Dust acoustic solitary waves and double layers in a dusty plasma with two-temperature trapped ions

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The combined effects of trapped ion distribution, two-ion-temperature, dust charge fluctuation, and dust fluid temperature are incorporated in the study of nonlinear dust acoustic waves in an unmagnetized dusty plasma. It is found that, owing to the departure from the Boltzmann ion distribution to the trapped ion distribution, the dynamics of small but finite amplitude dust acoustic waves is governed by a modified Korteweg–de Vries equation. The latter admits a stationary dust acoustic solitary wave solution, which has stronger nonlinearity, smaller amplitude, wider width, and higher propagation velocity than that involving adiabatic ions. The effect of two-ion-temperature is found to provide the possibility for the coexistence of rarefactive and compressive dust acoustic solitary structures and double layers. Although the dust fluid temperature increases the amplitude of the small but finite amplitude solitary waves, the dust charge fluctuation does the opposite effect. The present investigation should help us to understand the salient features of the nonlinear dust acoustic waves that have been observed in a recent numerical simulation study. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1643757]

I. INTRODUCTION

There has been a rapidly growing interest in the physics of dusty plasmas not only because dust is an omnipresent ingredient of our universe, but also because of its vital role in understanding different collective processes (mode modification, new eigenmodes, coherent structures, etc.) in astrophysical and space environments.[1–6] The consideration of charged dust grains in a plasma does not only modify the existing plasma wave spectra,[7,8] but also introduces a number of new novel eigenmodes, such as dust acoustic (DA) waves,[9,10] dust ion acoustic (DIA) waves,[11,12] dust lattice waves,[13,14] etc.

Rao *et al.*[9] first reported theoretically the existence of extremely low phase velocity (in comparison with the electron and ion thermal velocities) DA waves where the dust particle mass provides the inertia and the thermal pressures from the electrons and ions give rise to the restoring force. Rao *et al.*[9] have studied the DA solitary waves in an unmagnetized dusty plasma (with cold dust fluid) by using the reductive perturbation method. Motivated by the experimental observation[9] of such low phase velocity DA waves, Mamun *et al.*[15] have investigated nonlinear DA waves in a two-component unmagnetized dusty plasma consisting of a negatively charged cold dust fluid and Maxwellian ions.

On the other hand, Roychoudhury and Mukherjee[16] showed that the finite dust temperature restricts the region for the existence of nonlinear solitary waves. The dust temperature is important owing to the thermalization with the ions or orbital effects.[17] The effects of the dust fluid temperature and nonthermal distribution of ions drastically modify the properties of the large amplitude electrostatic solitary structures.[18] Also, El-Labany and El-Taibany[19] studied the effects of dust temperature, charge fluctuation and ion streaming on DA waves and double layers (DLs). The effects of finite dust temperature, dust charge fluctuation on DA waves and DLs in a warm dusty plasma system containing trapped and free electrons are investigated.[20]

Tagare[21] extended the model of Mamun *et al.*[15] to a dusty plasma consisting of cold dust particles and two-temperature isothermal ions and studied the existence and properties of DA solitary waves. Using the reductive perturbation theory and pseudopotential method, Xie *et al.*[22] studied the DA solitary waves and DLs in dusty plasma with variable dust charge and two-temperature ions, and they have shown that both compressive and rarefactive solitons as well as DLs exist. Also, the amplitudes of the dust solitary waves become smaller and the regime of Mach number is extended wider for the variable dust charge situation compared to the constant dust charge situation. On the other hand, the topics of nonlinear grain charge variation and electrostatic ion waves[23] have been reported by regarding dust grains as point charges, where the Debye length is much larger than the inter-grain distance.

On the other hand, Schekinov[24] studied analytically the nonlinear properties of DA waves in a dusty plasma consisting of cold dust grains of constant charge and nonisothermal ions. Mamun[25] studied nonlinear small amplitude DA waves considering nonisothermal ions. The effect of nonlinear dust

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grain charging on large amplitude electrostatic waves in a dusty plasma with trapped ions has been studied by Nejoh.[26] Kakati and Goswami[27] studied nonlinear shock-like DA waves considering nonisothermal ions and adiabatic dust charge variations using the reductive perturbation technique. El-Labany *et al.*[28] revisited the same problem and studied the critical density solitary waves and small amplitude DA waves in a hot dusty plasma with nonisothermal ions. Also, the effect of nonadiabatic dust charge variations on nonlinear DA waves with nonisothermal ions has been investigated by Ghosh *et al.*[29]

In most practical dusty plasma experiments, a gas flow is usually introduced, which can be charged quickly, while keeping relatively low temperature. It was found theoretically that two ion acoustic modes can propagate in two-ion plasmas.[22] Lakshami and Bharuthram[30] studied large amplitude rarefactive DA solitons in a plasma with Boltzmann distributed electrons, ion species at different temperatures and dust grains with constant charges. Roychoudhury and Chatterjee[31] studied arbitrary amplitude DLs in dusty plasma. They investigated the region of existence of DLs theoretically and numerically. The obliquely propagating solitary DA holes in a hot magnetized two-ion-temperature dusty plasma has been studied by Mamun.[32]

Recent numerical simulation studies[33] on linear and nonlinear DA waves exhibit a significant amount of ion trapping in the wave potential. Clearly, there is a departure from the Boltzmann ion distribution and one encounters vortex-like ion distributions in phase space. In this article, we investigate the properties of nonlinear DA waves by incorporating the effects of two-temperature nonisothermal ion distributions that have vortex-like distributions.[34–36] As the effects of second component of low-temperature species, dust temperature, and dust charge fluctuation, which have not been considered in these earlier investigations,[22,25,28] drastically modify the properties of electrostatic solitary structures,[37] in the present work we study the DA solitary structures in a warm unmagnetized dusty plasma which consists of a negatively charged extremely massive dust fluid, isothermal electrons and trapped ions of two different temperatures.

