=0.05$, and (b) the variation of the maximum Mach number coressponding to variations in σ_d and β with $\delta=10$ and $\beta_h=0.5$.

Now, we take off the small amplitude waves and go to investigate the effect of the dust temperature and dust charge fluctuation on large amplitude waves and the Sagdeev potential. Figures 4–6 show the dependence of the Sagdeev potential and maximum Mach number on the system parameters variation. Thus, we can observe that the inclusion of the charge fluctuation increases the allowed Mach number regime with respect to the constant dust charge. However, the admitted velocity regime is extended due to the inclusion of trapped particles especially for the flat topped distributed ones. Higher temperature dust particles cause an increment of the Mach number that also can be done by raising the ion temperature but with different responses and extend the admitted regime. By investigation in Figs. 5 and 6, we observe the dependence of the Sagdeev potential on wave velocity that varies from the subsonic region to the supersonic one. The admitted solution transit from the rarefactive feature to the compressive one due to increasing of the wave velocity. The minimum potential value, $\phi_{\min}$, decreases as δ increases. Also, The ion and dust temperatures increase the area of the negative Sagdeev potential that allows rarefactive solitons. This effect can be overcome by the inclusion of the trapped electrons that works on the opposite way.

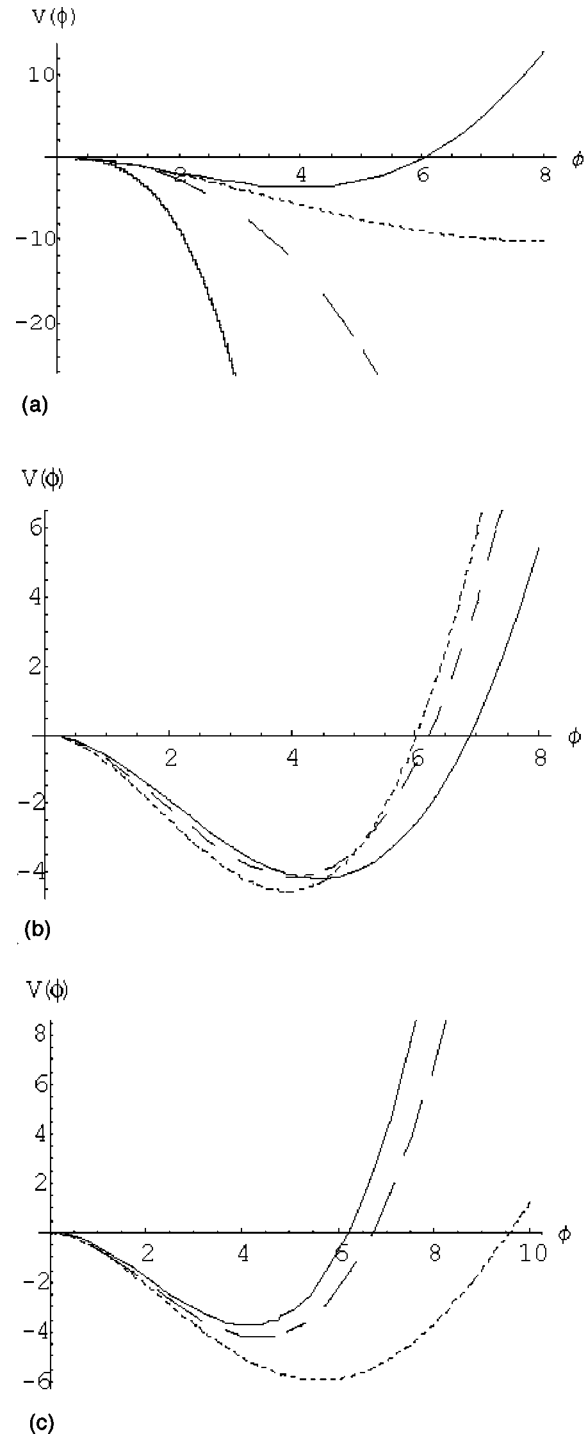

FIG. 5. The variation of the Sagdeev potential (a) against variation of the Mach number, where $\delta=10$, $\beta=0.5$, $\beta_h=0.3$, $\sigma_d=0$, $M=1.75$ (solid curve), $M=1.5$ (dotted curve), $M=1.25$ (dashed curve) and $M=0.9$ (lower curve), (b) against δ, where $\beta=0.5$, $\beta_h=0.1$, $\sigma_d=0$, $M=1.7$, $\delta=10$ (solid curve), $\delta=15$ (dashed curve), and $\delta=20$ (dotted curve) and (c) against β, where $\delta=10$, $\beta_h=0.3$, $\sigma_d=0$, $M=1.7$, $\beta=0.1$ (solid curve), $\beta=0.6$ (dashed curve), and $\beta=0.8$ (dotted curve).

Figure 7 shows the variation of the soliton solution; that deduced from the inclusion of the second order nonisothermality to our system, corresponding to system parameter variations. The allowed feature is the compressive one increases with any increase in the ion temperature or density.

