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Effect of dust-charge variation on dust acoustic solitary waves in a dusty plasma with trapped electrons

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Abstract. The effect of variable dust charge, dust temperature and trapped electrons on small-amplitude dust acoustic waves is investigated. It is found that both compressive and rarefractive solitons as well as double layers exist depending on the non-isothermality parameter. A modified Korteweg de Vries (MKdV) equation is derived. Critical cases, at which the nonlinear coefficient is approximately zero, are derived. In the vicinity of the critical values, KdV and further MKdV (FMKdV) equations are obtained. Employing quasipotential analysis, the Sagdeev potential equation has been derived. Because of the presence of free and trapped electrons, the plasma acoustic wave exhibits different features of various solitary waves. The Sagdeev potential equation, at a small amplitude, shows that the ordering of non-isothermality in the dusty plasma plays a unique role. In the case of a plasma with first-order non-isothermality, the Sagdeev potential equation shows the compressive solitary-wave propagation while, for a plasma with higher-order non-isothermality, the solution of this equation reveals the coexistence of compressive and rarefractive solitary waves. In addition, for certain plasma parameters, the solitary wave disappears and a double layer is expected. Again, with the better approximation in the Sagdeev potential equation, more features of solitary waves, known as spiky and explosive, along with the double layers, are also highlighted. The findings of this investigation may be useful in understanding laboratory plasma phenomena and astrophysical situations.

1. Introduction

Plasma coexisting with a finite size of charged dust particles, known as a dusty plasma, has received much attention in the last few years. The dusty plasma exists in astrophysical bodies and space environments such as cometary tails, planetary ring systems, interstellar and circumstellar clouds, asteroid zones [1–7] as well as in laboratory plasmas, such as in tokamak and low-temperature glow discharges, and in the fabrication of semiconductors using plasma-aided processes. The ubiquitous nature of the dusty plasmas has spurred many researchers to study the new features in the plasma dynamical system. Normally, the low-temperature plasma environment sustains negatively charged dust formed by the attachment of the electrons to the dust grains while radiation, photoionization and field emission might yield positively charged dust grains. If one ignores the dust charge fluctuation dynamics, the dusty plasma can be regarded as a multicomponent plasma with several ionic species [8, 9]. However, the existence of a solitary acoustic wave in a multicomponent plasma had been reported by Dwivedi [10], contrasting the observation on a dust acoustic (DA) wave by Rao et al. [11], where the dust-particle mass provides the inertia and the pressures of inertialess electrons and ions provide the restoring force. Relevant observations were also made by other authors mentioned above [8, 9]. The theoretical observations made by Rao et al. [11] encouraged Barkan et al. [12] to undertake an experimental study in laboratory dusty plasma. Later, Mamun et al. [13], Ma and Liu [14], Nejoh [15] and Xie et al. [16] have considered the dusty plasma with the dust-charge fluctuation augmented through $I_i + I_e = 0$, wherein the study of plasma acoustic waves reveals the rarefractive soliton features.

If streaming particles are injected in plasmas, we often find that they evolve towards a coherent trapped-particle state. This has been confirmed by experiments [17]. The onset of electron trapping is also seen in the formation of double layers [18] and computer simulation [19]. It is well known that the presence of trapped particles can significantly modify the wave propagation characteristics in collisionless plasmas [20]. In particular, as discussed in detail elsewhere [21], mode propagation in the presence of resonant particles for phase speeds near the thermal range is intrinsically nonlinear.

Although Xie et al. [16] studied the effect of adiabatic variation of dust charges on DA waves with isothermal electrons and ions and Das and Sarma [22] studied solitary waves in a dusty plasma with the inclusion of trapped electrons, neither paper includes the effect of dust temperature and charge fluctuation on DA waves in the presence of trapped electrons. The motivation of this paper is the study of the dynamics of various solitary waves in the presence of trapped electrons and Boltzmann ions that play as a neutrality background to the dusty plasma. The dynamics of the DA mode might reveal some new features. Based on this, we have considered an ideal dusty plasma dynamics in space to revisit the soliton features in an unmagnetized dusty plasma in the present study. In Sec. 2, we present the relevant equation governing the dynamics of the nonlinear DA waves. In Sec. 3, the modified Korteweg de Vries (MKdV) equation is derived for DA solitary waves using perturbation theory. In Sec. 4, two critical cases are discussed and the evolution equations are derived. Also, the condition under which the double layers can be formed is obtained. In Sec. 5, we derive the Sagdeev potential and investigate the existence of different solitary waves and double layers and also its tendency to produce small-amplitude solitons is investigated. Sec. 6 is devoted to the discussion and conclusion.

2. Governing equations

The dusty plasma we are going to study consists of three components; extremely massive and highly negatively charged warm adiabatic dust grains, Boltzmann-distributed ions together with free and trapped electrons. The charge neutrality at equilibrium requires

$$n_{\mathrm{io}} = n_{\mathrm{eo}} + Z_{\mathrm{do}} n_{\mathrm{do}}, \tag{1}$$

where n_{eo}, n_{io} and n_{do} are the unperturbed electron, ion and dust number densities, respectively, and Z_{do} is the unperturbed number of charges residing on the dust grain measured in units of the electron charge.

For one-dimensional low-frequency dust acoustic motions, we have the following equations for the warm dust fluid:

$$\frac{\partial n_\mathrm{d}}{\partial t} + \frac{\partial (n_\mathrm{d} u_\mathrm{d})}{\partial x} = 0, \tag{2}$$

$$\frac{\partial u_\mathrm{d}}{\partial t} + u_\mathrm{d}\frac{\partial u_\mathrm{d}}{\partial x} + \frac{1}{m_\mathrm{d} n_\mathrm{d}}\frac{\partial p_\mathrm{d}}{\partial x} + \frac{Q_\mathrm{d}}{m_\mathrm{d}}\frac{\partial \phi}{\partial x} = 0, \tag{3}$$

where n_d, u_d, p_d and m_d refer to the number density, the fluid velocity, the fluid pressure and the mass of the dust grain, respectively. Here, the dust-charge variable $Q_\mathrm{d} = -eZ_\mathrm{d}$, where $Z_\mathrm{d} > 0$ is the variable charge number of dust grains in units of the electron charge $-e$. The Poisson equation is given by

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(Z_\mathrm{d} n_\mathrm{d} + n_\mathrm{e} - n_\mathrm{i}), \tag{4}$$

where the ions are assumed to obey the Boltzmann distribution as

$$n_\mathrm{i} = n_\mathrm{io}\exp\left(-\frac{e\phi}{T_\mathrm{i}}\right).$$

In the dynamical system, some of the electrons are attached to the dust grains to form the charged dust grains, while some of the remaining electrons are bounced back and forth in the potential well, lose energy continuously and, as a result, become ultimately trapped electrons. In this case, the electron density is defined from the Vlasov equation consisting of free and trapped electrons. Following Schamel [20, 23, 24], the non-isothermality of the plasma is introduced through the electron densities that have the normalized form

$$\begin{aligned} n_\mathrm{e}(\phi) &= \int_{-\infty}^{\infty} f_\mathrm{e}(x,v)\,dv \\ &= n_\mathrm{eo}\left[\exp(\Gamma)\mathrm{erf}c(\sqrt{\Gamma}) + \frac{1}{\sqrt{\beta_\mathrm{h}}}\right. \\ &\quad \left.\times \begin{Bmatrix} \exp(\Gamma\beta_\mathrm{h})\mathrm{erf}(\sqrt{\Gamma\beta_\mathrm{h}}) & \text{for } \beta_\mathrm{h} \geq 0, \\ \dfrac{2}{\sqrt{\pi}}\exp\left(\Gamma\beta_\mathrm{h}\displaystyle\int_0^{\sqrt{-\Gamma\beta_\mathrm{h}}}\exp(X^2)\,dX\right) & \text{for } \beta_\mathrm{h} < 0. \end{Bmatrix}\right], \end{aligned}$$

where $\Gamma = e\phi/T_\mathrm{e}$ and $f_\mathrm{e}(x,v)$ and β_h represent the electron distribution function and the ratio of the free to the trapped electron temperatures, T_f/T_t, respectively. Considering the Maxwellian distribution, the Taylor expansion of the last equation, for $\phi \ll 1$, derives the electron density, n_e, as a linear combination of free and trapped electrons as

$$n_\mathrm{e} = n_\mathrm{eo}\left[1 + \Gamma - \tfrac{4}{3}b_1\Gamma^{3/2} + \tfrac{1}{2}\Gamma^2 - \tfrac{8}{15}b_2\Gamma^{5/2} + \tfrac{1}{6}\Gamma^3 + \cdots\right].$$