This article is organized as follows: The basic equations governing the dynamics of the nonlinear DA waves are presented in Sec. II. The modified KdV (MKdV) equation and its stationary solitary wave solution are derived in Sec. III. The MKdV equation is then generalized to include the effects of dust charge fluctuation, dust temperature and nonisothermal ion distribution. The formalisms of both rarefactive and compressive solitons as well as DLs are also obtained in the same section (Sec. III). Finally, a brief discussion is presented in Sec. IV.

II. BASIC EQUATIONS

We consider an unmagnetized dusty plasma consisting of massive, negatively charged dust grains, isothermal (Maxwellian) electrons and nonisothermal (trapped or bi-Maxwellian) ions. Thus, at equilibrium we have

$$n_{ilo}+n_{iho}=n_{eo}+Z_{do}n_{do}, \tag{1}$$

where n_{ilo}, n_{iho}, n_{eo}, and n_{do} are the unperturbed low-temperature ion, high-temperature ion, electron and dust number densities, respectively, and Z_{do} is the unperturbed number of electrons residing on the dust grain surface. The nonlinear dynamics of one-dimensional low phase speed (in comparison with ion thermal speed) DA waves in such a dusty plasma are described by[20,22]

$$\frac{\partial n_d}{\partial t}+\frac{\partial(n_d u_d)}{\partial x}=0, \tag{2}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}+3\sigma_d n_d\frac{\partial n_d}{\partial x}-Z_d\frac{\partial \phi}{\partial x}=0, \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2}=Z_d n_d+n_e-n_{il}-n_{ih}, \tag{4}$$

where n_d is the dust number density normalized by n_{do}; and n_e and n_{il} (n_{ih}) are the electron number density and the low (high) temperature ion number density, respectively, normalized by $n_{do}Z_{do}$. u_d is the dust fluid speed normalized by the DA speed $C_d=(Z_{do}T_{\mathrm{eff}}/m_d)^{1/2}$, ϕ is the electrostatic wave potential normalized by T_{eff}/e, Z_d is the number of electrons residing on the dust grain surface normalized by Z_{do}, $\sigma_d=(T_d/Z_{do}T_{\mathrm{eff}})$, $T_{\mathrm{eff}}=Z_{do}n_{do}[(n_{eo}/T_e)+(n_{ilo}/T_{il})+(n_{iho}/T_{ih})]^{-1}$, $\delta_1=n_{ilo}/n_{eo}$, $\delta_2=n_{iho}/n_{eo}$, and m_d is the dust particle mass. The space coordinate x and time t are normalized by the Debye length $\lambda_{Dd}=(T_{\mathrm{eff}}/4\pi Z_{do}n_{do}e^2)^{1/2}$ and the dust plasma period $\omega_{pd}^{-1}=(m_d/4\pi Z_{do}^2 n_{do}e^2)^{1/2}$, respectively. The electrons are assumed to have Boltzmann distribution. Thus, we can express n_e as

$$n_e=\frac{1}{\delta_1+\delta_2-1}\exp(\beta_1 s\phi), \tag{5}$$

where $\beta_1=T_{il}/T_e$, $s=T_{\mathrm{eff}}/T_{il}$, and T_e (T_{il}) is the thermal energy of electrons (low temperature ions). On the other hand, the ion number densities n_{ij} in the presence of trapped particles can be expressed as[34,35]

$$n_{ij}=\frac{n_{ijo}}{n_{do}Z_{do}}\left[\exp(\Gamma_j)[1-\mathrm{erf}(\sqrt{\Gamma_j})]+\frac{1}{\sqrt{\beta_j}}\left\{\begin{array}{ll}\exp\Gamma_j\beta_j\sqrt{\Gamma_j\beta_j} & \text{for } \beta_j\geqslant 0,\\ \dfrac{2}{\sqrt{\pi}}\exp(\Gamma_j\beta_j)\displaystyle\int_0^{\sqrt{-\Gamma_j\beta_j}}\exp(X^2)\,dX & \text{for } \beta_j<0.\end{array}\right\}\right],$$

where $\Gamma_j=-T_{\mathrm{eff}}\phi/T_{ij}$, β_j represent the ratio of the free ion temperature T_{ij} to the trapped ion temperature T_{tj}, and $j=l$ ($j=h$) for low (high) temperature ions. Now, for $\phi\ll 1$, we can approximate n_{ij} as

$$n_{ij} \simeq \frac{n_{ijo}}{n_{do}Z_{do}}\left[1+\Gamma_j-\frac{4}{3}b_{1j}\Gamma_j^{3/2}+\frac{1}{2}\Gamma_j^2-\frac{8}{15}b_{2j}\Gamma_j^{5/2}+\frac{1}{6}\Gamma_j^3+\dots\right],$$

where $b_{kj}=(1-\beta_j^k)/\sqrt{\pi}$. The cases $\beta_j=0$ and $\beta_j=1$ correspond to the plasma having the flat topped and Maxwellian distributions, respectively. Thus, for isothermal ions we put $b_{kj}=0$, whereas for the nonisothermal ions, we have $0<b_{kj}<1/\sqrt{\pi}$. Therefore, n_{ij} can be rewritten in the form

$$n_{ij}=\frac{n_{ijo}}{n_{do}Z_{do}}[\exp(\Gamma_j)-G_j(\Gamma_j)],$$

where[38,39] $G_j(\Gamma_j)=\Sigma_{k=1}^{\infty}[2^{(k+1)}b_{kj}(\Gamma_j)^{(2k+1)/2}/\Pi(2k+1)]$. Now, substituting Γ_j and j into our last equation we can express n_{il} and n_{ih} as

$$n_{il}=\frac{\delta_1}{\delta_1+\delta_2-1}[\exp(-s\phi)-G_l(-s\phi)], \tag{6}$$

$$n_{ih}=\frac{\delta_2}{\delta_1+\delta_2-1}[\exp(-s\beta\phi)-G_h(-s\beta\phi)], \tag{7}$$

$\beta_2=T_{ih}/T_e$ and $\beta=\beta_1/\beta_2$.

