Dust acoustic solitary waves and double layers in a dusty plasma with an arbitrary streaming ion beam

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The effects of variable dust charge, dust temperature and an arbitrary streaming ion beam on small amplitude dust acoustic waves are investigated. It is found that both compressive and rarefactive solitons as well as double layers exist. There exists a critical ion beam velocity below which the ion beam is unable to generate solitons. Korteweg–de Vries (KdV) equations with third and fourth-order nonlinearity at the critical values are derived and the properties of dust acoustic solitary waves are discussed. In the vicinity of the critical values, KdV-type with mixed nonlinearity is obtained. The quite dense positive ion, the dust temperature, the ion beam density, mass ratio and temperature govern the existence of dust acoustic waves. The findings of this investigation may be useful in understanding laboratory plasma phenomena and astrophysical situations. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1557912]

I. INTRODUCTION

There has been a rapidly growing interest in physics of dusty plasmas not only because of dust being an omnipresent ingredient of our universe, but also because of its vital role in understanding different collective processes (mode modification, new eigenmodes, coherent structures, etc.) in astrophysical and space environments.[1–6] The consideration of charged dust grains in a plasma does not only modify the existing plasma wave spectra,[7,8] but also introduces a number of new novel eigenmodes, such as dust acoustic (DA) waves,[9,10] dust ion acoustic (DIA) waves,[11,12] dust lattice waves,[13,14] etc.

Using a reductive perturbation theory, Xie *et al.*[15] derived small amplitude DA solitons with varying dust charges and they have shown that only rarefactive solitary waves exist when the Mach number lies within an appropriate regime depending on the system parameters. Also, the DA solitary waves and double layers in dusty plasma with variable dust charge and two-temperature ions were studied by Xie *et al.*[16] They have shown that both compressive and rarefactive solitons as well as double layers exist. Also, the amplitudes of the dust solitary waves become smaller and the regime of Mach number is extended wider for the variable dust charge situation with the case of constant dust charge. On the other hand, the ion beams in laboratory dusty plasmas have become indispensable in the field of materials processing such as etching, chemical vapor deposition and surface modification.[17] Such circumstances in plasma applications and the ease of realizing dusty plasmas on a laboratory scale have accelerated active studies on dust phenomena in plasmas. The topics of nonlinear grain charge variation and electrostatic ion waves[18,19] have been reported by regarding dust grains as point charges, where the Debye length is much larger than the inter-grain distance. The nonlinear dynamics of the dust charge and large amplitude DA waves in a plasma with an ion beam were studied by Nejoh.[20] He treated the beam ions as thermal without the inclusion of the particle streaming in his calculation of the beam current at the dust grain surface. Also he considered the beam fluid as isothermally distributed ones. In this paper, we focus our attention on the characteristics and behavior of the small amplitude electrostatic dust acoustic waves in an unmagnetized dusty plasma with an arbitrary streaming positive ion beam.

The manuscript is organized as follows. The basic equations describing the dusty plasma system under consideration, incorporating the contribution from the variable dust charge is given in Sec. II. In Sec. III, using current balance condition, the dependence of the dust charge on the plasma parameters, especially the relation of the dust charge variation to the plasma potential is obtained. In Sec. IV, using a reductive perturbation technique, the small amplitude DA solitary structures are studied with the inclusion of number of important effects as adiabatic dust fluid temperature, adiabatic variation of dust grain charges and ion beam. In Sec. V, two critical cases for the system are discussed and the modified Korteweg–de Vries (MKdV) and Korteweg–de Vries type (KdV type) equations with third, fourth-order, and mixed nonlinearity are obtained. Their solutions are also discussed. Section VI is devoted to the discussion and conclusion.

II. BASIC EQUATIONS

The dusty plasma we are studying, consists of four components; extremely massive, highly negatively charged dust grains, electrons, ions and positive ion beam. Charge neutrality at equilibrium reads

$$n_{io}+n_{bo}=n_{eo}+Z_{do}n_{do}, \tag{1}$$

where n_{io}, n_{bo}, n_{eo}, and n_{do} are the unperturbed ion, beam, electron, and dust number densities, respectively, and Z_{do} is the unperturbed number of charges residing on the dust grain measured in the unit of electron charge.

For one-dimensional low-frequency DA motions, we have the following nondimensional equations for the ion beam and warm dust fluids:[16]

$$\frac{\partial n_d}{\partial t}+\frac{\partial(n_d u_d)}{\partial x}=0, \tag{2}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}+3\sigma_d n_d\frac{\partial n_d}{\partial x}-Z_d\frac{\partial\phi}{\partial x}=0, \tag{3}$$

$$\frac{\partial n_b}{\partial t}+\frac{\partial(n_b v_b)}{\partial x}=0, \tag{4}$$

$$\frac{\partial v_b}{\partial t}+v_b\frac{\partial v_b}{\partial x}+3\mu_{bd}\sigma_b n_b\frac{\partial n_b}{\partial x}+\mu_{bd}\frac{\partial\phi}{\partial x}=0, \tag{5}$$

$$\frac{\partial^2\phi}{\partial x^2}=Z_d n_d+n_e-n_i-n_b, \tag{6}$$

where n_d and u_d refer to the number density and fluid velocity of the dust grain, respectively, and n_b and v_b are the corresponding beam parameters, respectively. All densities are normalized by $n_{do}Z_{do}$. The space coordinate x, time t, velocities and electrostatic potential ϕ are normalized by the Debye length $\lambda_{Dd}=(T_{\rm eff}/4\pi Z_{do}n_{do}e^2)^{1/2}$, the inverse dust plasma frequency $\omega_{pd}^{-1}=(m_d/4\pi Z_{do}^2 n_{do}e^2)^{1/2}$, the DA speed $C_d=(Z_{do}T_{\rm eff}/m_d)^{1/2}$ and $T_{\rm eff}/e$, respectively. The dimensionless number densities of electrons and ions are expressed as

$$n_e=\frac{n_{eo}}{Z_{do}n_{do}}\exp(\beta_1 s\phi),$$

$$n_i=\frac{n_{io}}{Z_{do}n_{do}}\exp(-s\phi).$$

Also, we introduce the following notations:

$$\sigma_d=\frac{T_d}{Z_{do}T_{\rm eff}},\quad \sigma_b=\frac{T_b}{T_{\rm eff}}=\frac{1}{s\beta_b},\quad \mu_{bd}=\frac{m_d}{m_b Z_{do}},$$

$$\delta_1=n_{io}/n_{eo},\quad \delta_2=n_{bo}/n_{eo},\quad \beta_1=\frac{T_i}{T_e},\quad \beta_b=\frac{T_i}{T_b},$$

$$s=\frac{T_{\rm eff}}{T_i}=\left(\frac{\delta_1+\delta_2-1}{\delta_1+\delta_2\beta_b+\beta_1}\right),$$

with temperature T_e for electrons, temperature T_i for ions and temperature T_b for beam in unit of energy, respectively, (m_d/m_b) are the mass of (dust/beam) particles, respectively.