The cases $\beta_\mathrm{h} = 1$ and $\beta_\mathrm{h} = 0$ correspond to the plasma having the Maxwellian and the flat-topped distributions, respectively. For an isothermal plasma, one can derive the electron density by imposing $b_1 = 0$ and $b_2 = 0$ whereas, for the non-isothermal plasma, we have $0 < b_1, b_2 < 1/\sqrt{\pi}$. Thus the non-isothermality of the plasma is expressed through the electron density, n_e, by the following modified form

$$n_\mathrm{e} = n_\mathrm{eo}[\exp(\Gamma) - G(\Gamma)],$$

where $G(\Gamma)=\sum_{k=1}^{n}[2^{(k+1)}b_k(\Gamma)^{(2k+1)/2}/\prod(2k+1)]$ with the non-isothermal parameter defined as $b_k=(1-\beta_{\mathrm{h}}^k)/\sqrt{\pi}$. Now we normalize all the physical quantities. We first introduce the effective temperature

$$T_{\mathrm{eff}}=\frac{T_{\mathrm{i}}T_{\mathrm{e}}}{(\mu T_{\mathrm{e}}+\nu T_{\mathrm{i}})},$$

where μ and ν are the normalized ion and electron number densities respectively. The densities of electrons and ions are normalized by $Z_{\mathrm{do}}n_{\mathrm{do}}$ and the dust density is normalized by n_{do}. Z_{d} is normalized by Z_{do}. The space coordinate x, time t, velocity u_{d}, pressure p_{d} and electrostatic potential ϕ are normalized by the Debye length $\lambda_{\mathrm{Dd}}=(T_{\mathrm{eff}}/4\pi Z_{\mathrm{do}}n_{\mathrm{do}}e^2)^{1/2}$, the inverse of the dust plasma frequency $\omega_{\mathrm{Pd}}^{-1}=(m_{\mathrm{d}}/4\pi Z_{\mathrm{do}}^2n_{\mathrm{do}}e^2)^{1/2}$, the dust acoustic speed $C_{\mathrm{d}}=(Z_{\mathrm{do}}T_{\mathrm{eff}}/m_{\mathrm{d}})^{1/2}$, $n_{\mathrm{do}}K_{\mathrm{B}}T_{\mathrm{d}}$ and T_{eff}/e, respectively. Therefore, we have the following set of basic equations in non-dimensional form:

$$\frac{\partial n_{\mathrm{d}}}{\partial t}+\frac{\partial(n_{\mathrm{d}}u_{\mathrm{d}})}{\partial x}=0, \tag{5}$$

$$\frac{\partial u_{\mathrm{d}}}{\partial t}+u_{\mathrm{d}}\frac{\partial u_{\mathrm{d}}}{\partial x}+3\sigma_{\mathrm{d}}n_{\mathrm{d}}\frac{\partial n_{\mathrm{d}}}{\partial x}-Z_{\mathrm{d}}\frac{\partial\phi}{\partial x}=0, \tag{6}$$

$$n_{\mathrm{i}}=\mu\exp(-s\phi), \tag{7}$$

$$n_{\mathrm{e}}=\nu\left[\exp(s\beta\phi)\mathrm{erf}c(\sqrt{s\beta\phi})+\frac{1}{\sqrt{\beta_{\mathrm{h}}}}\exp(\beta_{\mathrm{h}}s\beta\phi)\mathrm{erf}(\sqrt{\beta_{\mathrm{h}}s\beta\phi})\right] \tag{8}$$

The Poisson equation is given as

$$\frac{\partial^2\phi}{\partial x^2}=Z_{\mathrm{d}}n_{\mathrm{d}}+n_{\mathrm{e}}-n_{\mathrm{i}}, \tag{9}$$

where

$$\sigma_{\mathrm{d}}=\frac{T_{\mathrm{d}}}{Z_{\mathrm{do}}T_{\mathrm{eff}}},\quad \beta=\frac{T_{\mathrm{i}}}{T_{\mathrm{e}}}\quad\text{and}\quad s=\frac{1}{(\mu+\beta\nu)}.$$

Obviously, (1) leads to $\mu-\nu=1$.

Now, the dust-charge fluctuation included is determined by the charge current balance equation [25]:

$$\frac{\partial Q_{\mathrm{d}}}{\partial t}+u_{\mathrm{d}}\frac{\partial Q_{\mathrm{d}}}{\partial x}=I_{\mathrm{e}}+I_{\mathrm{i}}, \tag{10}$$

where Q_{d} is the dust-charge variable. The charge currents originating from electrons and ions reach the grain surface. Thus, the current-balance equation reads

$$I_{\mathrm{eo}}+I_{\mathrm{io}}\approx 0. \tag{11}$$

According to the well-known orbit-motion-limited probe model [1,6,7], we have the following expressions for electron and ion currents for spherical dust grains with radius r, normalized by $e\pi r^2(8T_{\mathrm{e}}/\pi m_{\mathrm{e}})^{1/2}$, as

$$I_{\mathrm{e}}=-n_{\mathrm{e}}\exp\left(\frac{e\Phi}{T_{\mathrm{e}}}\right)\quad\text{and}\quad I_{\mathrm{i}}=\alpha n_{\mathrm{i}}\left(1-\frac{e\Phi}{T_{\mathrm{i}}}\right),$$

where Φ denotes the dust grain surface potential relative to the plasma potential ϕ, $\alpha=(\beta/\mu_{\mathrm{i}})^{1/2}$, and $\mu_{\mathrm{i}}=m_{\mathrm{i}}/m_{\mathrm{e}}\approx 1840$. To compare our results with the previous ones, we introduce $\delta = \mu/\nu$ and the current-balance equation becomes

$$[\exp(s\beta[\Psi + \phi] - G(s\beta\phi)[\exp(s\beta\Psi)] = \alpha\delta(1 - s\Psi)\exp(-s\phi), \tag{12}$$

where $\Psi = e\Phi/T_{\text{eff}}$. Equation (12) is important for determining the dust charges $Q_{\text{d}} = C\Phi; C$ is the capacitance of a dust grain, i.e. $-eZ_{\text{d}} = rT_{\text{eff}}\Psi/e$. We have the normalized dust charges $Z_{\text{d}} = \Psi/\Psi_{\text{o}}$, where $\Psi_{\text{o}} = \Psi(\phi = 0)$ is the dust surface floating potential with respect to the unperturbed plasma potential at infinite place. Ψ_{o} can be determined from the following transcendental equation:

$$\exp(s\beta\Psi_{\text{o}}) - \alpha\delta(1 - s\Psi_{\text{o}}) = 0. \tag{13}$$

As can be seen, the dust charge is very sensitive to a small disturbance of ϕ around the unperturbed states. This point is very important to explain how the variable dust charge influences the shape of solitons and solitary waves.