We note that Z_d in (2) and (3) is not constant but varies with space and time. Thus, (1)–(3) are completed by the normalized dust grain charging equation.[40] However, the characteristic time for dust motion is of order of tens of milliseconds for micrometer-sized grains,[10] while the dust charging time is typically of order of 10^{-6} s. Therefore, on the hydrodynamic time scale, the dust charge can quickly reach local equilibrium, at which the currents from the electrons, low- and high-temperature ions to the dust are balanced. Thus, using the current balance equation[4,40] and the orbit-motion-limited probe model[41] we have

$$\alpha_1\delta_1(1-s\Psi)[\exp(-s\phi)-G_l(-s\phi)]+\alpha_2\delta_2(1-s\beta\Psi)[\exp(-s\beta\phi)-G_h(-s\beta\phi)]=\exp(s\beta_1[\Psi+\phi]), \tag{8}$$

where $\Psi=-Z_dZ_{d0}e^2/r_dT_{\text{eff}}$, $\alpha_{1,2}=(\beta_{1,2}/\mu_i)^{1/2}$, and $\mu_i=m_i/m_e$. We note that at equilibrium $Z_d=1$ and $\Psi=\Psi_o=-Z_{d0}e^2/r_dT_{\text{eff}}$, which can be determined by

$$\alpha_1\delta_1(1-s\Psi_o)+\alpha_2\delta_2(1-s\beta\Psi_o)=\exp(s\beta_1\Psi_o). \tag{9}$$

To show the dependence of Z_d on the physical parameters of the system we have numerically analyzed Eq. (8). The numerical results are displayed in Figs. 1 and 2. We have found some significant features in contrast with that studied in earlier works.[20,22] Figure 1 admits dust charge values for negative potential disturbance that does not appear previously. For positive plasma potential, increasing disturbance strength, first Z_d decreases quickly with a large slope, then gradually slows down with a smaller slope. However, for negative plasma potential disturbance, as its strength increases, Z_d decreases from one (which is the unperturbed dust charge number) to zero. As δ_1 increases, Z_d decreases for the potential disturbance up to the cutoff at $\phi\cong-3.6$. As β_1 increases, Z_d increases, while Z_d decreases as δ_2 increases. For a dusty plasma system containing one ion only, Z_d increases rapidly in the neighborhood of the unperturbed electrostatic potential to a greater value than the unperturbed one, thus it behaves like the dusty plasma contains two-temperature ions. Figure 2 shows the effect of the low-temperature trapped ion on the dust charge. We observed that both figures display distinct effects especially for the negative potential values. It is obvious from Fig. 2 that there exist two minimum values for either flat topped distribution or for particles that have velocity less than the thermal velocity.

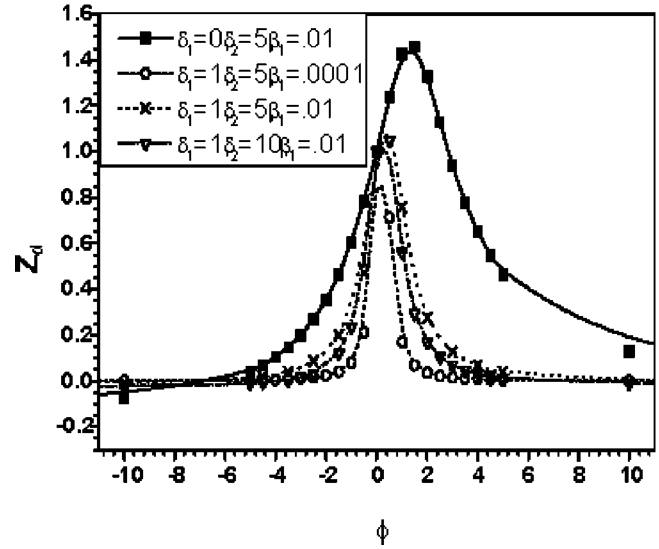

FIG. 1. Z_d is plotted against ϕ for different values of δ_1, δ_2 and β_1, where $\beta_2=0.1$, $\sigma_d=0$ and $\beta_l=\beta_h=1$.

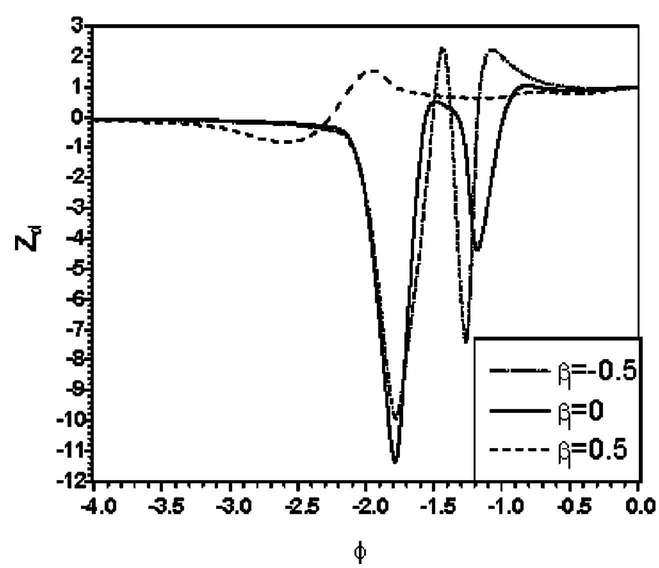

FIG. 2. Z_d is plotted against ϕ for different values of β_l, where $\delta_1=1$, $\delta_2=5$, $\beta_1=0.01$, $\beta_2=0.1$, $\sigma_d=0$, and $\beta_h=0.8$.

However, for particles that have velocity greater than the thermal one, Z_d has one minimum and one maximum in the negative potential region.