III. CHARGING OF DUST GRAINS

Dust particles are charged due to a variety of processes including the bombardment of the dust grain surface by background plasma electrons, ions and incident ion beams, photoelectron emission by ultraviolet (UV) radiation, ion sputtering, secondary electron production, etc. In low-temperature laboratory plasmas, dust particles are mainly negatively charged when any plasma electrons hitting the surface of the dust grains are attached to it and simply lost from the background plasma.[1] In general, the dust charge variable Q_d is determined by the charge current balance equation[21]

$$\frac{\partial Q_d}{\partial t}+u_d\frac{\partial Q_d}{\partial x}=I_e+I_i+I_b.$$

We notice that the characteristic time for dust motion is of order of tens of milliseconds for micrometersized grains,[10] while the dust charging time is typically of order of 10^{-8} s. Therefore, on the hydrodynamic time scale, the dust charge can quickly reach local equilibrium, at which the currents from the electrons, beam and ions to the dust are balanced. The current balance equation reads[4,21]

$$I_{eo}+I_{io}+I_{bo}\approx 0.$$

According to the well-known orbit-motion-limited probe model,[22] we have the following expressions for the electron and ion currents for spherical dust grains with radius r:

$$I_e=-e\pi r^2(8T_e/\pi m_e)^{1/2}n_e\exp\left(\frac{e\Phi}{T_e}\right)$$

and

$$I_i=e\pi r^2(8T_i/\pi m_i)^{1/2}n_i\left(1-\frac{e\Phi}{T_i}\right).$$

For ion beam that have an arbitrary streaming velocity v_o, we can express the ion beam current at grain surface[1,4,23] as

$$I_b=e\pi r^2(8T_b/\pi m_b)^{1/2}n_b\left[F_1(u_o)-F_2(u_o)\frac{e\Phi}{T_b}\right],$$

where Φ denotes the dust grain surface potential relative to the plasma potential ϕ and

$$F_1(u_o=v_o/\sqrt{2}v_{bth})$$
$$=\frac{\sqrt{\pi}}{4u_o}(1+2u_o^2)\mathrm{erf}(u_o)+0.5\exp(-u_o^2),$$

$$F_2(u_o)=\frac{\sqrt{\pi}}{2u_o}\mathrm{erf}(u_o),\quad v_{bth}=\sqrt{\frac{T_b}{m_b}}.$$

If $u_o=0$, we can get $F_1=F_2=1$ which tends to the case of nonstreaming beam that considered by Nejoh.[20] From the current balance equation, we have

$$\alpha_1\delta_1(1-s\Psi)\exp(-s\phi)+\alpha_2 n_b(\delta_1+\delta_2-1)(F_1-F_2 s\beta_b\Psi)$$
$$=\exp(s\beta_1[\Psi+\phi]), \tag{7}$$

where $\Psi=e\Phi/T_{\rm eff}$ and $\alpha_1=(\beta_1/\mu_i)^{1/2}$, $\alpha_2=(\beta_2/\mu_b)^{1/2}$, $\beta_2=\beta_1/\beta_b$, $\mu_{i,b}=m_{i,b}/m_e$.

Equation (7) is important for determining the dust charges due to the relation $Q_d=\check{C}\Phi$, where $\check{C}$ is the capacitance of dust grain. We have the normalized dust charges $Z_d=\Psi/\Psi_o$, where $\Psi_o=\Psi$ $(\phi=0)$ is the dust surface floating potential with respect to the unperturbed plasma potential at infinite place. Ψ_o can be determined from the following transcendental equation:

$$\alpha_1\delta_1(1-s\Psi_o)+\alpha_2\delta_2(F_1-s\beta_b F_2\Psi_o)=\exp(s\beta_1\Psi_o). \tag{8}$$

As can be seen, the dust charge is very sensitive to the small disturbance of ϕ around the unperturbed states. This point is very important for explanation how the variable dust charge influences the shape of solitons and solitary waves.

IV. DUST ACOUSTIC SOLITONS

In order to study the dynamics of small amplitude DA solitary waves in the presence of adiabatic variation of dust charges, we derive an evolution equation from Eqs. (2)–(6) by employing a reductive perturbation technique and the stretched coordinates[24] $\xi=\varepsilon^{1/2}(x-\lambda t)$, and $\tau=\varepsilon^{3/2}t$, where ε is a small parameter and λ is the solitary wave velocity normalized by C_d. The variables n_d, u_d, n_b, v_b, Z_d, and ϕ are then expanded as

$$n_d=1+\varepsilon n_{d1}+\varepsilon^2 n_{d2}+\varepsilon^3 n_{d3}+\cdots,$$
$$n_b=\nu+\varepsilon n_{b1}+\varepsilon^2 n_{b2}+\varepsilon^3 n_{b3}+\cdots,$$
$$u_d=\varepsilon u_{d1}+\varepsilon^2 u_{d2}+\varepsilon^3 u_{d3}+\cdots,$$
$$v_b=v_o+\varepsilon v_{b1}+\varepsilon^2 v_{b2}+\varepsilon^3 v_{b3}+\cdots,$$
$$Z_d=1+\varepsilon Z_{d1}+\varepsilon^2 Z_{d2}+\varepsilon^3 Z_{d3}+\cdots,$$
$$\phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+\cdots.$$

Substituting these expansions into Eqs. (2)–(6), and using $\Psi=\Psi_o Z_d$ in Eq. (7), then collecting terms of different powers of ε, in the lowest order we obtain

$$n_{b1}=\mu_{bd}\nu R_1\phi_1,\quad v_{b1}=\tilde{\lambda}\mu_{bd}R_1\phi_1,\quad Z_{d1}=\gamma_1\phi_1,$$
$$n_{d1}=-\phi_1 R_2,\quad u_{d1}=-\lambda\phi_1 R_2;\quad \tilde{\lambda}=\lambda-v_o \tag{9}$$

where $R_1=(\tilde{\lambda}^2-3\Lambda)^{-1}$, $R_2=(\lambda^2-3\sigma_d)^{-1}$; $\Lambda=\sigma_b\mu_{bd}\nu^2$ and the parameter γ_1 is derived in the Appendix.

The linear dispersion relation is given by

$$\gamma_1+\frac{s(\beta_1+\delta_1)}{(\delta_1+\delta_2-1)}=\nu\mu_{bd}R_1+R_2, \tag{10}$$

from which one can get a fourth-order algebraic equation in λ. If all the roots are real, each root would indicate a possible solitary wave. It is well-known that $v_o=0$ gives the stability of the waves that can be easily studied. However, in order to get the proper DA solitary waves with an ion beam, we have chosen the initial velocity of the ion beam, functionally depending on the system parameters in such a way that instability of the waves does not play a significant part.