3. Nonlinear dust acoustic (DA) waves

In order to study the dynamics of small-amplitude DA solitary waves in the presence of variation of dust charges, we derive an evolution equation from the system of (5)–(9), employing a reductive perturbation technique [26] by introducing the stretched coordinates [23] $\xi = \varepsilon^{1/4}(x - \lambda t)$ and $\tau = \varepsilon^{3/4}t$, where ε is a small parameter and λ is the solitary-wave velocity, normalized by C_{d}. The variables n_{d}, u_{d}, n_{b}, v_{b}, Z_{d} and ϕ are then expanded as

$$\begin{aligned}
n_{\text{d}} &= 1 + \varepsilon n_{\text{d}1} + \varepsilon^{3/2} n_{\text{d}2} + \varepsilon^2 n_{\text{d}3} + \varepsilon^{5/2} n_{\text{d}4} + \cdots,\\
u_{\text{d}} &= \varepsilon u_{\text{d}1} + \varepsilon^{3/2} u_{\text{d}2} + \varepsilon^2 u_{\text{d}3} + \varepsilon^{5/2} u_{\text{d}4} + \cdots,\\
Z_{\text{d}} &= 1 + \varepsilon Z_{\text{d}1} + \varepsilon^{3/2} Z_{\text{d}2} + \varepsilon^2 Z_{\text{d}3} + \varepsilon^{5/2} Z_{\text{d}4} + \cdots,\\
\phi &= \varepsilon\phi_1 + \varepsilon^{3/2}\phi_2 + \varepsilon^2\phi_3 + \varepsilon^{5/2}\phi_4 + \cdots.
\end{aligned}$$

Substituting these expansions into (5)–(9), using $\Psi = \Psi_{\text{o}} Z_{\text{d}}$ in (12) and then collecting terms of different powers of ε, in the lowest order we obtain

$$n_{\text{d}1} = -\phi_1 R, \quad u_{\text{d}1} = -\lambda\phi_1 R, \quad Z_{\text{d}1} = \frac{-(1+\beta)(1 - s\Psi_{\text{o}})\phi_1}{\Psi_{\text{o}}(1 + \beta(1 - s\Psi_{\text{o}}))} = \gamma_1\phi_1, \tag{14}$$

where $R = (\lambda^2 - 3\sigma_{\text{d}})^{-1}$. The linear dispersion relation is given by

$$\gamma_1 + 1 = R. \tag{15}$$

From this equation one can get an algebraic equation for λ, whose solution is given by

$$\lambda = [3\sigma_{\text{d}} + [\gamma_1 + 1]^{-1}]^{1/2}. \tag{16}$$

The next order in ε, $O(\varepsilon^{3/2})$, yields a system of equations in the subscripted-2 perturbed quantities. Eliminating these quantities, we get the MKdV equation

$$\frac{\partial\phi_1}{\partial\tau} + A\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} = 0, \tag{17}$$

where

$$B^{-1} = 2\lambda R^2, \quad A = B\left[\frac{2(s\beta)^{3/2}b_1}{(\delta - 1)} - \frac{3\gamma_2}{2}\right] \quad \text{and} \quad \gamma_2 = \frac{4\sqrt{s\beta^3}(1 - s\Psi_{\text{o}})b_1}{3\Psi_{\text{o}}(1 + \beta(1 - s\Psi_{\text{o}}))}.$$

To find the stationary solution of (17), we substitute $\eta = \xi - M\tau$ into (17) and, integrating twice, using the boundary conditions

$$\phi_1(\eta) \to 0, \quad \frac{d\phi_1(\eta)}{d\eta} \to 0, \quad \frac{d^2\phi_1(\eta)}{d\eta^2} \to 0 \quad \text{as } |\eta| \to \infty, \tag{18}$$

we get

$$\left(\frac{d\phi_1}{d\eta}\right)^2 = \frac{M\phi_1^2}{B}\left(1 - \frac{8A}{15M}\phi_1^{1/2}\right).$$

The one-soliton solution of (17) is given by

$$\phi_1 = \phi_{1\mathrm{m}} \mathrm{sech}^4[\eta/w_1], \tag{19}$$

where the amplitude $\phi_{1\mathrm{m}}$ and the width w_1 are given by $(15M/8A)^2$ and $4\sqrt{B/M}$, respectively.

4. Derivation of KdV and further MKdV (FMKdV) equations

The propagation of compressive solitons (that admitted only) depends on the sign of the nonlinear coefficient, A, of the MKdV equation. We can ensure that the dispersion coefficient of the MKdV equation, B, is always positive and thus the DA waves are compressive if $A > 0$. If $A \approx 0$, which corresponds to the so-called critical density ratio δ_c, the MKdV equation breaks down and one has to seek another equation suitable for describing the evolution of the system. When the physical parameters of the system make $A = 0$, we use the stretching coordinates [8, 27] $\xi = \varepsilon^{1/2}(x - \lambda t)$, $\tau = \varepsilon^{3/2} t$, and follow the procedure used before. We obtain the same relations as (14) for the lowest order of ε (coefficient of $O(\varepsilon^{3/2})$) and, to the order ε^2,we get

$$n_{\mathrm{d}2} = -R\phi_2, \quad u_{\mathrm{d}2} = -\lambda R\phi_2, \quad Z_{\mathrm{d}2} = \gamma_1\phi_2 + \gamma_2\phi_1^{3/2}, \tag{20}$$

$$[\gamma_1 - R + 1)]\phi_2 = \left[\frac{4(s\beta)^{3/2}b_1}{3(\delta - 1)} - \gamma_2\right]\phi_1^{3/2}. \tag{21}$$

If we consider the next-order in ε, $O(\varepsilon^{5/2})$, we obtain a system of equations in the subscripted-3 perturbed quantities. Solving this system with the aid of (12), (14) and (20), we obtain the following evolution equation

$$\frac{\partial\phi_1}{\partial\tau} + A\frac{\partial\phi_1^{1/2}\phi_2}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} + C\phi_1\frac{\partial\phi_1}{\partial\xi} = 0, \tag{22}$$

where

$$C = B\left[\frac{(\delta - 1)(\delta - \beta^2)}{(\delta + \beta)^2} + 3(\gamma_1 - [\lambda^2 + \sigma_\mathrm{d}]R^2)R - 2\gamma_3\right],$$

$$\gamma_3 = \frac{-(\delta - 1)(1 + \beta)^2(1 - s\Psi_\mathrm{o})}{2\Psi_\mathrm{o}(\delta + \beta)(1 + \beta(1 - s\Psi_\mathrm{o}))^2}.$$

Substituting $\eta = \xi - M\tau$ in (22) and integrating twice, using the boundary conditions (18), we get, for $A = 0$,

$$\left(\frac{d\phi_1}{d\eta}\right)^2 = \frac{M}{B}\phi_1^2\left(1 - \frac{C}{3M}\phi_1\right).$$

The one-soliton solution of (22) is given by

$$\phi_1 = \phi_{2\mathrm{m}} \mathrm{sech}^2[\eta/w_2], \tag{23}$$

where the amplitude $\phi_{2\mathrm{m}}$ and the width w_2 are given by $3M/C$ and $2\sqrt{B/M} = 0.5w_1$, respectively. Since $\gamma_1 \geq 0$, $\gamma_3 \geq 0$ and $M > 0$, (23) clearly indicates that only the rarefractive soliton waves exist.

On the other hand, when $A \to 0$ but $A \neq 0$, (23) would reduce to

$$\frac{\partial \phi_1}{\partial \tau} + D\phi_1^{1/2}\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + C\phi_1\frac{\partial \phi_1}{\partial \xi} = 0, \tag{24}$$

where we have used $A\phi_2 \to 2D\phi_1/3$ [28]. Substituting $\eta = \xi - M\tau$ in (24) and integrating twice, using the boundary conditions (18), we get

$$\frac{1}{2}\left(\frac{d\phi_1}{d\eta}\right)^2 = \frac{M\phi_1^2}{2B}\left(1 - \frac{8D\phi_1^{1/2}}{15M} - \frac{C\phi_1}{3M}\right) = -V(\phi_1, M). \tag{25}$$

Hence

$$V(\phi_1, M) = \frac{-M\phi_1^2}{2B} + \frac{4D\phi_1^{5/2}}{15B} + \frac{C\phi_1^3}{6B}. \tag{26}$$

For the formation of a double layer, we must have

$$V(\phi_\mathrm{m}, M) = 0, \quad \left(\frac{dV}{d\phi_1}\right)_{\phi_1=\phi_\mathrm{m}} = 0 \quad \text{and} \quad \left(\frac{d^2V}{d\phi_1^2}\right)_{\phi_1=\phi_\mathrm{m}} < 0. \tag{27}$$

These conditions imply that

$$\phi_\mathrm{m} = \left(\frac{4D}{5C}\right)^2 \quad \text{and} \quad M = \frac{-16D^2}{75C}. \tag{28}$$

Substituting for M and D in the relation (26), we obtain

$$V(\phi_1) = \frac{C\phi_1^2}{6B}\left(\phi_1^{1/2} - \phi_\mathrm{m}^{1/2}\right)^2. \tag{29}$$

From (25) and (29), we get

$$\left(\frac{d\phi_1}{d\eta}\right)^2 = \frac{-C\phi_1^2}{3B}\left(\phi_1^{1/2} - \phi_\mathrm{m}^{1/2}\right)^2.$$

The double-layer solution is

$$\phi_1 = \frac{\phi_\mathrm{m}}{4}(1 - \tanh[\eta/w])^2, \tag{30}$$

where

$$w = \frac{5}{D}\sqrt{-3BC}.$$

Obviously, the formation of the two types of DA double layers (compressive and rarefractive) depends on the signs of D and C.

