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On the higher-order solution of the dust-acoustic solitary waves in a warm magnetized dusty plasma with dust charge variation

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The higher-order contribution in reductive perturbation theory is studied for small-but-finite-amplitude dust-acoustic solitary waves in warm magnetized three-component dusty plasmas comprised of variational charged dust grains, isothermal ions, and electrons. The basic set of fluid equations is reduced to the Zakharov–Kuznetsov equation for the first-order perturbed potential and a linear inhomogeneous Zakharov–Kuznetsov-type equation for the second-order perturbed potential. Stationary solutions of both equations are obtained using a renormalization method. The effects of the higher-order contribution, external magnetic field, dust charge variation, dust grain temperature, ratios of temperature and density of positive ions to electrons, and directional cosine of the wave vector k along the x axis on the nature of the solitary waves are investigated. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1739235]

I. INTRODUCTION

An exciting area of the most recent research is the study of electrostatic and electromagnetic waves in dusty plasmas. One of these waves was discovered by Rao *et al.*,[1] which they referred to as the dust-acoustic wave. This wave is a very low-frequency acoustic-like wave, and the restoring force comes from the pressures of the inertialess electrons and ions, whereas the mass of the charged dust particles provides the inertia. The experimental result of Chu *et al.*,[2] as well as its interpretation by D'Angelo,[3] have for the first time proved conclusively the existence of such low-frequency wave modes. These conclusions have been validated by the independent experiments of Barkan *et al.*[4] and of Prabhuram and Goree.[5] Extensive work has been devoted to the study of dust-acoustic waves (DAWs) in an unmagnetized dusty plasma. For example, Mamun *et al.*[6] reported that only negative potential structures associated with nonlinear DAWs can exist in a two-component plasma of ions and dust particles. Laksmi *et al.*[7] carried out the kinetic, as well as fluid, analysis of nonlinear DAWs in a dusty plasma. Kinetic and fluid models lead to essentially the same results, limit of the dust thermal speed being much smaller than the dust-acoustic speed. Ma and Lui[8] showed that a dusty plasma could admit dust-acoustic soliton (DAS) on a very slow time scale involving the motion of dust grains, whose charge is self-consistently determined by local electron and ion currents.

Highly charged massive dust grains present in plasma may exhibit charge fluctuations in response to certain types of oscillations incorporated in the plasma. Under this situation, the grain charge becomes a time-dependent and self-consistent variable. The consequent modifications in the collective properties of dusty plasma in response to the variation of charge have been studied for various plasma systems.[9] For example, Nejoh[10] showed that the dust charge variation with parameters such as electrostatic potential, and electron and ion density would affect the characteristic collective motion of the plasma. Therefore, the effect of variation of the dust charges should play an important role. Xie *et al.*[11] derived small-amplitude DASs with varying dust charges and have shown that only rarefactive solitary waves exist when the Mach number lies within an appropriate regime depending on the system parameters. Moreover, the dust-acoustic solitary waves (DASWs) and double layers (DLs) in dusty plasma with variable dust charge and two-temperature ions were studied by Xie *et al.*,[12] and they have shown that both compressive and rarefactive solitons, as well as DLs, exist. In addition, the amplitudes of the DASWs become smaller and the regime of admitted Mach number is extended wider for the variable dust charge situation compared to the constant dust charge situation.

Up to now, most treatments of the DAWs have been done in an unmagnetized two- or three-component dusty plasma, although dusty plasma invariably occurs in the presence of an ambient magnetic field, both in space and in laboratory situations. On the other hand, it will be of great interest to see how DAW characteristics are modified in a magnetized plasma, especially in space environments, as in the Jovian magnetosphere and in the induced magnetosphere of comets.[13] Most investigations of dusty plasma in the presence of an external magnetic field have considered the dust charge as a point charge (see, e.g., Ref. 13 and references