The second order in ε yields a system of equations in the second-order perturbed quantities. Eliminating the second-order perturbed quantities, we get the standard KdV equation

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=0, \tag{11}$$

where

$$B^{-1}=2(\lambda R_2^2-C+\tilde{\lambda}\nu\mu_{bd}R_1^2),$$
$$A/B=s^2\left(\frac{-\beta_1^2+\delta_1}{\delta_1+\delta_2-1}\right)+R_2(3\gamma_1-3(\lambda^2+\sigma_d)R_2^2)$$
$$-2\gamma_2+3\mu_{bd}^2R_1^3\nu(\tilde{\lambda}^2+\Lambda),$$

and $C=[\alpha_2\delta_2\tilde{\lambda}\mu_{bd}R_1^2(F_1-s\beta_b F_2\Psi_o)/s\gamma_a\Psi_o]$ and γ_2 is derived in the Appendix. To find the stationary solution of Eq. (11), we substitute $\eta=\xi-M\tau$ into Eq. (11) and integrate twice, using the boundary conditions

$$\phi_1(\eta)\to 0,\quad \frac{d\phi_1(\eta)}{d\eta}\to 0,\quad \frac{d^2\phi_1(\eta)}{d\eta^2}\to 0\ \text{ as }\ |\eta|\to\infty, \tag{12}$$

to get

$$\left(\frac{d\phi_1}{d\eta}\right)^2=\frac{M\phi_1^2}{B}\left(1-\frac{A}{3M}\phi_1\right).$$

The one-soliton solution of (11) is given by

$$\phi_1=\phi_{1m}\,\mathrm{Sech}^2[\eta w_1], \tag{13}$$

where $\phi_{1m}=3M/A$ is the amplitude and $w_1^{-1}=2\sqrt{B/M}$ is the width. If for a given value of ion beam velocity, B is negative, the width of the soliton becomes imaginary and hence the ion beam will be unstable to excite DA waves in our system. Therefore, the condition for generation of DA waves is that the ion beam velocity must satisfy

$$\tilde{\lambda}\nu\mu_{bd}R_1^2+\lambda R_2^2>C. \tag{14}$$

Thus there is an ion beam velocity below which the ion beam does not generate DA soliton. The corresponding velocity is a function of beam–dust concentration, the initial surface potential, the dust temperature and beam temperature.

V. CRITICAL CASES

The propagation of compressive and rarefactive solitons depends on the sign of the nonlinear coefficient, A, of the KdV equation. If we assume that the dispersion coefficient of the KdV equations, B, is positive, thus the DA waves are (compressive/rarefactive) if $(A>0/A<0)$. When the phase velocity λ reaches the so-called λ_c; critical phase velocity, the nonlinear coefficient of the KdV equation vanishes $A=0$, and therefore, KdV equation breaks down and one has to seek for another equation suitable for describing the evolution of the system. This implies that the stretching coordinates mentioned above are not valid for this critical case and we use new stretching coordinates for obtaining the evolution equation describing the system.

When the plasma system parameters make $A=0$, we use the stretching coordinates[25] $\xi=\varepsilon(x-\lambda t),\tau=\varepsilon^3 t$, and expand the dependent variables in the same manner as before. Substituting them into Eqs. (2)–(6), and using $\Psi=\Psi_o Z_d$ in Eq. (7), then collecting terms in different powers of ε, we obtain the same relations as (9) for the lowest order of ε (coefficient of ε^2), and to the order of ε^3, we get

$$n_{b2}=\mu_{bd}\nu R_1\{\phi_2+G_1[\phi_1^2/2]\},$$
$$n_{d2}=-R_2\{\phi_2-J_1[\phi_1^2/2]\},$$
$$v_{b2}=\tilde{\lambda}\mu_{bd}R_1\{\phi_2+h_1[\phi_1^2/2]\},$$
$$u_{d2}=-\lambda R_2\{\phi_2-K_1[\phi_1^2/2]\},$$
$$Z_{d2}=\gamma_1\phi_2+\gamma_2\phi_1^2, \tag{15}$$

where

$$G_1=3\mu_{bd}[\tilde{\lambda}^2+\Lambda]R_1^2,\quad H_1=\mu_{bd}R_1^2[\tilde{\lambda}^2+9\Lambda],$$

$$J_1=-\gamma_1+3[\lambda^2+\sigma_d]R_2^2,\quad K_1=-\gamma_1+[\lambda^2+9\sigma_d]R_2^2.$$

If we consider the next-order in ε, we obtain a system of equations in the third-order perturbed quantities. Solving this system with the aid of Eqs. (9) and (15), we finally obtain the MKdV equation

$$\frac{\partial\phi_1}{\partial\tau}+A\frac{\partial\phi_1\phi_2}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}+F\frac{\partial\phi_1^3}{\partial\xi}=0, \tag{16}$$

where

$$F/B=\frac{-s^3}{6}\left(\frac{\beta_1^3+\delta_1}{\delta_1+\delta_2-1}\right)+\frac{1}{6}(8\gamma_2+3\gamma_1^2)R_2-\gamma_3$$
$$-3(\lambda^2+\sigma_d)R_2^3\gamma_1+\frac{1}{2}(30\lambda^2\sigma_d+5\lambda^4+9\sigma_d^2)R_2^5$$
$$+\frac{1}{2}(30\tilde{\lambda}^2\Lambda+5\tilde{\lambda}^4+9\Lambda^2)\nu\mu_{bd}^3R_1^5,$$

where γ_3 is derived in the Appendix. Substituting $\eta=\xi-M\tau$ in Eq. (16) and integrating twice, using the boundary conditions; (11), we get for $A=0$

$$\left(\frac{d\phi_1}{d\eta}\right)^2=\frac{M}{B}\phi_1^2\left(1-\frac{F}{2M}\phi_1^2\right).$$

The one-soliton solution of Eq. (16) is given by

$$\phi_1=\phi_{2m}\,\mathrm{Sech}[\eta w_2], \tag{17}$$

where the amplitude $\phi_{2m}=\pm\sqrt{2M/F}$, and the width $w_2^{-1}=\sqrt{B/M}=0.5w_1^{-1}$. Obviously, the physically reasonable solitons, correspond to the condition $F>0$, and in this case both compressive and rarefactive solitons are also allowed to coexist.

On the other hand, when $A\rightarrow 0$ but $A\neq 0$, Eq. (16) would reduce to

$$\frac{\partial\phi_1}{\partial\tau}+D\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}+F\frac{\partial\phi_1^3}{\partial\xi}=0, \tag{18}$$

where we have used $A\phi_2\rightarrow D\phi_1/2$.[16,25,26] Substituting $\eta=\xi-M\tau$ in Eq. (16) and integrating twice, using the boundary conditions; (11), we get

$$\frac{1}{2}\left(\frac{d\phi_1}{d\eta}\right)^2=\frac{M\phi_1^2}{2B}\left(1-\frac{D}{3M}\phi_1-\frac{F}{2M}\phi_1^2\right)=-V(\phi_1,M). \tag{19}$$

Hence

$$V(\phi_1,M)=\frac{-M\phi_1^2}{2B}+\frac{D}{6B}\phi_1^3+\frac{F}{4B}\phi_1^4. \tag{20}$$

For the formation of double layer, we must have

$$V(\phi_m,M)=0,\quad \left(\frac{dV}{d\phi_1}\right)_{\phi_1=\phi_m}=0\ \text{ and }\ \left(\frac{d^2V}{d\phi_1^2}\right)_{\phi_1=\phi_m}<0. \tag{21}$$