Now, if the nonlinear coefficient of the KdV equation vanishes, $C \approx 0$, the KdV equation breaks down also and one has to look for another equation suitable for describing the evolution of the system. This implies that both of the stretching coordinates used before become invalid and instead of them we use the new stretching coordinates $\xi = \varepsilon^{3/4}(x - \lambda t)$ and $\tau = \varepsilon^{9/4} t$. We obtain the linear relation, (15), for the lowest order, and for the next order of ε we get the same relations (14) and (20). Also, we can obtain the third-order perturbed quantities as

$$\begin{gathered} n_{d3} = R\{-\phi_3 + E_1\phi_1^2\}, \quad u_{\mathrm{d}3} = \lambda R\{-\phi_3 + E_2\phi_1^3\}, \\ Z_{\mathrm{d}3} = \gamma_1\phi_3 + \tfrac{3}{2}\gamma_2\phi_2\phi_1^{1/2} + \gamma_3\phi_1^2 \end{gathered} \tag{31}$$

and

$$\begin{aligned}[1 + \gamma_1 - R]\phi_3 = {} & \left[\frac{2(s\beta)^{3/2}b_1}{\delta - 1} - \frac{3}{2}\gamma_2\right]\phi_1^{1/2}\phi_2 \\ & + \left[\frac{(\delta - 1)(\delta - \beta^2)}{2(\delta + \beta)^2} + \frac{3}{2}(\gamma_1 + [\lambda^2 + \sigma_{\mathrm{d}}]R^2)R - \gamma_3\right]\phi_1^2,\end{aligned}$$

where

$$E_1 = -\tfrac{1}{2}\gamma_1 + \tfrac{3}{2}[\lambda^2 + \sigma_{\mathrm{d}}]R^2 \quad \text{and} \quad E_2 = E_1 - \lambda R^2.$$

If we continue to the next order of $\varepsilon, O(\varepsilon^{5/2})$, we get a system of equations in the subscripted-4 perturbed quantities. Eliminating the perturbed quantities with subscript 4, we obtain a FMKdV equation

$$\frac{\partial\phi_1}{\partial\tau} + F\phi_1^{3/2}\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} = 0, \tag{32}$$

where

$$F/B = \tfrac{1}{6}\left[21\gamma_2 R - 15\gamma_4 + 8(s\beta)^{5/2}\nu b_2\right],$$

$$\begin{aligned}\gamma_4 = {} & \frac{-4(s\beta)^{3/2}(1 - s\Psi_{\mathrm{o}})}{15\Psi_{\mathrm{o}}(1 + \beta(1 - s\Psi_{\mathrm{o}}))^3}\{5b_1(1 - \beta^2(1 - s\Psi_{\mathrm{o}})(\beta(1 - s\Psi_{\mathrm{o}}) + 2) \\ & + 2\beta b_2(1 + \beta(1 - s\Psi_{\mathrm{o}}))^2\}\end{aligned}$$

and

$$Z_{\mathrm{d}4} = \gamma_1\phi_4 + \tfrac{3}{8}\gamma_2\left(\phi_2^2\phi_1^{-1/2} + 4\phi_1^{1/2}\phi_3\right) + 2\gamma_3\phi_1\phi_2 + \gamma_4\phi_1^{5/2}.$$

Substituting $\eta = \xi - M\tau$ in (32) and integrating twice, using the boundary conditions (18), we get

$$\left(\frac{d\phi_1}{d\eta}\right)^2 = \frac{M\phi_1^2}{B}\left(1 - \frac{8F}{35M}\phi_1^{3/2}\right).$$

The one-soliton solution of (32) is given by

$$\phi_1 = \phi_{3\mathrm{m}}\operatorname{sech}^{4/3}[\eta/w_3], \tag{33}$$

where the amplitude $\phi_{3\mathrm{m}}$ and the width w_3 are given by $(35M/8F)^{2/3}$ and $(4/3)\sqrt{B/M} = w_1/3$, respectively.

5. Quasipotential analysis and large amplitude solitary waves

Now we turn our attention to investigate the properties of large-amplitude DA solitary waves. We assume that all variables in (5)–(9) depend only on a single variable

$\zeta = x - Mt$, where ζ is normalized by $\lambda_{\rm Dd}$ and M is the Mach number (solitary-wave velocity divided by $C_{\rm d}$). In this stationary frame and using $\Psi = \Psi_{\rm o} Z_{\rm d}$, (5) and (6) can be integrated to give the following expression for dust number density

$$n_{\rm d} = \frac{1}{\sqrt{1 + 2(V_{\rm d}(\phi)/(M^2 - 3\sigma_{\rm d}))}}, \tag{34}$$

where we have imposed the appropriate boundary conditions for localized disturbances, *viz.* $\phi \longrightarrow 0, n_{\rm d} \longrightarrow 1, u_{\rm d} \longrightarrow 0, n_{\rm eo} \longrightarrow \nu, n_{\rm i} \longrightarrow \mu$ as $\zeta \longrightarrow \pm\infty$, and

$$V_{\rm d}(\phi) = \int_0^\phi Z_{\rm d}\, d\phi = \frac{1}{\Psi_{\rm o}} \int_0^\phi \Psi(\phi)\, d\phi. \tag{35}$$

Substituting for the normalized number densities of ion and electrons, the dust charge number expression $Z_{\rm d} = \Psi/\Psi_{\rm o}$ and the dust number density, (34), into the Poisson equation (7) and integrating it, imposing the boundary conditions for localized solutions, namely $\phi \longrightarrow 0$ and $d\phi/d\zeta \longrightarrow 0$ as $\zeta \longrightarrow \pm\infty$, we get

$$\frac{1}{2}\left(\frac{d\phi}{d\zeta}\right)^2 + V(\phi) = 0, \tag{36}$$

where the Sagdeev quasipotential reads

$$\begin{aligned} V(\phi) = {}& \frac{1}{\beta s(\delta - 1)}\left[1 - \exp(\beta s\phi)\mathrm{erf}c(\sqrt{\beta s\phi}) - \frac{1}{\sqrt{\beta_{\rm h}^3}}\exp(\beta_{\rm h}\beta s\phi)\mathrm{erf}(\sqrt{\beta_{\rm h}\beta s\phi})\right. \\ & \left. + 2\sqrt{\frac{\beta s\phi}{\pi}}\left(\frac{1}{\beta_{\rm h}} - 1\right)\right] + \{1 - \sqrt{1 + 2(V_{\rm d}(\phi)/(M^2 - 3\sigma_{\rm d}))}\}(M^2 - 3\sigma_{\rm d}) \\ & + \frac{\delta}{s(\delta - 1)}(1 - \exp(-s\phi)). \end{aligned} \tag{37}$$

In order to have solitary-wave solutions, the quasipotential must satisfy the following conditions. (i) $V(\phi) \to 0, dV(\phi)/d\phi \to 0$ and $d^2V(\phi)/d\phi^2 < 0$ at $\phi = 0$; i.e. the fixed point at the origin is unstable; (ii) there exists a non-zero $\phi_{\rm m}$, the maximum (or minimum) value of ϕ, at which $V(\phi_{\rm m}) \geq 0$ and (iii) $V(\phi) < 0$ when ϕ lies between 0 and $\phi_{\rm m}$. From (37), condition (i) leads to

$$M > M_{\rm l} = \sqrt{\frac{1}{\gamma_1 + 1} + 3\sigma_{\rm d}}. \tag{38}$$

This fixes the lower limit of M, which is equivalent to the value of λ obtained in Sec. 3. The upper limit of M for which negative solitary waves exist can be found from the condition $V(\phi_{1\,\rm min}) = 0$, where $\phi_{1\,\rm min}$ is the minimum value of ϕ for which the dust density $n_{\rm d}$ is real, i.e. $V_{\rm d}(\phi_{1\,\rm min}) = -(M^2 - 3\sigma_{\rm d})/2$, e.g. $V_{\rm d0}(\phi_{1\,\rm min}) \equiv \phi_{1\,\rm min} = -(M^2 - 3\sigma_{\rm d})/2$ in the case of constant dust charge.