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therein). However, the study of dust grains in a magnetic field has been investigated in connection with the charging of satellites and rockets in Earth's ionosphere and magnetosphere.[14–16] In general, the presence of an external magnetic field makes a dusty plasma anisotropic. Hence, charging currents to a spherical dust grain is different in different directions. In the presence of a very strong magnetic field, the orbits of the magnetized plasma particles are confined to one dimension along the field lines, as if they are "glued" to the magnetic field lines. Hence, the perturbed field does not come into play, and the problem of charging currents becomes independent of the magnetic field.[17] Tsytovich *et al.*[18] showed that when the magnetic field becomes larger than a critical value (about 4 kG), the electron gyroradius is equal to the electron collection radius on dust grains. Only fast-magnetized electrons would then be involved in the charging process, while a fraction of low-energy electrons would be reflected backwards along the magnetic field direction. Hence, the cross section for magnetized electrons is changed, resulting in lowering the electron current by a factor of 4 compared to that in the absence of a magnetic field. In addition, at this value of the magnetic field strength, the ion gyroradius is still much larger than the ion dust attraction size, the ions will be attracted to the dust grain with approximately the same rate, and the ion current on the grain will then remain the same as in an unmagnetized plasma. For a much stronger magnetic field in plasma, the ion gyroradius becomes smaller than the dust size. Here, both the electron and ion currents are modified due to the strong magnetization of the plasma particles. Tsytovich *et al.*[18] treated numerically the problem of a very strong magnetic field in variational charged dusty plasma and reported the dependence of dust charge on the external magnetic field strength, as well as on the parameter $\xi = \sqrt{m_i T_i / m_e T_e}$, where $m_i/m_e(T_i/T_e)$ is the ion-to-electron mass (temperature) ratio. They found that the dust charge in a strong magnetic field could be substantially larger (up to 12 times) than that in the absence of the magnetic field (or in a weak magnetic field). Later, Salimullah *et al.*[17] used the kinetic theory to examine the currents of electrons and ions to a spherical dust grain in a uniform, strongly magnetized dusty plasma. They found that the external magnetic field reduces the charging current, thereby decreasing the dust charge fluctuation damping of a low-frequency electrostatic wave in a dusty plasma.

Zakharov and Kuznetsov[19] made the first attempt to model a soliton in a magnetized three-dimensional system. They obtained a three-dimensional differential equation, which is known as the ZK equation. Many authors have studied the ZK equation (see, e.g., Ref. 13 and references therein) using a reductive perturbation theory.[20] The ZK equation contains the lowest-order nonlinearity and dispersion, and it consequently can describe a wave of only small amplitude. As the wave amplitude increases, the width and velocity of a soliton deviate from the prediction of the ZK equation, in dicating the breakdown of the ZK approximation. To describe the DAWs of larger amplitude, the higher-order nonlinear and dispersive effects have to be taken into account. To this end, the higher-order approximation of the reductive perturbation theory has been known to be a powerful tool.[21] To the authors' knowledge, no efforts have been made to study the effect of higher-order corrections in the reductive perturbation theory in the presence of both variational warm dust charge and external magnetic field. Thus, in this paper, we study, qualitatively, the propagation characteristics of higher-order nonlinear DASWs in a collisionless, magnetized, three-component dusty plasma consisting of warm variational charged dust grains, isothermal ions, and electrons.

The outline of the paper is as follows. In Sec. II, we write down the basic set of fluid equations describing the model. The nonlinear DASWs are investigated by derivation of the ZK equation for the first-order perturbed potential and the linear inhomogeneous Zakharov–Kuznetsov-type (LIZKT) equation for the second-order perturbed potential. We apply the renormalization method introduced by Kodama and Taniuti[22] to reductive perturbation theory to obtain the stationary solutions of both equations. Section III is devoted to discussion and conclusions.

II. MODEL AND DERIVATIONS

We consider a three-component dusty plasma consisting of warm variational charged dust grains, isothermal ions and electrons in the presence of an external magnetic field $\mathbf{B}_0 = B_0\hat{x}$. The basic normalized equations governing the dusty plasma dynamics are written in the following form:[13]

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} - Z_d \nabla \phi + Z_d(\mathbf{u} \times \Omega \hat{\mathbf{x}}) + \frac{5}{3}\frac{\sigma_d}{n^{1/3}} \nabla n = 0, \tag{2}$$

$$\nabla^2 \phi = Z_d n + n_e - n_i. \tag{3}$$

The inertia of ions and electrons are neglected; hence, the ion and electron densities are governed by Boltzmann distribution as