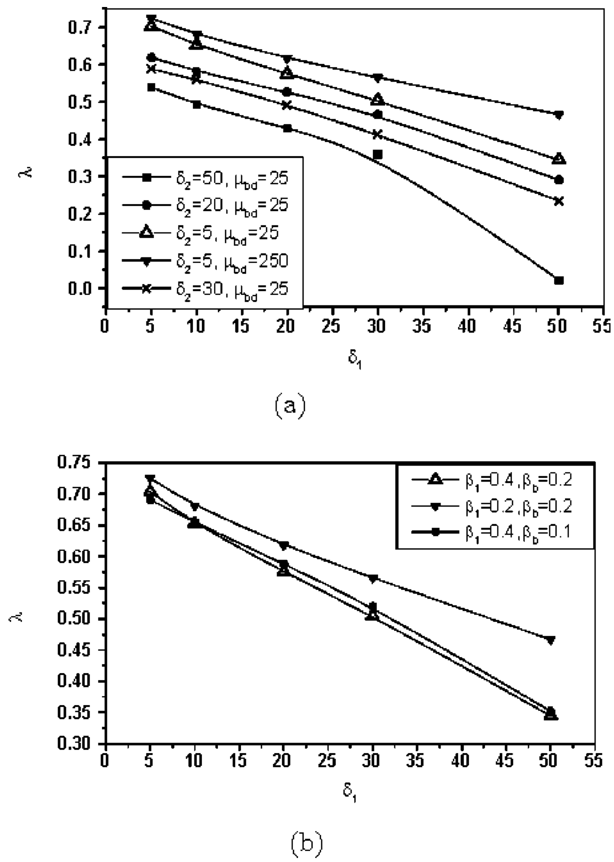

FIG. 1. Shows the variation of phase velocity λ; (a) vs δ_2 and δ_1, where $\beta_1=0.4$, $\beta_b=0.2$, $v_o=0.1$, (b) vs β_1 and β_b, where $\delta_2=5$, $\mu_{bd}=25$, $v_0=0.1$.

The conditions (21) ensure that the particle will remain at rest at $\phi_1=\phi_m$ and no reflection will occur. These conditions imply

$$\phi_{1m}=\frac{-D}{3F}\ \text{ and }\ M=\frac{-D^2}{18F}. \tag{22}$$

Substituting for M and D into the relation (20), we obtain

$$V(\phi_1)=\frac{\phi_1^2F}{4B}(\phi_1-\phi_m)^2. \tag{23}$$

From Eqs. (19) and (23), we get

$$\left(\frac{d\phi_1}{d\eta}\right)^2=\frac{-\phi_1^2F}{2B}(\phi_1-\phi_m)^2.$$

Then, the double layer solution is

$$\phi_1=\frac{\phi_m}{2}(1-\mathrm{Tanh}[\eta w]), \tag{24}$$

where

$$w^{-1}=\frac{2\sqrt{-2BF}}{D}.$$

Obviously this double layer solution exists only when the system parameters make the condition $F<0$ is fulfilled. Here we get the two types of DA double layers depending on the sign of D.

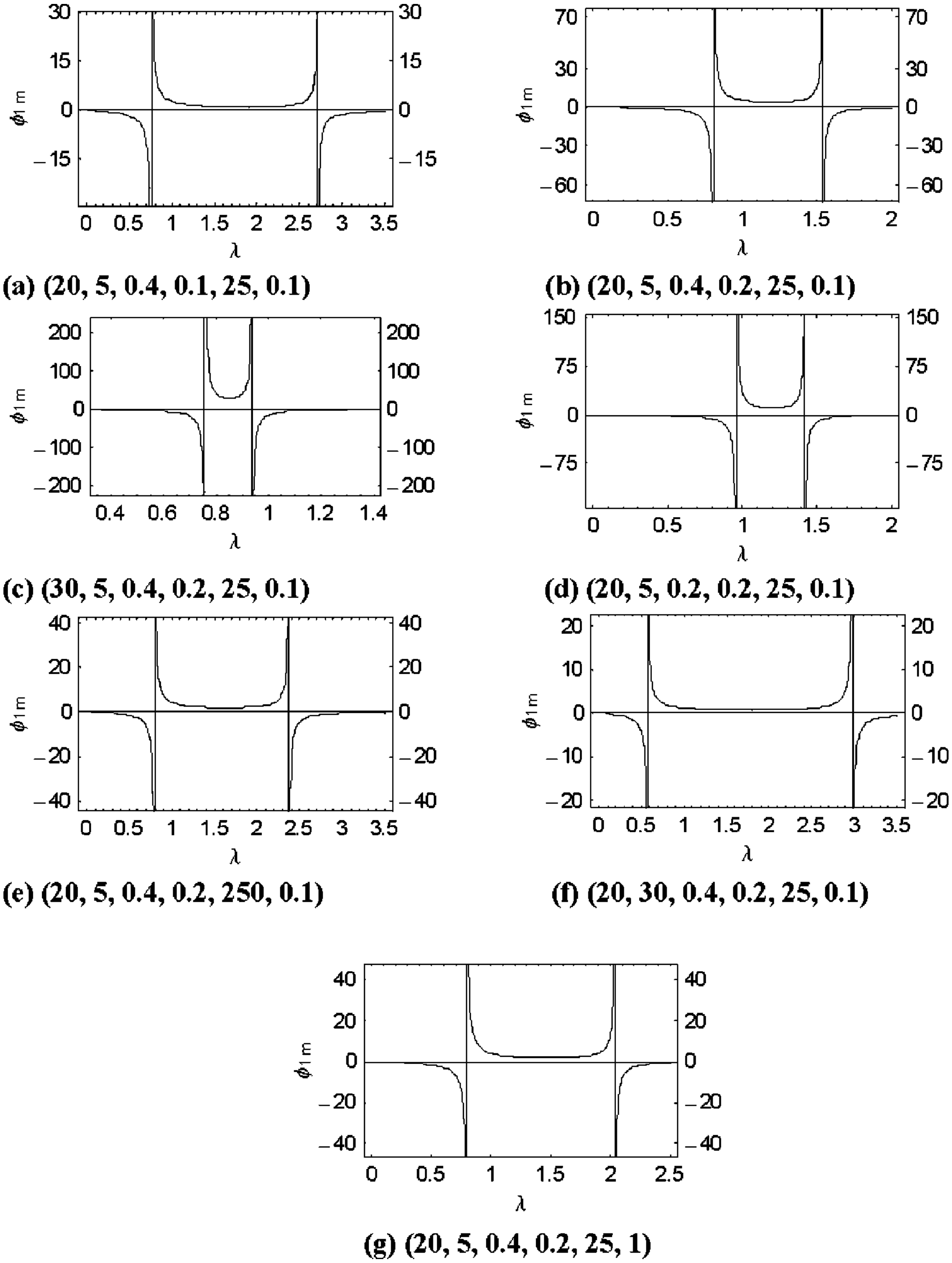

FIG. 2. Shows the existence of the rarefactive and compressive solitons of the KdV equation and its variation corresponding to the system parameter variation attached with each graph, the used brackets are in the form $(\delta_1, \delta_2, \beta_1, \beta_b, \mu_{bd}, v_0)$.