The critical upper limit of Mach number $M_{\rm cr}$ for which positive plasma potential solitary waves exist can be found from the condition $V(\phi_{1\,\rm max}) = 0$, where $\phi_{1\,\rm max}$ is the maximum value of ϕ; meanwhile $V(\phi)$ is tangent to the ϕ-axis for which $dV(\phi_{1\,\rm max})/d\phi = 0$, i.e. $\phi_{1\,\rm max}$ and $M_{\rm cr}$ should be determined by the following relations:

$$\frac{1}{\beta s(\delta-1)}\left[1-\exp(\beta s\phi)\mathrm{erf}c(\sqrt{\beta s\phi})-\frac{1}{\sqrt{\beta_{\mathrm{h}}^3}}\exp(\beta_{\mathrm{h}}\beta s\phi)\mathrm{erf}(\sqrt{\beta_{\mathrm{h}}\beta s\phi})\right.$$
$$\left.+2\sqrt{\frac{\beta s\phi}{\pi}}\left(\frac{1}{\beta_{\mathrm{h}}}-1\right)\right]+\frac{\delta}{s(\delta-1)}(1-\exp(-s\phi))$$
$$+\left\{1-\sqrt{1+2(V_{\mathrm{d}}(\phi)/(M^2-3\sigma_{\mathrm{d}}))}\right\}(M^2-3\sigma_{\mathrm{d}})=0$$

and

$$\frac{\delta}{(\delta-1)}\exp(-s\phi)-\frac{1}{(\delta-1)}\left[\exp(\beta s\phi)\mathrm{erf}c(\sqrt{\beta s\phi})+\frac{1}{\sqrt{\beta_{\mathrm{h}}}}\exp(\beta_{\mathrm{h}}\beta s\phi)\mathrm{erf}(\sqrt{\beta_{\mathrm{h}}\beta s\phi})\right]$$
$$+\frac{Z_{\mathrm{d}}}{\sqrt{1+2(V_{\mathrm{d}}(\phi)/(M^2-3\sigma_{\mathrm{d}}))}}=0.$$

The coupled transcendental equations can be solved numerically for the largest positive potential amplitude and upper Mach number where the positive plasma potential solitary waves should exist.

The quasipotential can be obtained from (37), where $V_{\mathrm{d}}(\phi)$ is obtained numerically from (35).

If we expand the expression for the quasipotential around $\phi=0$, for small amplitudes we will recover all the results of the small-amplitude DA soliton obtained by the reduced perturbation technique in Sec. 4. For example, for small ϕ, we can write to the order of $O(\phi^{5/2})$

$$V_{\mathrm{d}}(\phi)=\phi+\frac{\gamma_1}{2}\phi^2+\frac{2\gamma_2}{5}\phi^{5/2}.$$

Substituting for $V_{\mathrm{d}}(\phi)$ and the dimensionless number density of dust, electrons and ions in (37) we have

$$V(\phi)=\frac{1}{2}\{(M^2-3\sigma_{\mathrm{d}})^{-1}-1-\gamma_1\}\phi^2+\frac{2}{5}\left[\frac{4(s\beta)^{3/2}b_1}{3(\delta-1)}-\gamma_2\right]\phi^{5/2}.$$

Equation (36) with (34) can be rewritten as

$$\left(\frac{d\phi}{d\zeta}\right)^2=a_1\phi^2-a_2\phi^{5/2}, \tag{39}$$

where $a_1=M/B$ and $a_2=8A/15B$. Equation (39) has the soliton solution as in (19). For slightly larger values of ϕ, the Sagdeev potential can be given as

$$V(\phi)=\frac{1}{2}\{(M^2-3\sigma_{\mathrm{d}})^{-1}-1-\gamma_1\}\phi^2+\frac{2}{5}\left[\frac{4(s\beta)^{3/2}b_1}{3(\delta-1)}-\gamma_2\right]\phi^{5/2}$$
$$+\phi^3\left(\frac{(\delta-1)(\delta-\beta^2)}{6(\delta+\beta)^2}+\frac{1}{2}\gamma_1(M^2-3\sigma_{\mathrm{d}})^{-1}-\frac{1}{3}\gamma_3-\frac{1}{2}(M^2-3\sigma_{\mathrm{d}})^{-2}\right)$$

and we can write (36) as

$$\left(\frac{d\phi}{d\zeta}\right)^2=a_1\phi^2-a_2\phi^{5/2}-a_3\phi^3, \tag{40}$$

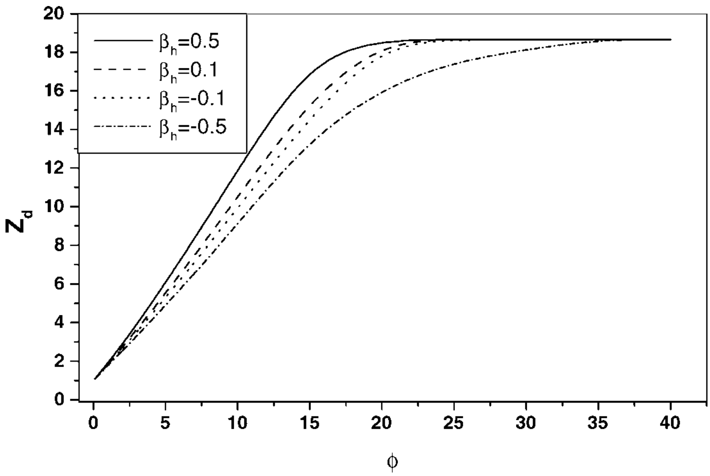

Figure 1. $Z_{\rm d}$ against plasma potential disturbance ϕ for different $\beta_{\rm h}$ values with $\delta = 10$ and $\beta = 0.5$.

where $a_3 = C/3B$. Equation (40) has a shock-like wave solution as in (30). For $a_2 = 0$, we have a soliton solution like (23). Other types of solitons, *viz.* spiky-type solitary waves, collapsible waves, etc., can be obtained by taking higher-order terms and using the so-called 'tanh' method [29] or the modified extended tanh method [30]. Since the expression for $V(\phi)$ derived in (37) is exact, one can expand it up to any desirable power of ϕ and then can obtain all different types of solitary waves depending on the non-isothermal parameter, dust temperature and the charge-variation effect obtained by perturbation theory. If we expand $Z_{\rm d}$ to higher powers of ϕ as

$$Z_{\rm d} = 1 + \gamma_1\phi + \gamma_2\phi^{3/2} + \gamma_3\phi^2 + \gamma_4\phi^{5/2},$$

the Sagdeev potential can be given as

$$\begin{aligned} V(\phi) = {} & \frac{1}{2}\{(M^2 - 3\sigma_{\rm d})^{-1} - 1 - \gamma_1\}\phi^2 + \frac{2}{5}\left[\frac{4(s\beta)^{3/2}b_1}{3(\delta-1)} - \gamma_2\right]\phi^{5/2} \\ & + \phi^3\left(\frac{(\delta-1)(\delta-\beta^2)}{6(\delta+\beta)^2} + \frac{1}{2}\gamma_1(M^2-3\sigma_{\rm d})^{-1} - \frac{1}{3}\gamma_3 - \frac{1}{2}(M^2-3\sigma_{\rm d})^{-2}\right) \\ & + \phi^{7/2}\left[\frac{2}{5}\gamma_2(M^2-3\sigma_{\rm d})^{-1} - \frac{2}{7}\gamma_4 + \frac{16(s\beta)^{5/2}b_2}{105(\delta-1)}\right]. \end{aligned}$$