III. NONLINEAR DUST ACOUSTIC WAVES

To study the dynamics of small-amplitude DA solitary waves in the presence of adiabatic variation of dust charges and trapped ions of two different temperatures, we employ a reductive perturbation technique. We introduce the stretched coordinates[34,42] $\xi=\varepsilon^{1/4}(x-\lambda t)$ and $\tau=\varepsilon^{3/4}t$, where ε is a small parameter and λ is the solitary wave velocity normalized by C_d. The variables n_d, u_d, Z_d and ϕ are expanded as

$$n_d=1+\varepsilon n_{d1}+\varepsilon^{3/2}n_{d2}++\varepsilon^2 n_{d3}+\varepsilon^{5/2}n_{d4}+\cdots,$$

$$\begin{aligned} u_d&=\varepsilon u_{d1}+\varepsilon^{3/2}u_{d2}+\varepsilon^2 u_{d3}+\varepsilon^{5/2}u_{d4}+\cdots,\\ Z_d&=1+\varepsilon Z_{d1}+\varepsilon^{3/2}Z_{d2}+\varepsilon^2 Z_{d3}+\varepsilon^{5/2}Z_{d4}+\cdots,\\ \phi&=\varepsilon\phi_1+\varepsilon^{3/2}\phi_2+\varepsilon^2\phi_3+\varepsilon^{5/2}\phi_4+\cdots. \end{aligned} \tag{10}$$

Now, substituting these expansions into Eqs. (2)–(8) and collecting the terms of different powers of ε, in the lowest order, we obtain

$$\begin{aligned} n_{d1}&=-R\phi_1,\\ u_{d1}&=-\lambda R\phi_1,\\ Z_{d1}&=\gamma_1\phi_1, \end{aligned} \tag{11}$$

where $R=(\lambda^2-3\sigma_d)^{-1}$ and

$$\gamma_1=\frac{-\{\alpha_1\delta_1(1+\beta_1)(1-s\Psi_o)+\alpha_2\delta_2(\beta+\beta_1)(1-s\beta\Psi_o)\}}{\Psi_o[\alpha_1\delta_1\{1+\beta_1(1-s\Psi_o)\}+\alpha_2\delta_2(\beta+\beta_1\{1-s\beta\Psi_o\})]}.$$

The linear dispersion relation is given by

$$\gamma_1+\frac{s(\beta_1+\delta_1+\delta_2\beta)}{(\delta_1+\delta_2-1)}=R. \tag{12}$$

It agrees with that obtained by Xie *et al.*[22] with $\sigma_d=0$ and it does not depend on the nonisothermal parameter. It is seen that λ is proportional to the square root of σ_d. The dust temperature increases the velocity and the contrary occurs with the effect of dust charge variation where $\gamma_1>0$.

The next order in $\varepsilon, O(\varepsilon^{3/2})$, yields a system of equations that leads to the modified Korteweg–de Vries (MKdV) equation

$$\frac{\partial\phi_1}{\partial\tau}+A\sqrt{-\phi_1}\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=0, \tag{13}$$

where

$$B=1/2\lambda R^2,$$

$$A=B\left[\frac{2s^{3/2}(\delta_1 b_{1l}+\delta_2 b_{1h}\beta^{3/2})}{(\delta_1+\delta_2-1)}-\frac{3}{2}\gamma_2\right],$$

$$\gamma_2=\frac{4\sqrt{s}[\alpha_1\delta_1(1-s\Psi_o)b_{1l}+\alpha_2\delta_2\beta^{3/2}b_{1h}(1-s\beta\Psi_o)]}{3\Psi_o[\alpha_1\delta_1(1+\beta_1(1-s\Psi_o))+\alpha_2\delta_2(\beta+\beta_1(1-s\beta\Psi_o))]}.$$

Now, using the boundary conditions

$$\phi_1(\eta)\to 0,\quad \frac{d\phi_1(\eta)}{d\eta}\to 0,\quad \frac{d^2\phi_1(\eta)}{d\eta^2}\to 0 \quad\text{as}$$

$$|\eta|\to\infty \qquad \eta=\xi-M\tau, \tag{14}$$

the stationary solution of Eq. (13) is given by

$$\phi_1=\phi_{1m}\mathrm{sech}^4[\eta/w_1], \tag{15}$$

where the amplitude ϕ_{1m} and the width w_1 are given by $-(15M/8A)^2$ and $4\sqrt{B/M}$, respectively. The soliton solution has amplitude smaller than that obtained in the isothermal case while the width becomes wider. Equation (15) admits only a rarefactive soliton. Figures 3–5 show the dependence of ϕ_{1m} on the lower-temperature trapped ions parameters. One observed that (i) ϕ_{1m} increases as δ_2 increases, but it decreases as δ_1 increases; (ii) ϕ_{1m} increases rapidly for $\beta_1<10^{-3}$, reaches a maximum value, and then it begins to decrease as β_1 increases; and (iii) ϕ_{1m} decreases rapidly as β_l increases. It might have a compressive soliton form if the particles have an isothermal population. (iv) It is obvious, from the expression of ϕ_{1m}, that it increases as σ_d increases.

The variation of the soliton width due to variation of the second low-temperature ion parameters has the same feature of λ variation except for the dust temperature variation. Figure 6 shows that λ increases as σ_d increases, but the width decreases. So, the inclusion of the dust temperature decreases the soliton width.