Now, if the nonlinear coefficient of the MKdV equation vanishes, $F=0$, therefore, MKdV equation breaks down also and one has to look for another equation suitable for describing the evolution of the system. This implies that both the stretching coordinates used before become invalid and instead of them we use the new stretching coordinates[25] $\xi = \varepsilon^{3/2}(x-\lambda t), \tau=\varepsilon^{9/2}t$, and expand those dependent variables as before. Substituting them into Eqs. (2)–(6), and using $\Psi=\Psi_o Z_d$ in Eq. (7), then collecting the terms of different powers of ε, we obtain the linear relation, Eq. (9) for the lowest order, and for the next order of ε we get the same relations as Eq. (15). Also, we can obtain the third-order perturbed quantities as

$$n_{b3}=\mu_{bd}\nu R_1\{\phi_3+G_1\phi_1\phi_2+G_2\phi_1^3\}, \tag{25}$$

$$v_{b3}=\mu_{bd}\tilde{\lambda}R_1\{\phi_3+H_1\phi_1\phi_2+H_2\phi_1^3\}, \tag{26}$$

$$n_{d3}=R_2\{-\phi_3+J_1\phi_1\phi_2+J_2\phi_1^3\}, \tag{27}$$

$$u_{d3}=\lambda R_2\{-\phi_3+K_1\phi_1\phi_2+K_2\phi_1^3\}, \tag{28}$$

$$Z_{d3}=\gamma_1\phi_3+2\gamma_2\phi_1\phi_2+\gamma_3\phi_1^3, \tag{29}$$

where

$$G_2=(30\tilde{\lambda}^2\Lambda+5\tilde{\lambda}^4+9\Lambda^2)\left(\frac{\mu_{bd}^2}{2}\right)R_1^4,$$

$$H_2=\frac{1}{2}(30\tilde{\lambda}^2\Lambda+\tilde{\lambda}^4+45\Lambda^2)\mu_{bd}^2R_1^4,$$

$$J_2=\frac{3}{2}\gamma_1[\lambda^2+\sigma_d]R_2^2-\frac{1}{3}\gamma_2$$

$$-\frac{1}{2}(30\lambda^2\sigma_d+5\lambda^4+9\sigma_d^2)R_2^4,$$

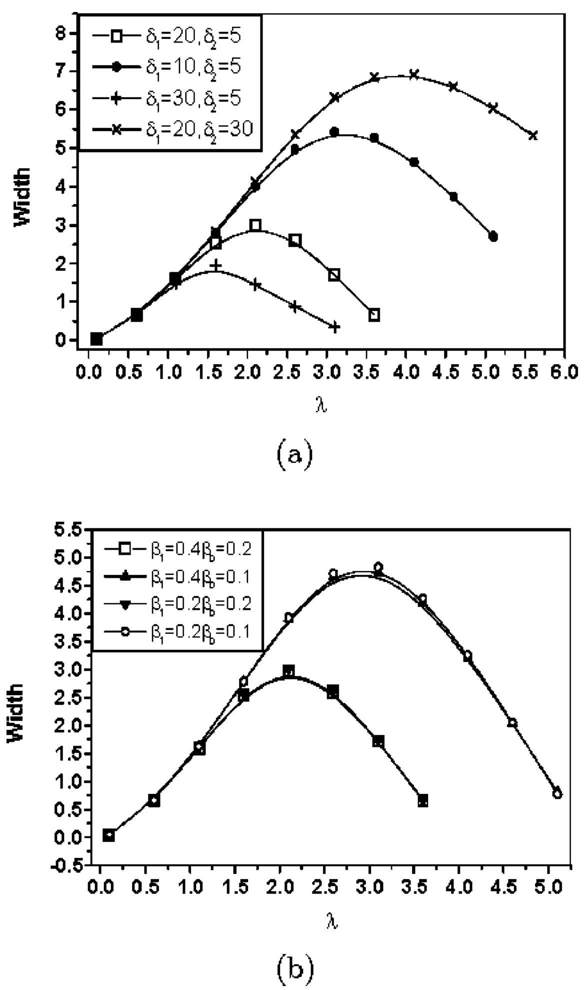

FIG. 3. Shows the variation of the width of both the rarefactive and compressive solitons (a) vs δ_2, δ_1 and λ, where $\beta_1=0.4$, $\beta_b=0.2$, $\mu_{bd}=25$, $v_0=0.1$ and (b) vs β_1, β_b, and λ, where $\delta_1=20$, $\delta_2=5$, $\mu_{bd}=25$, $v_0=0.1$.

$$K_2=\frac{1}{2}\gamma_1[\lambda^2+9\sigma_d]R_2^2-\frac{1}{3}\gamma_2 -\frac{1}{2}(30\lambda^2\sigma_d+\lambda^4+45\sigma_d^2)R_2^4,$$

and γ_3 is derived in the Appendix. If we continue to the next order of ε, we get a system of equations in the fourth-order perturbed quantities. Solving that system yields to a new type of KdV equation, that can be called KdV type

$$\frac{\partial\phi_1}{\partial\tau}+H\frac{\partial\phi_1^4}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=0, \tag{30}$$

where

$$\begin{aligned}H/B=&\frac{s^4}{24}\left(\frac{\beta_1^4-\delta_1}{\delta_1+\delta_2-1}\right)+\gamma_4+R_2\Bigg[\frac{-5}{4}\gamma_3+J_2\gamma_1\\&+\frac{\gamma_2}{2}\{5(\lambda^2+\sigma_d)R_2^2-\gamma_1\}+\frac{3}{8}\gamma_1^2(\lambda^2+\sigma_d)R_2^2\\&-\frac{3\gamma_1}{4}(30\lambda^2\sigma_d+5\lambda^4+9\sigma_d^2)R_2^4\Bigg]\\&+\frac{5}{8}R_2^6(7\lambda^6+105\lambda^4\sigma_d+189\lambda^2\sigma_d^2+27\sigma_d^3)]\\&-\frac{5}{8}R_1^7\mu_{bd}^3[(7\tilde{\lambda}^6+105\tilde{\lambda}^4\Lambda+189\tilde{\lambda}^2\Lambda^2+27\Lambda^3)],\end{aligned}$$

where γ_4 is derived in the Appendix. Substituting $\eta=\xi-M\tau$ in Eq. (30) and integrating twice, using the boundary conditions; we get

$$\left(\frac{d\phi_1}{d\eta}\right)^2=\frac{M}{B}\phi_1^2\left(1-\frac{H}{5M}\phi_1^3\right).$$

The one-soliton solution of (30) is given by

$$\phi_1=\phi_{3m}\,\mathrm{Sech}^{2/3}[\eta w_3], \tag{31}$$

where the amplitude and the width are given by $\phi_{3m}=(5M/2H)^{1/3}$ and $w_3^{-1}=\frac{2}{3}\sqrt{B/M}=w_1^{-1}/3$, respectively.