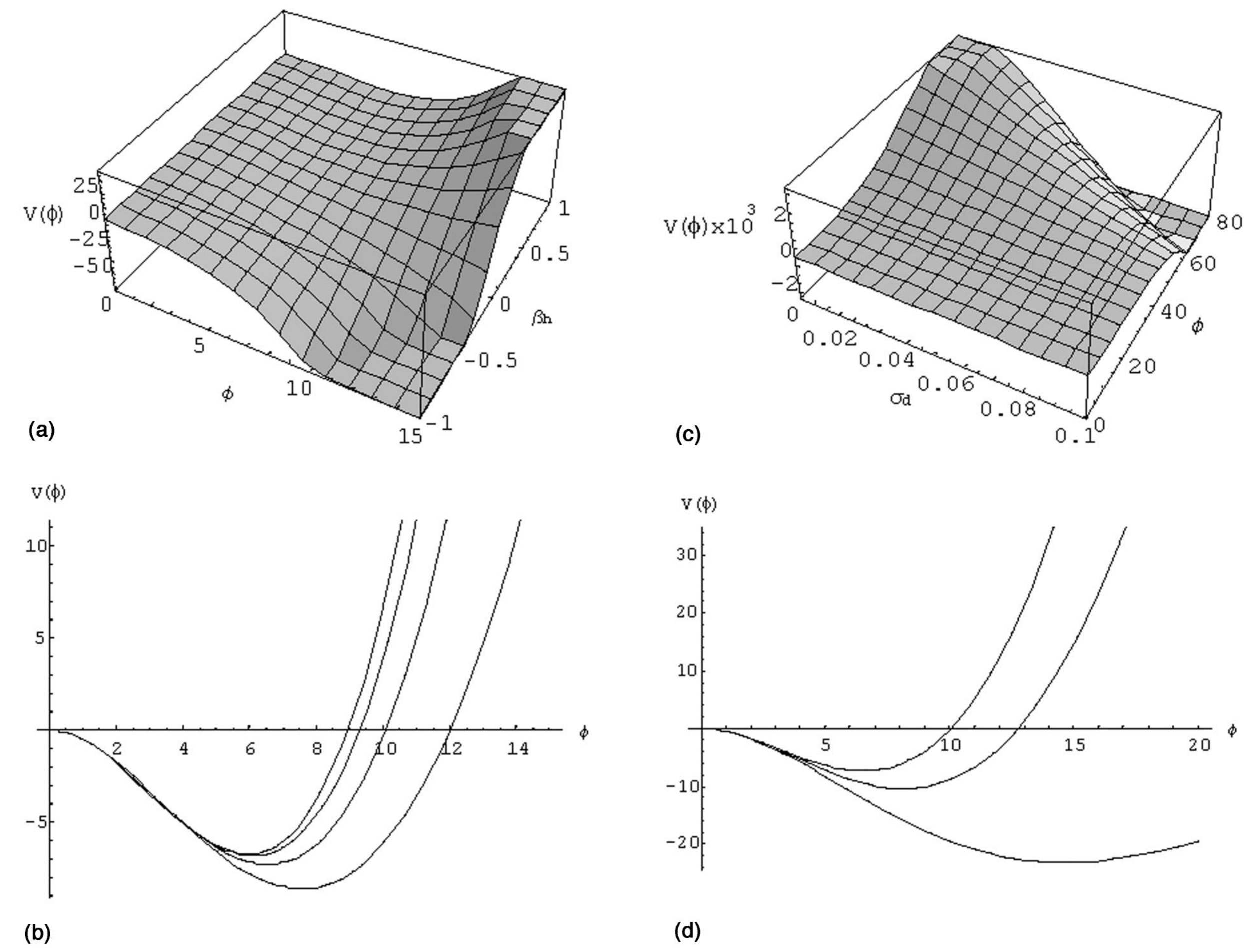

FIG. 6. The variation of the Sagdeev potential against β_h in (a) and (b), where $\delta=5$, $\beta=0.5$, $\sigma_d=0$, $M=1.7$, the variation of $\beta_h=0.1-0.7$, in (b), in the step of 0.2 from the lower curve to the upper one, and the variation of the Sagdeev potential against σ_d in (c,d), where $\delta=5$, $\beta=0.5$, $\beta_h=0.3$, $M=1.7$ and the variation of $\sigma_d=0-0.1$, in (d), in the step of 0.05 from the upper curve to the lower one.

The contrary is done by increase of the nonisothermal parameter.

Thus, we can conclude that the dust temperature and charge fluctuation effects must be included to investigate the dusty plasma systems containing trapped electrons. These effects drastically change all the solitary waves and double layers observed in such system. On the other hand, the Mach number lies within an appropriate regime depending on the system parameters. The results obtained in this paper agree exactly with those obtained by Xie *et al.