In this case, (36) becomes

$$\left(\frac{d\phi}{d\zeta}\right)^2 = a_1\phi^2 - a_2\phi^{5/2} - a_3\phi^3 - a_4\phi^{7/2}, \tag{41}$$

where $a_4 = 8F/35B$. If $a_2 = a_3 = 0$, then we have a soliton solution like (33). Otherwise, we can reform (41), using $\phi = \theta^2$, as

$$\left(\frac{d\theta}{d\zeta}\right)^2 = a\theta^2(\overline{\phi} - \theta)^3, \tag{42}$$

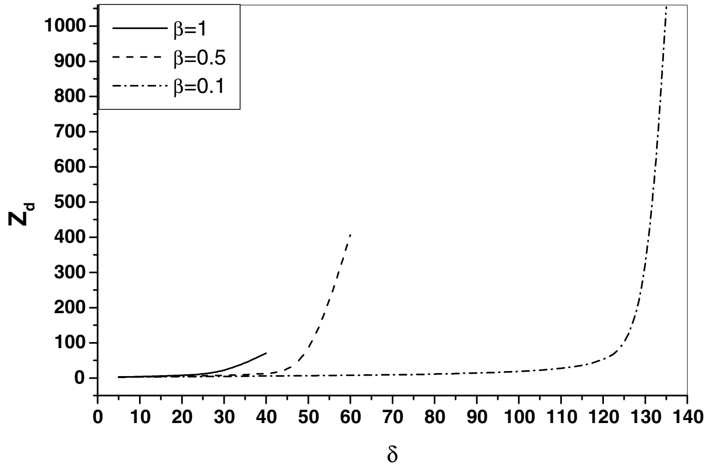

Figure 2. $Z_{\rm d}$ against δ for different β values with $\phi = 2$ and $\beta_{\rm h} = 0.9$.

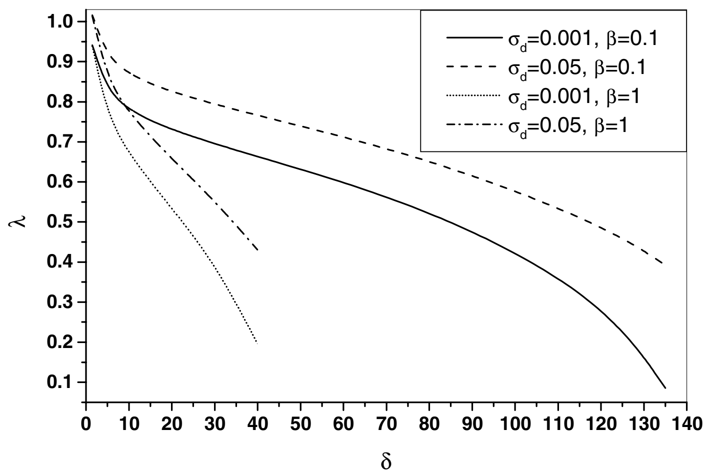

Figure 3. The variation of phase velocity λ against system parameter variations.

where $\overline{\phi} = -a_3/3a_4, a = a_4/4$ and $a_3^2 = 3a_2a_4$. Equation (42) can be solved for the soliton profile and the solution can be obtained as an implicit function of η in the following form

$$\phi(\zeta) = \overline{\phi}^2 \left(1 - \tanh^2\left[\left(\frac{\overline{\phi}}{\overline{\phi} - \sqrt{\phi(\zeta)}}\right)^{1/2} - \sqrt{a\overline{\phi}^3}\frac{\zeta}{2}\right]\right)^2. \tag{43}$$

Note that $\phi(\zeta)$ occurs on both the left- and right-hand sides of (43). The solution of (43) gives a profile of a spiky solitary wave defined in the region $0 < \phi(\zeta) < \sqrt{\overline{\phi}}$ while, for the region defined as $\phi < 0$, the soliton solution can be obtained in a similar manner and it has an explosive solitary-wave profile in the plasma acoustic dynamics. Thus one can proceed by taking the nonlinear term to any power of

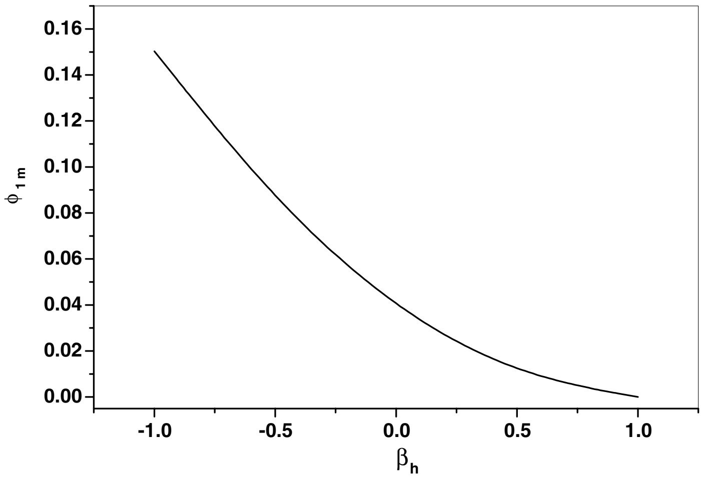

Figure 4. $\phi_{1\mathrm{m}}$ is plotted against β_{h} for $\delta = 20, \beta = 1$ and $\sigma_{\mathrm{d}} = 0.005$.

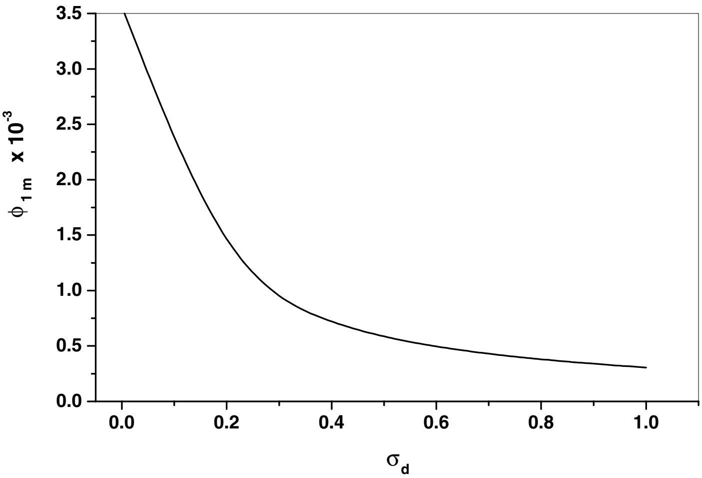

Figure 5. $\phi_{1\mathrm{m}}$ is plotted against σ_{d} for $\delta = 20, \beta = 1$ and $\beta_{\mathrm{h}} = 0.7$.

ϕ and then can investigate different natures of the solitary waves under different approximations.