VI. DISCUSSION AND CONCLUSION

In this system, the ordering $m_d\gg m_i\gg m_e$ holds, as obtained in laboratory plasmas. Typical laboratory plasma frequencies are 10^2 Hz: 10^{5-6} Hz: 10^{9-10} Hz, and have roughly the same ordering as the mass ratios. Thus, the inclusion of the mass ratios is equal to considering the collective motion of dust grain particles[17,19,20] when we define the velocities of the electrons, positive ions, and dust grains in equilibrium as v_{eo}, v_{io}, and v_{deo}, and assume that $v_{eth}\gg v_{ph}\gg v_{io}, v_o\gg v_{dth}$ where v_{eth} (v_{dth}) and v_{ph} are the electron (dust) thermal velocity and phase velocity of the dust acoustic waves, respectively. The dust thermal velocity, in general, is much less than the wave phase velocity because of the massive dust grains. The ion-beam velocity v_o is assumed to be less than the phase velocity $v_{ph}\simeq(Z_{do}T_e/m_d)^{1/2}$. Since the dust acoustic instability is brought about by the condition[27–30] $v_o>v_{ph}$, the dust acoustic instability does not occur in our system. It is also assumed that the ion-beam velocity is less than the beam thermal velocity. It is noted that, in the case where $m_d=1.6\times10^{-21}$ kg, $Z_{do}=10^3$, $Te=1$ eV, $Ti=0.4$ eV, the dust thermal velocity is $v_{dth}=1.45$ m/s, the DA velocity, $C_d=(Z_{do}T_{\mathrm{eff}}/m_d)^{1/2}\simeq280$ m/s and the wave phase velocity $v_{ph}\simeq(Z_{do}T_e/m_d)^{1/2}=300$ m/s. If we assume K+ ions as the ion-beam component, the beam thermal velocity is $v_{bth}\simeq2.2\times10^3$ m/s. When the beam velocity is 28 m/s, we obtain $v_o=0.1$ because the velocity is normalized by C_d. In the following, we use the parameters $\mu_i=1836$, $m_b=40\,m_i$ for K+ ions, also, we assume that $\sigma_d\simeq0$. Figure 1 shows the variation of the phase velocity, λ, corresponding to different system parameter variations. It shows that:

(i) λ decreases as the ratio of ion density to electron density, δ_1 or the ratio of beam density to electron density, δ_2 increases;

(ii) λ deceases as the ratio of ion temperature to electron temperature, β_1 increases;

(iii) for small δ_1, λ increases as the ratio of beam temperature to electron temperature, β_b increases. As δ_1 and β_b increase, λ decreases;

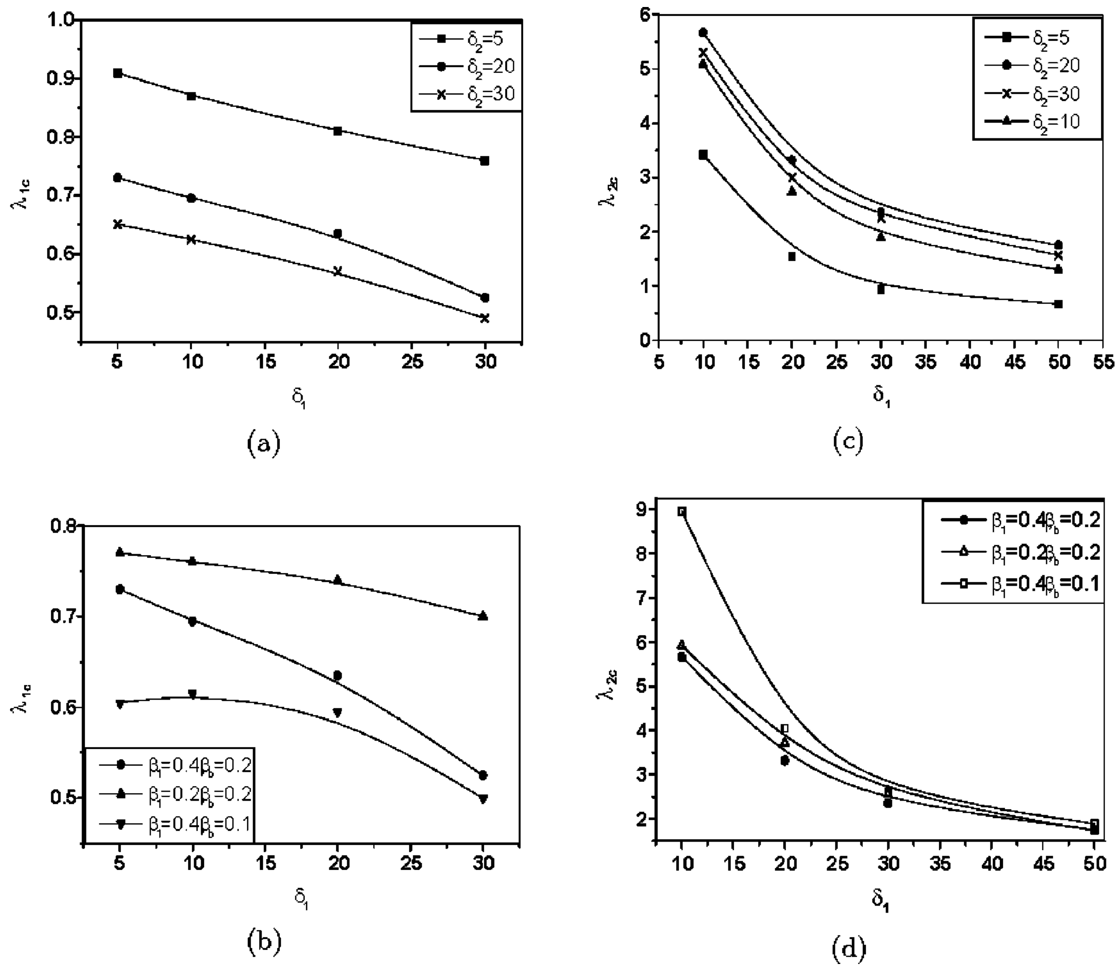

FIG. 4. Shows the variation of the two critical phase velocities (λ_{1c},λ_{2c}); in (a,c) vs δ_1, δ_2, where $\beta_1=0.4$, $\beta_b=0.2$, and in (b,d) vs β_1,β_b, where $\delta_1=20$, $\delta_2=5$, $\mu_{bd}=25$ and $v_0=0.1$.

(iv) λ increases as the ratio of dust mass to beam mass μ_{bd}, increases.

Now, we study the variation of the KdV solitons, with its different two types as the system parameter vary. Figure 2 shows the ranges of existence of the rarefactive soliton and the compressive soliton of the KdV equation, Eq. (11). We can observe that there are two critical phase velocities at which KdV fails to describe the system that force us to seek another equation to describe the system at these critical cases, that has been derived and so-called MKdV, Eq. (16). Figure 3 shows the variation of the width of the two types of KdV solitons due to various system parameter variations. It shows that there are two distinct behaviors:

(1) below the second critical phase velocity, λ_{2c}, the width of the solitons

— (increase/decrease) as (λ or δ_2/δ_1 or β_b) increases

— does not change with variation in β_1;

(2) above λ_{2c}, the width of the solitons;

— (increase/decrease) as ($\delta_2/\delta_1,\beta_2$ or λ) increases

— decrease slightly as β_1 increases.