*[20] (by neglecting the

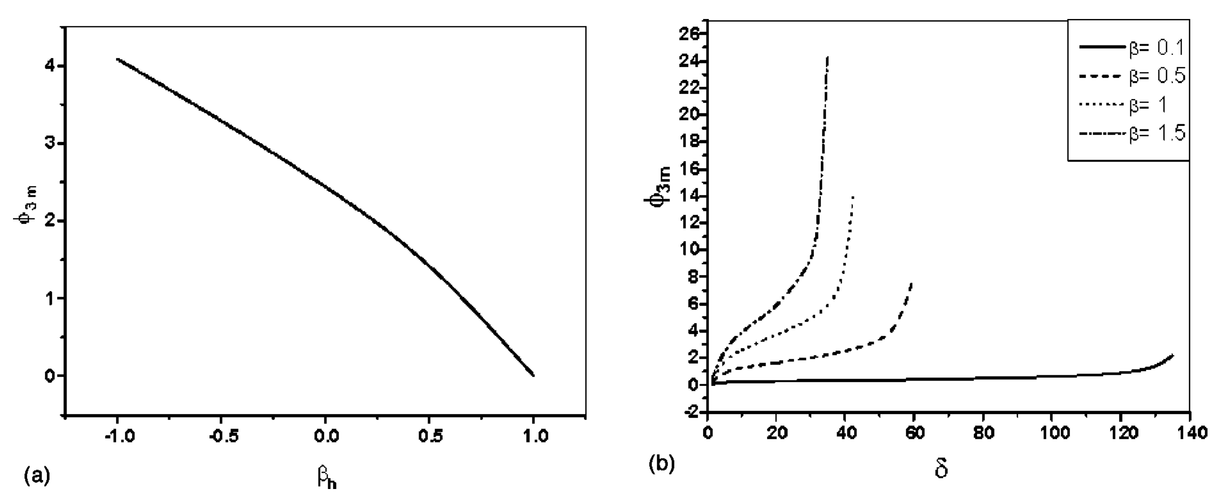

FIG. 7. ϕ_{3m} is plotted (a) against β_h for $\delta=20$, $\beta=1$, $\sigma_d=0.001$ and (b) against δ for $\sigma_d=0.001$ and $\beta_h=-0.7$ and different β values.

nonisothermal effect and dust temperature), and that obtained by Das *et al.*[29,30] (by neglecting the effect of dust temperature and variant charge). However, these results disagree with the lower limit of the wave velocity that obtained by Ma and Liu.[19] Also, it is the first time to consider the effect of the dust temperature and variation of dust charges in the presence of trapped electrons. We can consider this study as a modification and a generalization of these previous works.

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