$$n_i = \mu \exp(-S\phi), \tag{4}$$

$$n_e = \nu \exp(\beta S \phi), \tag{5}$$

where $\mu[=n_i^{(0)}/Z_d^{(0)} n^{(0)}]$ and $\nu[=n_e^{(0)}/Z_d^{(0)} n^{(0)}]$ are the normalized ion and electron number density, respectively. $n_i^{(0)}$, $n_e^{(0)}$, and $n^{(0)}$ are the unperturbed ion, electron, and dust number densities, respectively. n and $\mathbf{u}$ refer to the number density and fluid velocity of the dust grains, respectively. The densities of electrons and ions are normalized by $Z_d^{(0)} n^{(0)}$ and the dust grains density is normalized by $n^{(0)}$. Z_d is normalized by $Z_d^{(0)}$. The space coordinates [x, y, and z], time t, dust-cyclotron frequency Ω, fluid velocity $\mathbf{u}$, and electrostatic potential ϕ are normalized by the Debye length $\lambda_{Dd} = (T_{\text{eff}}/4\pi e^2 n^{(0)} Z_d^{(0)})^{1/2}$, the inverse dust plasma frequency $\omega_{\text{pd}}^{-1} = (m_d/4\pi e^2 n^{(0)} Z_d^{(0)^2})^{1/2}$, dust plasma frequency $\omega_{\text{pd}} = (4\pi e^2 n^{(0)} Z_d^{(0)^2}/m_d)^{1/2}$, the dust-acoustic speed $C_{\text{DA}} = (Z_d^{(0)} T_{\text{eff}}/m_d)^{1/2}$, and T_{eff}/e, respectively. We introduce the following notations:

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$$\beta=\frac{T_i}{T_e},\quad S=\frac{1}{\mu+\beta\nu},\quad T_{\rm eff}=\frac{T_iT_e}{\mu T_e+\nu T_i},$$

$$\text{and}\quad \sigma_d=\frac{T_d}{Z_d^{(0)}T_{\rm eff}},$$

where T_e, T_i, and T_d are the electron, ion, and dust temperatures, respectively.

We assume that the charging of the dust grains arises from plasma currents due to the electrons and ions reaching the grain surface. In this case, the dust grain charge variable Q_d is determined by the charge current balance equation[10]

$$\frac{\partial Q_d}{\partial t}+u\cdot\nabla Q_d=I_e+I_i. \tag{6}$$

Assuming that (i) the streaming velocities of the electrons and ions are much smaller than their thermal velocities,[10] and (ii) the condition under which the Orbit Motion Limited (OML) theory in a magnetized plasma is still valid, the magnetic field is strong (about 4 kG) enough to make the electron gyroradius is equal to the electron collection radius on dust grains (i.e., the electron current decreases by a factor of 4 compared to that in the absence of a magnetic field), and the ion gyroradius is still much larger than the ion dust attraction size (i.e., the ion current on the grain will remain the same as in an unmagnetized plasma).[18] Therefore, we have the following expressions for the electron and ion currents for spherical grains of radius r:

$$I_e=-\frac{1}{4}e\pi r^2(8T_e/\pi m_e)^{1/2}n_e\exp\left(\frac{e\Phi}{T_e}\right),$$

$$I_i=e\pi r^2(8T_i/\pi m_i)^{1/2}n_i\left(1-\frac{e\Phi}{T_i}\right),$$

where Φ denotes the dust grain surface potential related to the plasma potential ϕ. At equilibrium, equating I_e+I_i to zero and using the expressions of I_e and I_i in Eq. (6), we get

$$\alpha\delta(1-S\psi)\exp(-S\phi)-\exp(S\beta[\psi+\phi])=0, \tag{7}$$

where $\psi=e\Phi/T_{\rm eff}$, $\alpha=\sqrt{\beta/\mu_i}$, $\mu_i=m_i/m_e\approx1840$, $\delta=4\mu/\nu=4n_{i0}/n_{e0}$, $\mu=\delta/(\delta-1)$, $\nu=1/(\delta-1)$, and $S=(\delta-1)/(\delta+\beta)$.

Equation (7) is important for determining the dust charges due to the relation $Q_d=C\Phi$, where C is the capacitance of dust grains ($C=r$). That is to say, $-eZ_d=rT_{\rm eff}\psi/e$, from which we have the normalized dust charge $Z_d=\psi/\psi_0$, where $\psi_0=\psi(\phi=0)$ is the dust surface floating potential with respect to the unperturbed plasma potential at infinite place. ψ_0 can be determined by the following relation:

$$\alpha\delta(1-S\psi_0)-\exp(S\beta\psi_0)=0. \tag{8}$$

As can be seen, the dust charge is very sensitive to the small disturbance of ϕ around the unperturbed state. This point is very important for explanation how the variable dust charge influence the shape of solitary waves.[11]