In Fig. 4 we study the variation of the two critical phase velocity, (λ_{1c},λ_{2c}), corresponding to various system parameter variations. It shows that

(i) (λ_{1c} and/or λ_{2c}) decrease as δ_1 or β_1 increases but in different manner;

(ii) as β_b increases ($\lambda_{1c}/\lambda_{2c}$) (increases/decreases), respectively;

(iii) λ_{2c} increases corresponding to variation in δ_2 for small δ_2. For higher δ_2, it decreases as δ_2 increases. On the other hand, λ_{1c} decreases as δ_2 increases. Figure 5 show the variation of the soliton solutions of the MKdV equation and the KdV type equations, around the critical phase velocities, corresponding to system parameter variations, remember that both the two types coexisted at the critical phase velocities. Also, it illustrates the existence and the ability of each equation to describe the system at critical cases. Since at some critical phase velocities, MKdV fails to describe the system that motive to seek a higher evolution equation that can describe the system, Eq. (30). we can observe that;

(iv) ϕ_{2m} increases as δ_1,δ_2, or β_1 increases, but it decreases as β_b increases;

(v) ϕ_{3m} decreases as δ_1 or β_1 increases, but it increases as β_b increases;

(vi) For smaller δ_1 and δ_2 values MKdV equation is sufficient to describe the system at λ_{1c} and KdV type equation is able to describe the system at λ_{2c}. For higher δ_1 and

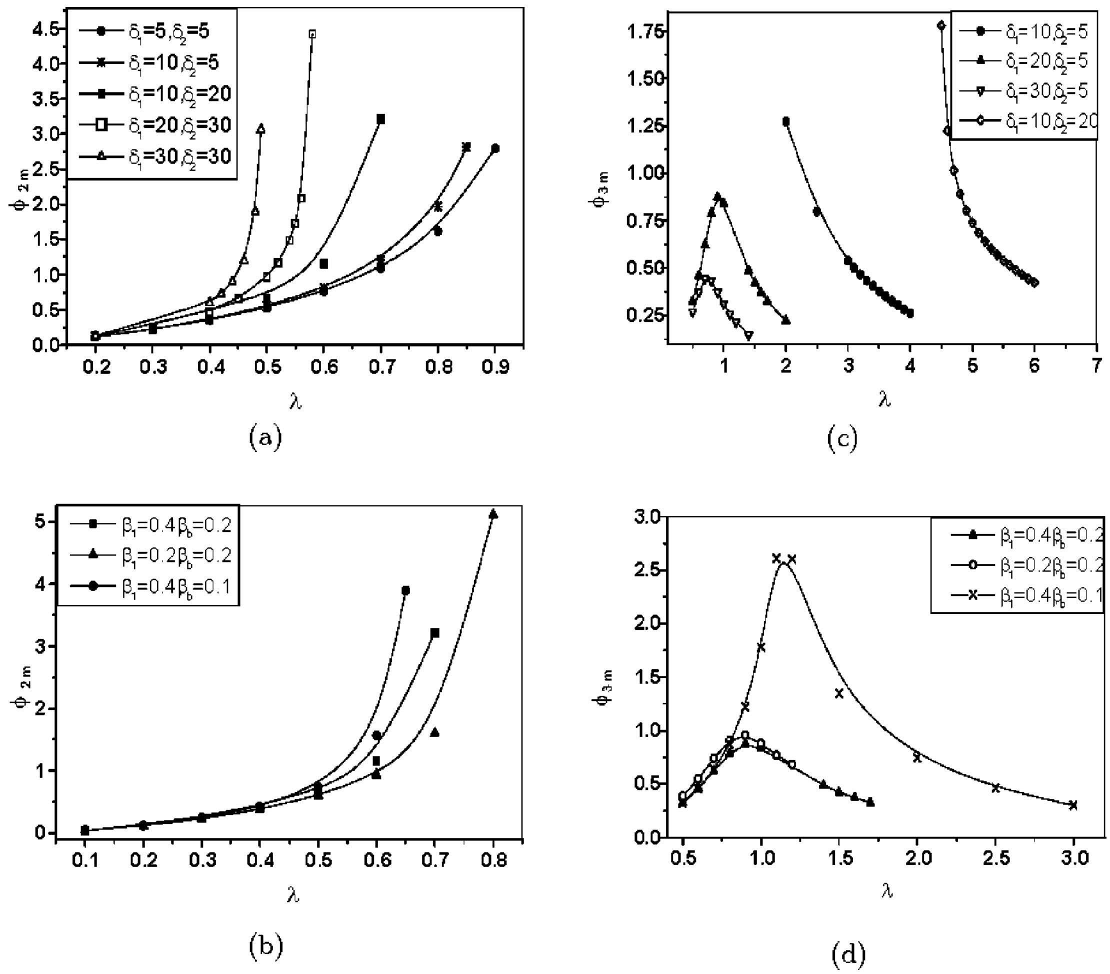

FIG. 5. (a) Shows the variation of the soliton solution of the MKdV equations vs δ_1, δ_2, where $\beta_1 = 0.4$, $\beta_b = 0.2$, (b) its variations vs β_1, β_b, where $\delta_1 = 10$, $\delta_2 = 20$. (c) Shows the variation of the soliton solution of the KdV type equation vs δ_1, δ_2, where $\beta_1 = 0.4$, $\beta_b = 0.2$, and (d) its variations vs β_1, β_b, where $\delta_1 = 20$, $\delta_2 = 5$, $\mu_{bd} = 25$, $v_0 = 0.1$.

δ_2 values MKdV equation is sufficient to describe the system at λ_{1c} and λ_{2c}, the contrary for smaller or moderate values of δ_2 that KdV type equation is sufficient to describe the system at λ_{1c} and λ_{2c}. For smaller δ_2 and higher δ_2 values KdV type equation is sufficient to describe the system at λ_{2c}.

In this paper we have investigated the effect of nonlinear dust charging on DA waves and double layers in a dusty plasma consisting of warm dust fluid, isothermal electrons and an arbitrary streaming ion beam. We have derived KdV Eq. (11) and obtained DA solitary waves. We get both the compressive and rarefactive DA solitons. At some critical phase velocities KdV equation fails to describe the system that forced us to apply a new stretching and then we get a MKdV equation with cubic nonlinearity, Eq. (16) that admits coexistence of both compressive and rarefactive solitons. On the other hand, there exist some critical points that make the nonlinear coefficients of the KdV and MKdV equations become zero, then they also fail to describe the system and we replace the previous two stretching variables with a newer one. Applying this stretching leads us to get the KdV type Eq. (30) that covers and solve this problem and allow only compressive soliton. Also, we get the condition under which double layers with two different types can existed. Thus, DA solitons exist in the region where DA double layers do not exist.

Also, the existence and dependence of each solitary solution corresponding to various system parameter variations are investigated. From this investigation, we can observe the violence effect of the arbitrary streaming velocity of the beam and the dust charging fluctuation on the discussed system. Thus, we can conclude that the quite dense positive ion, the dust temperature, the ion beam density, mass ratio and temperature govern the collective motion of dust grains.

On the other hand, we point out that the model considered here is structurally unstable, in the sense that a small change in the parameters or inclusion of small additional effects will not produce just a small change in the solution, but completely change its nature, for example, from double layer to solitary wave. Considering such a viewpoint, the results presented here are highly charged, heavy, micrometeresized dust grains. As nonlinear DA waves with an ion beam were present in dusty plasma which have been observed in laboratory plasmas, they may serve as a source of improvement in the etching rate of plasma processing. This investigation would be effective for understanding the properties of grain charging and small amplitude DA waves with an arbitrary streaming positive ion beam.