The solution of the MKdV equation admits only rarefactive solitons. The DA waves are rarefactive if $A<0$. If one puts $A=0$ that corresponds to $\beta_l=\beta_h=1$ or δ_c, one can get an equation which can be solved numerically to obtain the relation between δ_1 and δ_2. The MKdV equation is, therefore, inadequate, and one has to find another equation in

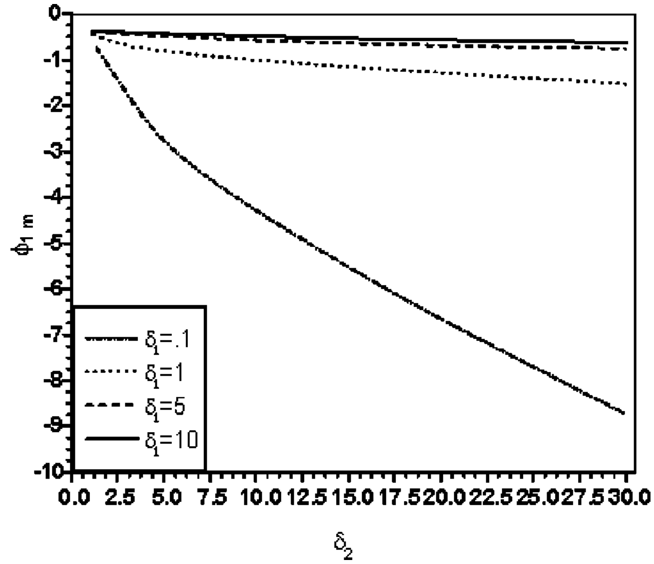

FIG. 3. ϕ_{1m} is plotted against δ_2 for different values of δ_1, where $\beta_1=0.0001$, $\beta_2=0.1$, $\sigma_d=0$, $\beta_l=0.2$, and $\beta_h=0.8$.

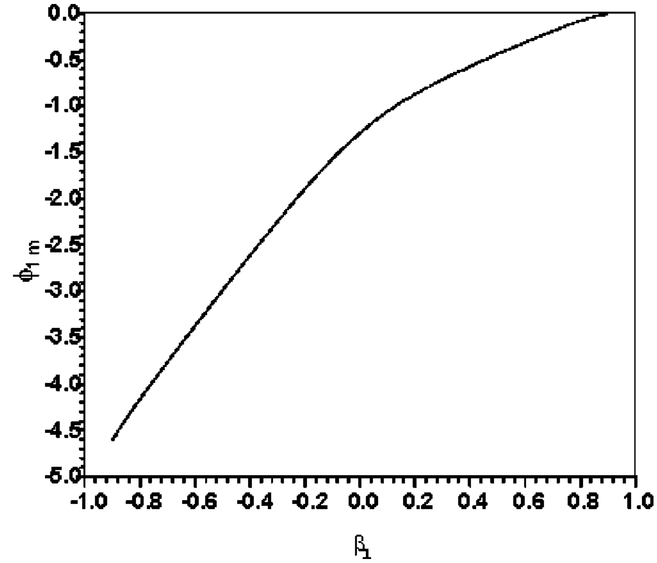

FIG. 5. ϕ_{1m} is plotted against β_l, where $\delta_1=1$, $\delta_2=5$, $\beta_1=0.0001$, $\beta_2=0.1$, $\sigma_d=0$, and $\beta_h=0.95$.

order to study the nonlinear properties of DA waves. We use the stretched coordinates[20,34–36,43] $\xi=\varepsilon^{1/2}(x-\lambda t)$, $\tau=\varepsilon^{3/2}t$, and follow the same procedure used before. Accordingly, for the lowest order of ε we obtain the relations (11), and for next order of ε we get

$$n_{d2}=-R\phi_2,\quad u_{d2}=-\lambda R\phi_2,$$

$$Z_{d2}=\gamma_1\phi_2+\gamma_2(-\phi_1)^{3/2}, \tag{16}$$

$$\left[\gamma_1+\frac{s(\beta_1+\delta_1+\delta_2\beta)}{(\delta_1+\delta_2-1)}-R\right]\phi_2 =\left[\frac{4s^{3/2}(\delta_1 b_{1l}+\delta_2 b_{1h}\beta^{3/2})}{3(\delta_1+\delta_2-1)}-\gamma_2\right](-\phi_1)^{3/2}. \tag{17}$$

For the next order in ε, $O(\varepsilon^2)$, we obtain a set of equations, which, after making use of Eqs. (11) and (16), yields

$$\frac{\partial\phi_1}{\partial\tau}+A\frac{\partial}{\partial\xi}\{(-\phi_1)^{1/2}\phi_2\}+B\frac{\partial^3\phi_1}{\partial\xi^3}+C\phi_1\frac{\partial\phi_1}{\partial\xi}=0, \tag{18}$$

where

$$C=B\left[\frac{s^2(\delta_1-\beta_1^2+\beta^2\delta_2)}{(\delta_1+\delta_2-1)}+3R(\gamma_1-[\lambda^2+\sigma_d]R^2)-2\gamma_3\right]$$

and

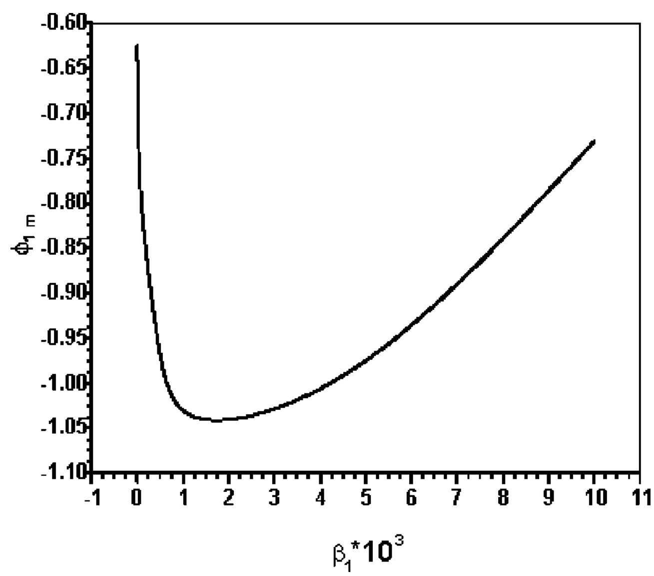

FIG. 4. ϕ_{1m} is plotted against β_1, where $\delta_1=1$, $\delta_2=5$, $\beta_2=0.1$, $\sigma_d=0$, $\beta_l=0.2$, and $\beta_h=0.8$.