For small but finite amplitude, Eqs. (1)–(8) can be analyzed using the reductive perturbation theory. In order to find a suitable choice of scaling for the independent variables, Infeld and Rowlands[23] used the linear dispersion argument to obtain the following stretching for the independent variables:

$$X=\in^{1/2}(x-\lambda t),\quad Y=\in^{1/2}y,\quad Z=\in^{1/2}z,\quad \text{and}\ T=\in^{3/2}t, \tag{9}$$

where λ is the phase velocity of the DASWs to be determined later, and $\in$ measures the size of the perturbation amplitude. Furthermore, the plasma parameters $\Theta\equiv[n,n_i,n_e,u_x,\phi,Z_d]$ are expanded as power series of $\in$ as

$$\Theta=\sum_{s=0}^{\infty}\in^s\Theta^{(s)}, \tag{10}$$

with the conditions

$$n^{(0)}=1,\quad n_i^{(0)}=\mu,\quad n_e^{(0)}=\nu,\quad u_x^{(0)}=\phi^{(0)}=0,\quad Z_d^{(0)}=1, \tag{11}$$

while $u_{y,z}$ are expanded as

$$u_{y,z}=\in^{3/2}u_{y,z}^{(1)}+\in^2u_{y,z}^{(2)}+\in^{5/2}u_{y,z}^{(3)}+\in^3u_{y,z}^{(4)}+\cdots. \tag{12}$$

The charge neutrality condition in the dusty plasma is always maintained through the relation

$$\mu-\nu=1. \tag{13}$$

Substituting Eqs. (9)–(13) into Eqs. (1)–(5), and using $Z_d=\psi/\psi_0$ in Eq. (7), the lowest-order in $\in$ yields

$$n^{(1)}=\frac{1}{\Delta}\phi^{(1)},\quad u_x^{(1)}=\frac{\lambda}{\Delta}\phi^{(1)},\quad u_y^{(1)}=\frac{-\lambda^2}{\Omega\Delta}\frac{\partial\phi^{(1)}}{\partial Z},$$
$$u_z^{(1)}=\frac{\lambda^2}{\Omega\Delta}\frac{\partial\phi^{(1)}}{\partial Y},\quad Z_d^{(1)}=\gamma_1\phi^{(1)}, \tag{14}$$

and Poisson's equation gives the linear dispersion relation

$$\lambda^2=\tfrac{5}{3}\sigma_d+(1+\gamma_1)^{-1}, \tag{15}$$

where

$$\gamma_1=\frac{-(1+\beta)(1-S\psi_0)}{\psi_0[1+\beta(1-S\psi_0)]}\quad\text{and}\quad\Delta=\frac{5}{3}\sigma_d-\lambda^2.$$

If we consider the next-order in $\in$, we obtain a system of equations in the second-order perturbed quantities. Solving this system with the aid of Eqs. (14) and (15), we finally obtain the ZK equation as

$$\frac{\partial\phi^{(1)}}{\partial T}+AB\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial X}+\frac{A}{2}\frac{\partial^3\phi^{(1)}}{\partial X^3}+\frac{AD}{2}\left(\frac{\partial^3\phi^{(1)}}{\partial X\partial Y^2}+\frac{\partial^3\phi^{(1)}}{\partial X\partial Z^2}\right)=0, \tag{16}$$

where

$$A=\frac{\Delta^2}{\lambda},$$

$$B=\frac{1}{2}\left[\frac{3\lambda^2-\frac{5}{9}\sigma_d}{\Delta^3}-\frac{3\gamma_1}{\Delta}+\frac{(\delta-1)(\delta-\beta)^2}{(\delta+\beta)^2}-2\gamma_2\right],$$

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and $D=1+\frac{\lambda^4}{\Omega^2\Delta^2}$.

The second-order perturbed quantities can be calculated, with the aid of (14), as

$$n^{(2)}=I_1\phi^{(1)^2}+I_2\frac{\partial^2\phi^{(1)}}{\partial X^2}+I_3\left(\frac{\partial^2\phi^{(1)}}{\partial Y^2}+\frac{\partial^2\phi^{(1)}}{\partial Z^2}\right)+I_4\phi^{(2)}, \tag{17a}$$

$$u_x^{(2)}=J_1\phi^{(1)^2}+J_2\frac{\partial^2\phi^{(1)}}{\partial X^2}+J_3\left(\frac{\partial^2\phi^{(1)}}{\partial Y^2}+\frac{\partial^2\phi^{(1)}}{\partial Z^2}\right)+J_4\phi^{(2)}, \tag{17b}$$