6. Discussion and conclusion

In this paper, by employing the reductive perturbation technique, we have studied the effects of adiabatic variation of charges, dust temperature and trapped electrons on DA solitary waves in an unmagnetized dusty plasma. The current neutrality from ions and electrons on the dust grains causes an adiabatic variation of dust charges, which modifies the shape of the DA solitary waves. We have derived the MKdV (17) and the admitted compressive DA solitary waves has been obtained. At some critical phase velocities, the MKdV equation fails to describe the system. This forced us to apply a new stretching and we obtained a KdV equation, (22),

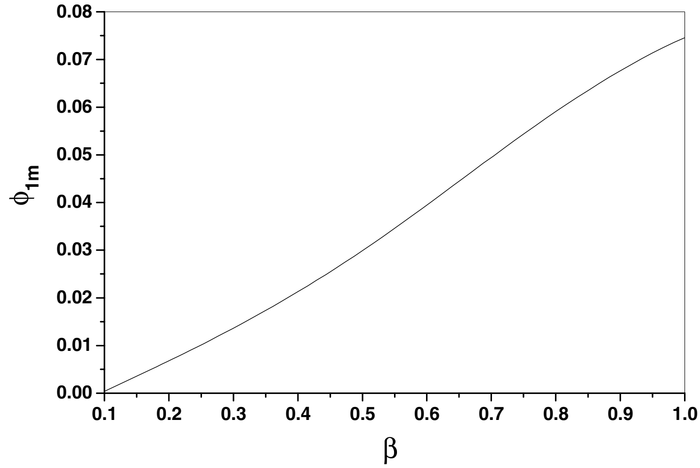

Figure 6. ϕ_{1m} is plotted against β for $\delta = 30, \beta_h = -0.7$ and $\sigma_d = 0.001$.

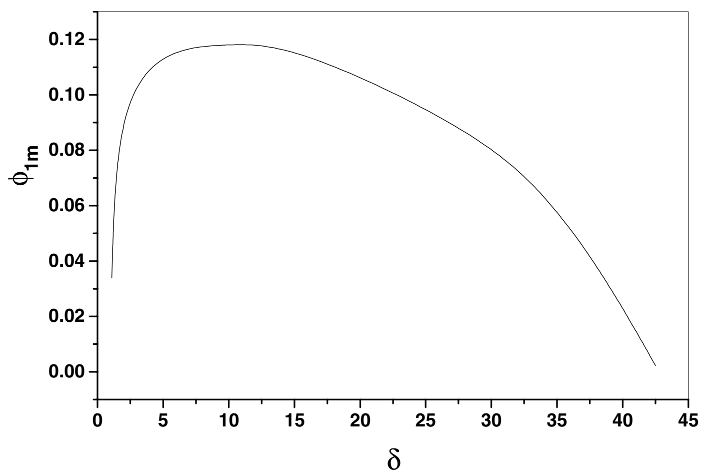

Figure 7. ϕ_{1m} is plotted against δ for $\sigma_d = 0.001, \beta = 1$ and $\beta_h = -0.7$.

that admits a rarefractive soliton only. On the other hand, there exist some critical points that make the nonlinear coefficients of the KdV and MKdV equations nearly zero, and thus they also fail to describe the system and we replaced the previous two stretching variables with a newer one. Applying this stretching leads to a FMKdV equation, (32), that covers and solves this problem and allows only a compressive soliton. Also, we derive the condition under which double layers with two different types can exist. Thus, DA solitons exist in the region where DA double layers do not exist. The dependence of amplitude, width, velocity and dust charge on system parameter variations is investigated in the following figures.

Figures 1 and 2 illustrate the dependence of Z_d on the system parameter variations. They show that, for a positive plasma potential ϕ with an increasing disturbance strength, Z_d increases first quickly with a large slope, then gradually slows down with a smaller slope and finally reaches a constant value. As β_h increases from its negative value to a positive one, Z_d increases. Also, as β increases, Z_d increases.

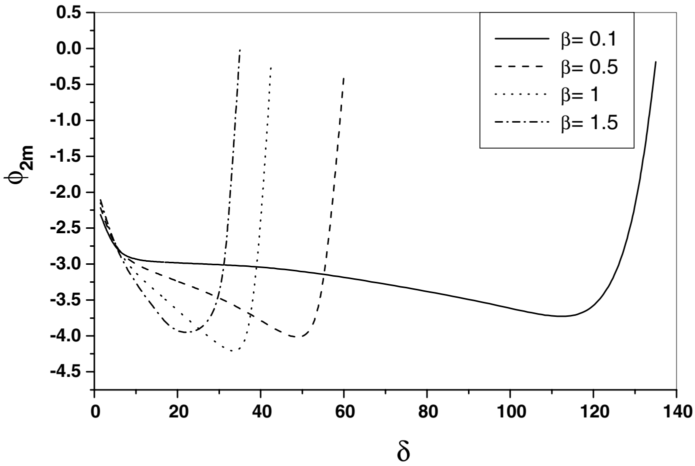

Figure 8. $\phi_{2\mathrm{m}}$ is plotted against δ for $\sigma_{\mathrm{d}} = 0.001$ and different β values.

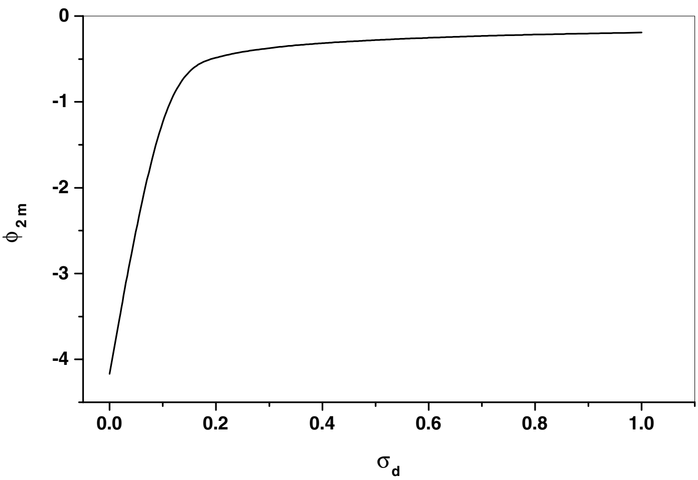

Figure 9. $\phi_{2\mathrm{m}}$ is plotted against σ_{d} for $\delta = 25$ and $\beta = 1$.

On the other hand, Z_{d} has a nearly fixed value for a variation in δ, except near the maximum admitted δ, which we call $\delta_{\max}$, where it increases rapidly.

Figure 3 shows the variation of λ with δ. It shows that λ decreases as δ or β increases. On the contrary, λ increases as σ_{d} increases.

The variation of $\phi_{1\mathrm{m}}$ with physical parameters is shown in Figs 4–7, from which we conclude that:

- $\phi_{1\mathrm{m}}$decreases as σ_{d} or β_{h} increases. On the contrary, it increases as β increases.
- For small δ, $\phi_{1\mathrm{m}}$ increases rapidly as δ increases and then, for moderate or higher values of δ, $\phi_{1\mathrm{m}}$ decreases as δ increases.

Figures 8 and 9 show the variation of the rarefractive soliton solution versus various system parameter variations. They show that the amplitude $\phi_{2\mathrm{m}}$ decreases rapidly as σ_{d} increases and then it behaves like the exponential decay function.

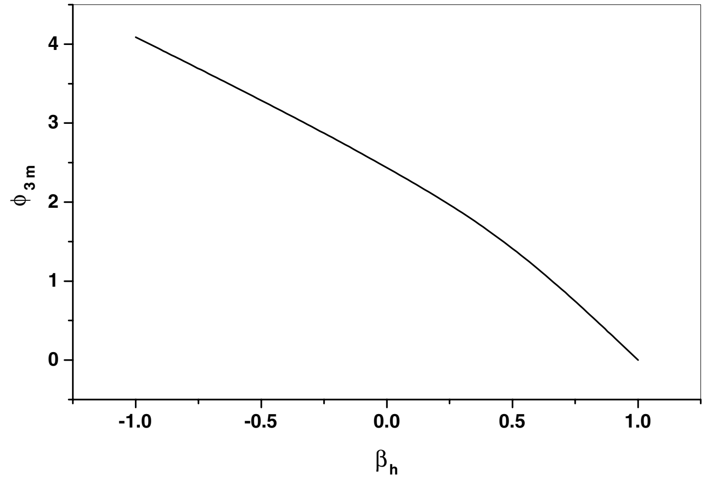

Figure 10. $\phi_{3\mathrm{m}}$ is plotted against β_{h} for $\delta = 20, \beta = 1$ and $\sigma_{\mathrm{d}} = 0.001$.