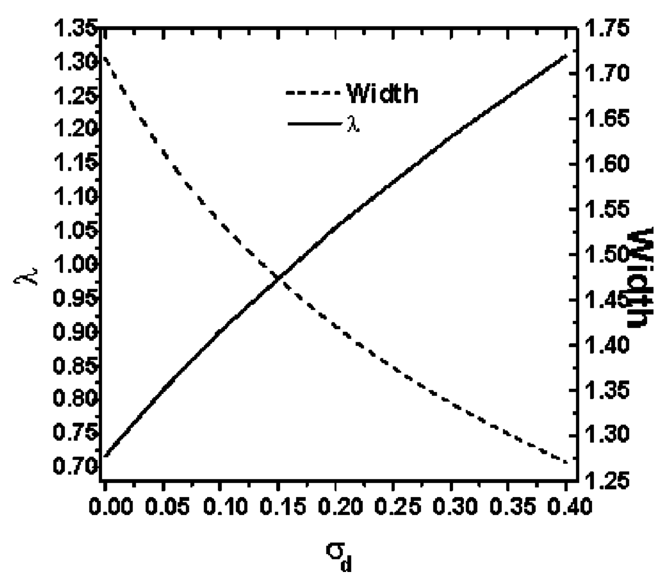

FIG. 6. λ and width are plotted against σ_d, where $\delta_1=1$, $\delta_2=10$, $\beta_1=0.1$, $\beta_2=0.5$, $\beta_l=0.2$, and $\beta_h=0.8$.

$$\gamma_3=\frac{\gamma_d}{\Psi_o\gamma_a},\quad \gamma_d=\gamma_{d1}+\gamma_{d2}+\gamma_{d3},$$

$$\gamma_{d1}=\frac{s}{2}\{\alpha_1\delta_1(1-\beta_1^2)(1-s\Psi_o)+\alpha_2\delta_2(\beta^2-\beta_1^2)\times(1-s\beta\Psi_o)\}.$$

$$\gamma_{d2}=\gamma_1 s\Psi_o[\alpha_1\delta_1(1-\beta_1^2(1-s\Psi_o))+\alpha_2\delta_2(\beta^2-\beta_1^2(1-s\beta\Psi_o))],$$

$$\gamma_{d3}=-\frac{s(\gamma_1\beta_1\Psi_o)^2}{2}[\alpha_1\delta_1(1-s\Psi_o)+\alpha_2\delta_2(1-s\beta\Psi_o)].$$

The one-soliton solution of Eq. (18), $A=0$, is given by

$$\phi_1=\phi_{2m}\,\mathrm{sech}^2[\eta/w_2], \tag{19}$$

where the amplitude ϕ_{2m} and the width w_2 are given by $3M/C$ and $2\sqrt{B/M}=0.5\,w_1$, respectively. Since $\gamma_1\geqslant 0$, $\gamma_3\geqslant 0$ and $M>0$, Eq. (19) clearly indicates that both rarefactive and compressive solitons exist. The width of the soliton becomes narrower than that of the MKdV equation. One can observe that the inclusion of the second trapped ion species admits the existence of the two kinds of solitons. On the other hand, when[20] $A\phi_2\rightarrow -2D\phi_1/3$, Eq. (18) would reduce to

$$\frac{\partial\phi_1}{\partial\tau}+D(-\phi_1)^{1/2}\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}+C\phi_1\frac{\partial\phi_1}{\partial\xi}=0. \tag{20}$$

Equation (20) has the same form as Eq. (49) of Ref. 34, the only differences between Eq. (20) here and Eq. (49) of Ref. 34 are in coefficients D, B, and C. If D is $O(1)$, Eq. (20) reduces to Eq. (49) of Ref. 34, and the $\frac{1}{4}$ scaling prevails accounting for a balance of the increased nonlinearity and dispersion. On the other hand, if D is $O(\varepsilon^{1/2})$, the nonlinearity is weakened and becomes comparable to the ordinary hydrodynamic nonlinearity represented by the C-term, and the $\frac{1}{2}$ scaling is appropriate.[34] The scaling, hence, reflects the strength of the nonlinearity accounting for different solitary wave solutions. The most general solitary wave solution in which both nonlinearities are taken into account is given by Eq. (48) of Ref. 34. Now, substituting $\eta=\xi-M\tau$ in Eq. (20) and integrating twice, using (14), we get

$$\frac{1}{2}\left(\frac{d\phi_1}{d\eta}\right)^2=\frac{M\phi_1^2}{2B}\left(1-\frac{8D(-\phi_1)^{1/2}}{15M}-\frac{C\phi_1}{3M}\right)=-V(\phi_1,M). \tag{21}$$

Hence

$$V(\phi_1,M)=\frac{-M\phi_1^2}{2B}+\frac{4D(-\phi_1)^{5/2}}{15B}+\frac{C\phi_1^3}{6B}. \tag{22}$$

For the formation of DLs, we must satisfy the following conditions:

$$V(\phi_m,M)=0,\quad \left(\frac{dV}{d\phi_1}\right)_{\phi_1=\phi_m}=0\quad\text{and}$$

$$\left(\frac{d^2V}{d\phi_1^2}\right)_{\phi_1=\phi_m}<0. \tag{23}$$

Using $y^2=-\phi_1$ in Eq. (22), we have

$$V(y,M)=\frac{-My^4}{2B}+\frac{4Dy^5}{15B}-\frac{Cy^6}{6B},$$

which can be rewritten as

$$V(y)=\frac{-Cy^4}{6B}(y-y_m)^2, \tag{24}$$

where the conditions (23) imply

$$y_m=\frac{4D}{5C}\quad\text{and}\quad M=\frac{16D^2}{75C}. \tag{25}$$

Substituting M and D into the relation (21), we can transform into

$$\left(\frac{dy}{d\eta}\right)^2=\frac{Cy^2}{12B}(y-y_m)^2.$$

Thus, the DL solution is

$$\phi_1=-\frac{y_m^2}{4}[1-\tanh(\eta/w)]^2, \tag{26}$$

where

$$w=\frac{5}{D}\sqrt{3BC}.$$

Here, rarefactive DA DLs are admitted only contrary to the case studied in recent studies[19,20] with the inclusion of ion beam or trapped electrons.