$$u_y^{(2)}=\frac{\lambda^3}{\Omega^2\Delta}\frac{\partial^2\phi^{(1)}}{\partial X\partial Y},\quad u_z^{(2)}=\frac{\lambda^3}{\Omega^2\Delta}\frac{\partial^2\phi^{(1)}}{\partial X\partial Z},$$

$$Z_d^{(2)}=\gamma_1\phi^{(1)}+\gamma_2\phi^{(1)2}, \tag{17c}$$

where

$$I_1=\left\{\frac{(\delta-1)(\delta-\beta^2)}{2(\delta+\beta)^2}-\frac{\gamma_1}{\Delta}-\gamma_2\right\},\quad I_2=I_3=1,$$

$$I_4=\frac{1}{\Delta},$$

$$J_1=\left\{\frac{(5\sigma_d+3\lambda^2)}{6\lambda}I_1-\frac{(5\sigma_d+9\lambda^2)}{36\Delta^2\lambda}-\frac{\gamma_1}{4\lambda}\right\},$$

$$J_2=\frac{5\sigma_d+3\lambda^2}{6\lambda},$$

$$J_3=\left\{\frac{5\sigma_d+3\lambda^2}{6\lambda}-\frac{3\lambda^3}{6\Omega^2\Delta}\right\},\quad J_4=\frac{\lambda}{\Delta},$$

and $\gamma_2=\frac{-S(1+\beta)^2(1-S\psi_0)}{2\psi_0[1+\beta(1-S\psi_0)]^3}$.

If we consider the next-order of $\in$, we obtain a system of equations. Solving this system with the aid of Eqs. (14)–(17), we finally obtain the LIZKT equation for the second-order perturbed potential $\phi^{(2)}$:

$$\begin{aligned}\frac{\partial\phi^{(2)}}{\partial T}&+AB\frac{\partial}{\partial X}(\phi^{(1)}\phi^{(2)})+\frac{A}{2}\frac{\partial^3\phi^{(2)}}{\partial X^3}+\frac{AD}{2}\left(\frac{\partial^3\phi^{(2)}}{\partial X\partial Y^2}+\frac{\partial^3\phi^{(2)}}{\partial X\partial Z^2}\right)\\
&=-H_1\phi^{(1)^2}\frac{\partial\phi^{(1)}}{\partial X}-H_2\phi^{(1)}\frac{\partial^3\phi^{(1)}}{\partial X^3}-H_3\left(\phi^{(1)}\frac{\partial^3\phi^{(1)}}{\partial X\partial Y^2}+\phi^{(1)}\frac{\partial^3\phi^{(1)}}{\partial X\partial Z^2}\right)-H_4\left(\frac{\partial\phi^{(1)}}{\partial X}\frac{\partial^2\phi^{(1)}}{\partial X^2}\right)\\
&\quad-H_5\left(\frac{\partial\phi^{(1)}}{\partial X}\frac{\partial^2\phi^{(1)}}{\partial Y^2}+\frac{\partial\phi^{(1)}}{\partial X}\frac{\partial^2\phi^{(1)}}{\partial Z^2}\right)-H_6\left(\frac{\partial\phi^{(1)}}{\partial Y}\frac{\partial^2\phi^{(1)}}{\partial X\partial Y}+\frac{\partial\phi^{(1)}}{\partial Z}\frac{\partial^2\phi^{(1)}}{\partial X\partial Z}\right)-H_7\frac{\partial^5\phi^{(1)}}{\partial X^5}\\
&\quad-H_8\left(\frac{\partial^5\phi^{(1)}}{\partial X^3\partial Y^2}+\frac{\partial^5\phi^{(1)}}{\partial X^3\partial Z^2}\right)-H_9\left(\frac{\partial^5\phi^{(1)}}{\partial X\partial Y^4}+\frac{\partial^5\phi^{(1)}}{\partial X\partial Z^4}+2\frac{\partial^5\phi^{(1)}}{\partial X\partial Y^2\partial Z^2}\right),\end{aligned} \tag{18}$$

where H_i $(i=1,2,3,...,9)$ is given in the Appendix.