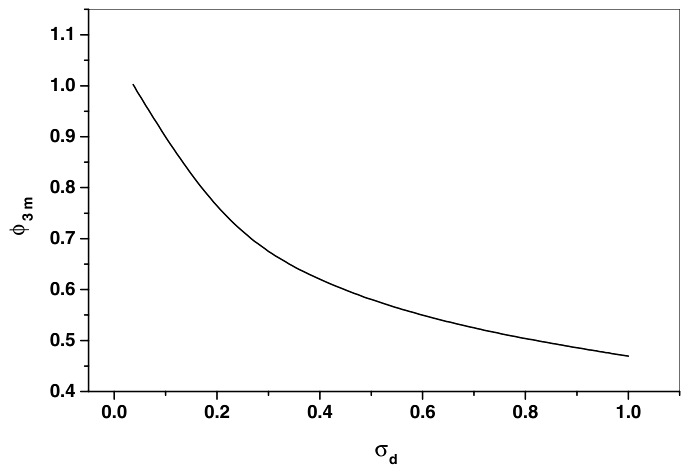

Figure 11. $\phi_{3\mathrm{m}}$ is plotted against σ_{d} for $\delta = 20, \beta = 1$ and $\beta_{\mathrm{h}} = 0.7$.

On the other hand, $\phi_{2\mathrm{m}}$ decreases slowly as δ increases; however, nearest to δ_{max}, it increases rapidly (note the similar profiles while changing β and compare them with the $\mathrm{Z_d}$ profile). Figure 8 shows also that as β increases $\phi_{2\mathrm{m}}$ decreases.

After precise investigation, we can see a remarkable similarity of the curves. This should not be surprising if we recognize the fact that it is an indication of an approximate similarity law of the system. From the expressions (12) and (13) we find that the system parameter $\gamma = \alpha\delta = (n_{\mathrm{io}}/n_{\mathrm{eo}})\sqrt{(T_{\mathrm{i}}m_{\mathrm{e}}/T_{\mathrm{e}}m_{\mathrm{i}})}$, which determines the approximate similarity law of the system, is very important. If we perform a scale transform $\beta^{'} = \ell\beta, \delta^{'} = \delta/\sqrt{\ell}$ such that $\alpha^{'} = \sqrt{\ell}\alpha$ and $\alpha^{'}\delta^{'} = \alpha\delta$ are kept constant, the solutions $\Psi^{'}$ and $\Psi^{'}_{\mathrm{o}}$ are almost equivalent to the original Ψ and Ψ_{o}. However, since the parameters $\beta, \beta_{\mathrm{h}}$ and s can still modify the quantities of the Mach number and the peak amplitude $\phi_{1\mathrm{m}}$ and Z_{d1}, this similarity is in a rough sense as shown in Fig. 8.

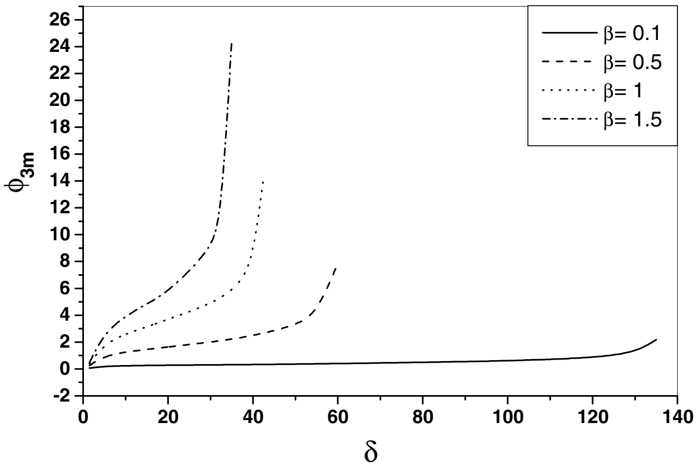

Figure 12. $\phi_{3\mathrm{m}}$ is plotted against δ for $\sigma_{\mathrm{d}} = 0.001$ and $\beta_{\mathrm{h}} = -0.7$ and different β values.

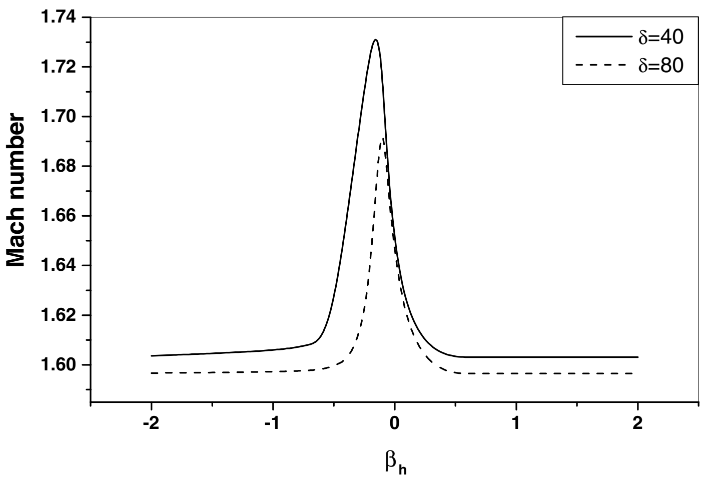

Figure 13. The variation of the upper limit of the Mach number with β_{h} for two different values of δ with $\beta = 0.5$ and $\sigma_{\mathrm{d}} = 0.05$.

Figures 10, 11 and 12 show that $\phi_{3\mathrm{m}}$ decreases as β_{h} or σ_{d} increases but in a different manner. $\phi_{3\mathrm{m}}$ increases as δ or β increases. It increases rapidly as it becomes closest to $\delta_{\max}$.

Figures 13 and 14 illustrate the dependence of the upper limit of the Mach number on the variation of the system parameters. One can observe that M_{cr} changes drastically with any β_{h} variation. M_{cr} decreases as δ increases, but it increases as β or σ_{d} increases.

On the other hand, Sagdeev's quasipotential approach is extremely suitable for studying large-amplitude solitary waves in plasma. One can derive all the one-soliton results of perturbation methods and can compare them with the exact results obtained by the quasipotential (also called the pseudopotential) method. The pseudopotential is derived for the preceding system. A mechanism was found

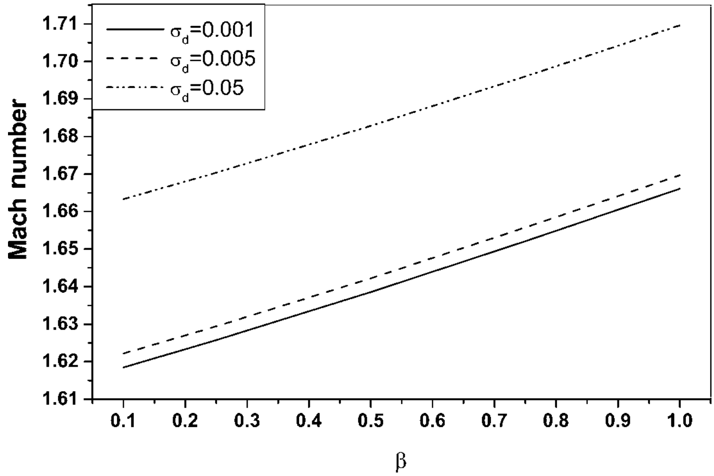

Figure 14. The variation of the upper limit of the Mach number corresponding to variations in σ_{d} and β with $\delta = 10$ and $\beta_{\mathrm{h}} = 0.5$.

by which one can obtain a power series in ϕ for the pseudopotential. It was found that different ordering gives rise to different types of solitons. In the case of higher-order nonlinearity both explosive solitary waves, where the energy in the soliton is conserved, and the collapse of the soliton, where the energy is not conserved in the wave profile, were found. Moreover, as the exact Sagdeev potential is obtained, one can expand it up to any desired power of ϕ. The modification in the amplitude and the width of the solitary-wave structures due to the inclusion of the effects of reflected electrons, dust temperature and charge fluctuation is investigated. It has been shown that the non-isothermality plays an important role in the Sagdeev potential to yield the various solitary waves. This investigation would be effective for understanding the properties of grain charging and the dynamics of DA waves in the presence of trapped electrons.

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