Now, if the nonlinear coefficient of the KdV equation vanishes, $C=0$, i.e., the KdV equation fails to describe the system successfully. This failure forces us to look for another equation which is suitable for describing the evolution of the system. Figure 7 shows the relation between δ_1 and δ_2 corresponding to two values of β_1 and for $C=0$. It shows that δ_2 increases as δ_1 or β_1 increases. Instead of the stretching used before we have to use higher stretching coordinates of the perturbation theory,[20] $\xi=\varepsilon^{3/4}(x-\lambda t)$ and $\tau=\varepsilon^{9/4}t$. We obtain the linear relation, Eq. (12), for the lowest order, and for the next orders of ε we get the same relations (11), (16) and (17). The third order perturbed quantities can be obtained as

$$n_{d3}=R\{-\phi_3+E_1\phi_1^2\},u_{d3}=\lambda R\{-\phi_3+E_2\phi_1^2\},$$
$$Z_{d3}=\gamma_1\phi_3+\tfrac{3}{2}\gamma_2\phi_2(-\phi_1)^{1/2}+\gamma_3\phi_1^2, \tag{27}$$

and

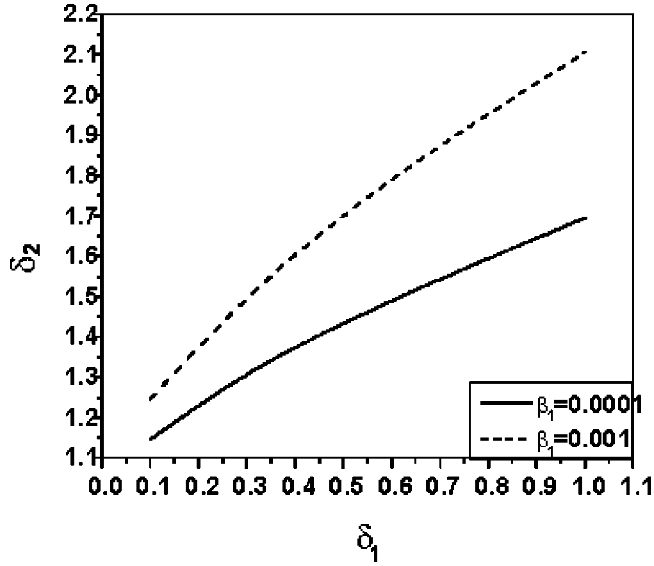

FIG. 7. The variation of δ_1 and δ_2 corresponding to $C=0$, where $\sigma_d=0$ and $\beta_l=\beta_h=1$.

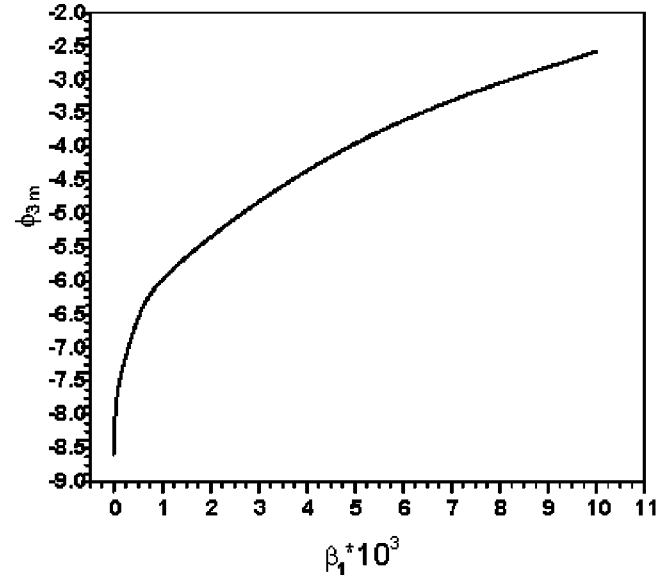

FIG. 8. ϕ_{3m} is plotted against β_1, where $\delta_1=1$, $\delta_2=5$, $\beta_2=0.1$, $\sigma_d=0$, $\beta_l=0.2$, and $\beta_h=0.8$.

$$\phi_3=\left[\gamma_1+\frac{s(\beta_1+\delta_1+\delta_2\beta)}{(\delta_1+\delta_2-1)}-R\right]^{-1}\times\left\{\left[\frac{s^2(\delta_1-\beta_1^2+\beta^2\delta_2)}{2(\delta_1+\delta_2-1)}+\frac{3}{2}R(\gamma_1-[\lambda^2+\sigma_d]R^2)-\gamma_3\right]\phi_1^2+\left[\frac{2s^{3/2}(\delta_1 b_{1l}+\delta_2 b_{1h}\beta^{3/2})}{(\delta_1+\delta_2-1)}-\frac{3}{2}\gamma_2\right](-\phi_1)^{1/2}\phi_2\right\}, \tag{28}$$

where

$$E_1=-\tfrac{1}{2}\gamma_1+\tfrac{3}{2}[\lambda^2+\sigma_d]R^2,\quad E_2=E_1-\lambda R^2.$$