APPENDIX

In this appendix, we derive expressions for the perturbative quantities of normalized dust charge number associated with the corresponding perturbative quantities of normalized local plasma potential. From Eqs. (7) and (8) we have, for the first-order of ε

$$Z_{d1}=\frac{\gamma_b\phi_1}{\Psi_o\gamma_a}=\gamma_1\phi_1,$$

$$\gamma_a=\beta_1\exp(s\beta_1\Psi_o)+\alpha_1\delta_1+\alpha_2\delta_2\beta_bF_2),$$

and

$$\gamma_b=-[\alpha_1\delta_1(1-s\Psi_o)-(\alpha_2\delta_2\mu_{bd}R_1 \times(F_1-s\beta_bF_2\Psi_o)/s)+\beta_1\exp(s\beta_1\Psi_0)].$$

For the second-order of ε, we have

$$Z_{d2}=\frac{1}{\gamma_a s\Psi_o}\left\{\frac{-1}{2}s^2\beta_1^2\exp(s\beta_1\Psi_o)[\phi_1+\Psi_oZ_{d1}]^2 -\alpha_2\Psi_o(\delta_1+\delta_2-1) \cdot s\beta_bZ_{d1}n_{b1}F_2+\alpha_2(F_1-s\beta_bF_2\Psi_o) \times(\delta_1+\delta_2-1)n_{b2}+s\phi_2[\gamma_b-C\gamma_a\Psi_0/\tilde{\lambda}R_1] +\frac{1}{2}s^2\alpha_1\delta_1\phi_1[[1-s\Psi_o]\phi_1+2Z_{d1}\Psi_o]\right\},$$

from which we can derive the following form:

$$Z_{d2}=\gamma_1\phi_2+\gamma_2\phi_1^2,\ \gamma_2=\frac{\gamma_c}{\Psi_o\gamma_a},$$

with

$$\gamma_c=\gamma_{c1}+\gamma_{c2}+\gamma_{c3},$$

where

$$\gamma_{c1}=\frac{s}{2}[-\beta_1^2\exp(s\beta_1\Psi_o)+\alpha_1\delta_1(1-s\Psi_o) +C\gamma_a\Psi_oG_1/2R_1\tilde{\lambda}],$$

$$\gamma_{c2}=s\gamma_1\Psi_o[-\beta_1^2\exp(s\beta_1\Psi_o)+\alpha_1\delta_1 -(\alpha_2\delta_2\beta_b\mu_{bd}R_1F_2/s)],$$

$$\gamma_{c3}=-s(\beta_1\gamma_1\Psi_o)^2\exp(s\beta_1\Psi_o)/2.$$

Furthermore, for the third-order of ε, we have

$$Z_{d3}=\frac{1}{\gamma_a s\Psi_o}\Bigg\{\alpha_1\delta_1s^2\phi_1\phi_2[1-s\Psi_o]+s\phi_3[\gamma_b-C\gamma_a\Psi_0/R_1\tilde{\lambda}]+\{(\delta_1+\delta_2-1)\}[-\alpha_2s\beta_b\Psi_oF_2(Z_{d1}n_{b2}+Z_{d2}n_{b1}) +\alpha_2(F_1-s\beta_bF_2\Psi_o)n_{b3}]-\frac{1}{2}s^3Z_{d1}\phi_1^2\Psi_o\alpha_1\delta_1+\alpha_1\delta_1s^2\Psi_o(Z_{d2}\phi_1+Z_{d1}\phi_2) -\frac{1}{6}s^3\phi_1^3(\alpha_1\delta_1[1-s\Psi_o])-s^2\beta_1^2\exp(s\beta_1\Psi_o)\left([\phi_2+\Psi_oZ_{d2}][\phi_1+\Psi_oZ_{d1}]+\frac{1}{6}s\beta_1[\phi_1+\Psi_oZ_{d1}]^3\right)\Bigg\}.$$

After straightforward calculations, with the aid of Eqs. (9) and (15), the last equation can be rewritten as

$$Z_{d3}=\gamma_1\phi_3+2\gamma_2\phi_2\phi_1+\gamma_3\phi_1^3,$$

with

$$\gamma_3=\frac{\gamma_d}{\Psi_o\gamma_a},\quad \gamma_d=\gamma_{d1}+\gamma_{d2}+\gamma_{d3}+\gamma_{d4}+\gamma_{d5}+\gamma_{d6},$$

$$\gamma_{d1}=\frac{-s^2}{6}[\beta_1^3\exp(s\beta_1\Psi_o)+\alpha_1\delta_1(1-s\Psi_o)] +\gamma_a\Psi_0CG_2/R_1\tilde{\lambda},$$

$$\gamma_{d2}=-\frac{\gamma_1\Psi_o}{2}[s^2\alpha_1\delta_1+\alpha_2\delta_2\mu_{bd}R_1\beta_bF_2G_1 +s^2\beta_1^3\exp(s\beta_1\Psi_o)],$$

$$\gamma_{d3}=\frac{\gamma_2\gamma_{c2}}{\gamma_1},\quad \gamma_{d4}=-s\gamma_1\gamma_2(\beta_1\Psi_o)^2\exp(s\beta_1\Psi_o),$$

$$\gamma_{d5}=s\beta_1\gamma_{d4}\gamma_1/2\gamma_2,\quad \gamma_{d6}=\Psi_os\beta_1\gamma_{d4}\gamma_1^2/6\gamma_2.$$

Similarly, we can derive the form of Z_{d4} that has the form of

$$Z_{d4}=\gamma_1\phi_4+2\gamma_2(\phi_3\phi_1+\phi_2^2)+3\gamma_3\phi_1^2\phi_2+\gamma_4\phi_1^4,$$

$$\gamma_4=\frac{\gamma_e}{\gamma_a\Psi_o},\quad \gamma_e=\sum_{i=1}^{11}\gamma_{ei},$$

$$\gamma_{e1}=\frac{S^3}{24}\{\alpha_1\delta_1[1-s\Psi_o]-\beta_1^4\exp(s\beta_1\Psi_o)\} +\frac{5}{8}(C\gamma_a\Psi_oR_1^5\mu_{bd}^3/\tilde{\lambda})[(7\tilde{\lambda}^6+105\tilde{\lambda}^4\Lambda +189\tilde{\lambda}^2\Lambda^2+27\Lambda^3)],$$

$$\gamma_{e2}=\frac{S^3\Psi_o\gamma_1}{6}\{\alpha_1\delta_1-\beta_1^4\exp(s\beta_1\Psi_o)\}$$
$$-\alpha_2\delta_2\mu_{bd}R_1\Psi_o\beta_b\gamma_1F_2G_2,$$
$$\gamma_{e3}=\gamma_2\gamma_{d2}/\gamma_1,\quad \gamma_{e4}=\gamma_{c2}\gamma_3/\gamma_1,\quad \gamma_{e5}=\gamma_{d4}\gamma_2/2\gamma_1,$$
$$\gamma_{e6}=\gamma_3\gamma_{d4}/\gamma_2,\quad \gamma_{e7}=s\beta_1\gamma_{d4},$$
$$\gamma_{e8}=s^2\beta_1^2\gamma_{d4}\gamma_1/4\gamma_2,$$
$$\gamma_{e9}=s\beta_1\gamma_{d4}\gamma_1\Psi_o/2,\quad \gamma_{e10}=s^2\beta_1^2\gamma_{d4}\gamma_1^2\Psi_o/6\gamma_2,$$
$$\gamma_{e11}=s^2\beta_1^2\gamma_{d4}\gamma_1^3\Psi_o/24\gamma_2.$$

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