If we continue to the next order of ε, we get a system of equations in the subscripted 4 perturbed quantities. Eliminating the perturbed quantities with subscript 4, we obtain a further MKdV (FMKdV) equation

$$\frac{\partial\phi_1}{\partial\tau}+F(-\phi_1)^{3/2}\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=0, \tag{29}$$

where

$$F/B=\frac{1}{6}\left[21\gamma_2 R-15\gamma_4+\frac{8s^{5/2}(\delta_1 b_{2l}+\delta_2 b_{2h}\beta^{5/2})}{(\delta_1+\delta_2-1)}\right],$$

$$\gamma_4=\frac{\gamma_e}{\Psi_o\gamma_a},\quad \gamma_e=\gamma_{e1}+\gamma_{e2}+\gamma_{e3}+\gamma_{e4},$$

$$\gamma_{e1}=-\tfrac{8}{15}\sqrt{s^3}[\alpha_1\delta_1(1-s\Psi_o)b_{2l}+\alpha_2\delta_2\beta^{5/2}b_{2h}(1-s\beta\Psi_o)],$$

$$\gamma_{e2}=-\tfrac{4}{3}\gamma_1\Psi_o\sqrt{s^3}[\alpha_1\delta_1 b_{1l}+\alpha_2\delta_2\beta^{5/2}b_{1h}],$$

$$\gamma_{e3}=\frac{\gamma_{d2}\gamma_2}{\gamma_1},\quad \gamma_{e4}=\frac{2\gamma_{d3}\gamma_2}{\gamma_1}.$$

$$Z_{d4}=\gamma_1\phi_4+\tfrac{3}{8}\gamma_2[\phi_2^2(-\phi_1)^{-1/2}+4(-\phi_1)^{1/2}\phi_3]+2\gamma_3\phi_1\phi_2+\gamma_4(-\phi_1)^{5/2}.$$

The one-soliton solution of Eq. (29) is given by

$$\phi_1=\phi_{3m}\mathrm{sech}^{4/3}[\eta/w_3], \tag{30}$$

where the amplitude ϕ_{3m} and the width w_3 are given by $(35M/8F)^{2/3}$ and $(\frac{4}{3})\sqrt{B/M}=w_1/3$, respectively.

Figures 8 and 9 show the variation of ϕ_{3m} for different parametric regimes. It is shown that (i) ϕ_{3m} decreases rapidly as β_1 increases for $\beta_1<10^{-3}$, then it decreases gradu-

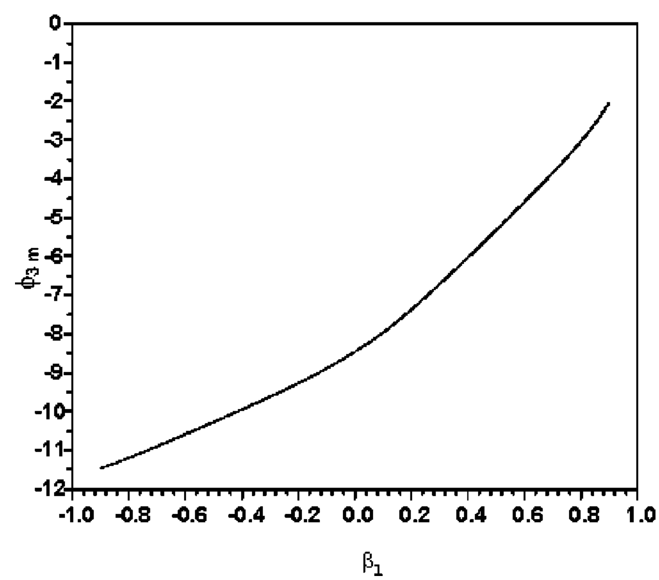

FIG. 9. ϕ_{3m} is plotted against β_l, where $\delta_1=1$, $\delta_2=5$, $\beta_1=0.0001$, $\beta_2=0.1$, $\sigma_d=0$, and $\beta_h=0.95$.

ally with small slope for $\beta_1 > 10^{-3}$. (ii) It decreases rapidly as β_l increases. Also, ϕ_{3m} has the same features as ϕ_{1m} corresponding to the same variation of δ_1, δ_2 and σ_d but with a larger rarefactive amplitude (not shown).

IV. DISCUSSION

We have studied the effects of the dust charge variation on the small-amplitude DA waves in dusty plasmas having two-temperature trapped ions. The modified KdV equation and FMKdV equation with high-order nonlinear terms, as well as the DL solution, are also obtained. We have found that the large dust-charge fluctuation induced by the low-temperature ions is the physical reason why two-temperature-ions dusty plasmas can admit the transition of soliton and/or solitary waves for dust density profile from the rarefactive type to the compressive type when the system parameter changes. It is also found that the presence of lower-temperature ions plays a crucial role in the coexistence of both compressive and rarefactive waves, as well as the DL. Our numerical results also confirm that the positive plasma potential soliton should exist in the dusty plasma with relative low-temperature or/and small number density regime of lower-temperature ion.

It is found that the soliton width becomes narrower compared with the case of constant dust charge. The modification introduced by the variable dust charge on the amplitude and width of the solitons are analyzed. Finally, it is pointed out that the approximate similarity law that El-Labany and El-Taibany predicted[20] is no longer valid for the present system.

The results that we obtained from this investigation may be summarized as follows: (i) As δ_1 increases, Z_d, ϕ_{1m}, rarefactive ϕ_{2m} and ϕ_{3m} decrease; but λ and compressive ϕ_{2m} increase. (ii) As δ_2 increases, λ, Z_d and compressive ϕ_{2m} decrease; however, ϕ_{1m}, rarefactive ϕ_{2m} and ϕ_{3m} increase. (iii) As β_1 increases, Z_d and compressive ϕ_{2m} increase but rarefactive ϕ_{2m} decreases. ϕ_{1m} first increases very rapidly for $\beta_1 < 10^{-3}$; then, for $\beta_1 > 10^{-3}$, it decreases with a smaller slope. On contrary, ϕ_{3m} decreases very rapidly for $\beta_1 < 10^{-3}$, then the decrement slope becomes smaller one. (iv) As β_l increases, ϕ_{1m} and ϕ_{3m} decrease. Z_d is strongly affected, especially for negative potential region and through the transition from subthermal particles to superthermal ones. (v) As σ_d increases, λ, ϕ_{1m}, compressive ϕ_{2m} and ϕ_{3m} increase but rarefactive ϕ_{2m} and the soliton width decrease.

This work agrees exactly with the results of Xie *et al.*[22] by neglecting the dust temperature and trapped ions and also with the results of Mamun[32] on neglecting the nonisothermal parameters. It may be added that the effects of obliqueness, external magnetic field and inhomogeneity in plasma density on these solitary structures, and their instabilities, are also important problems, but beyond the scope of our present work.

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