Now, the basic set of fluid Eqs. (1)–(8) is reduced to ZK [Eq. (16)] for $\phi^{(1)}$, and a LIZKT [Eq. (18)] for $\phi^{(2)}$ of which the source term (inhomogeneous term) is described by a known function of $\phi^{(1)}$. However, Eq. (18) has a resonant term that gives rise to a secular solution. Thus, to eliminate the secular behavior, we use the renormalization method, which was introduced by Kodama and Taniuti.[22] According to this method, Eqs. (16) and (18) are modified to

$$\frac{\partial\phi^{(1)}}{\partial T}+AB\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial X}+\frac{A}{2}\frac{\partial^3\phi^{(1)}}{\partial X^3}+\frac{AD}{2}\left(\frac{\partial^3\phi^{(1)}}{\partial X\partial Y^2}+\frac{\partial^3\phi^{(1)}}{\partial X\partial Z^2}\right)+\frac{\delta\vartheta}{l}\frac{\partial\phi^{(1)}}{\partial X}=0, \tag{19}$$

$$\frac{\partial\phi^{(2)}}{\partial T}+AB\frac{\partial}{\partial X}(\phi^{(1)}\phi^{(2)})+\frac{A}{2}\frac{\partial^3\phi^{(2)}}{\partial X^3}+\frac{AD}{2}\left(\frac{\partial^3\phi^{(2)}}{\partial X\partial Y^2}+\frac{\partial^3\phi^{(2)}}{\partial X\partial Z^2}\right)+\frac{\delta\vartheta}{l}\frac{\partial\phi^{(2)}}{\partial X}=\Lambda^{(2)}(\phi^{(1)})+\frac{\vartheta^{(1)}}{l}\frac{\partial\phi^{(1)}}{\partial X}, \tag{20}$$

where $\Lambda^{(2)}$ represents the right-hand side of Eq. (18). The parameter $\delta\vartheta$ in Eqs. (19) and (20) can be determined from the condition that the resonant term in $\Lambda^{(2)}$ is canceled out by the term $(\delta\vartheta/l)(\partial\phi^{(1)}/\partial X)$.[24]

Let us introduce the variable

$$\eta=lX+mY+nZ-(\vartheta+\delta\vartheta)T, \tag{21}$$

where $\vartheta+\delta\vartheta=M-1=\Delta M$, with M the Mach number. l, m, and n are the direction cosines: $(l^2+m^2+n^2=1)$. Integrating with respect to the variable η and using the vanishing

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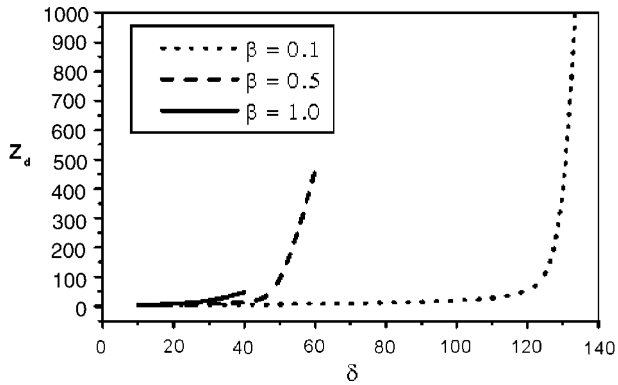

FIG. 1. Z_d vs δ for different values of β.

boundary condition for $\phi^{(1)}_{(\eta)}$ and $\phi^{(2)}_{(\eta)}$ and their derivatives up to second-order for $|\eta|\to\infty$, we obtain from (19) and (20), the equations

$$\frac{d^2\phi^{(1)}}{d\eta^2}+\frac{1}{F}\left(\frac{AB}{2}\phi^{(1)}-\frac{\vartheta}{l}\right)\phi^{(1)}=0, \tag{22}$$

$$\frac{d^2\phi^{(2)}}{d\eta^2}+\frac{1}{F}\left(AB\phi^{(1)}-\frac{\vartheta}{l}\right)\phi^{(2)}=\frac{1}{lF}\int_{-\infty}^{\eta}\left[\Lambda^{(2)}(\phi^{(1)})+\vartheta^{(1)}\frac{d\phi^{(1)}}{d\eta}\right]d\eta, \tag{23}$$

where $F=\frac{1}{2}A[l^2+D(m^2+n^2)]$.

The one-soliton solution of Eq. (22) is given by

$$\phi^{(1)}=\phi_0\,\mathrm{sech}^2(\eta/\omega), \tag{24}$$

where the amplitude $\phi_0=3\vartheta/ABl$, and the width $\omega=2\sqrt{lF/\vartheta}$.

In order to cancel out the resonant term in $\Lambda^{(2)}(\phi^{(1)})$, we set $\delta\vartheta=16lFE_4\omega^{-4}$.

The soliton solution of Eq. (23) is given by

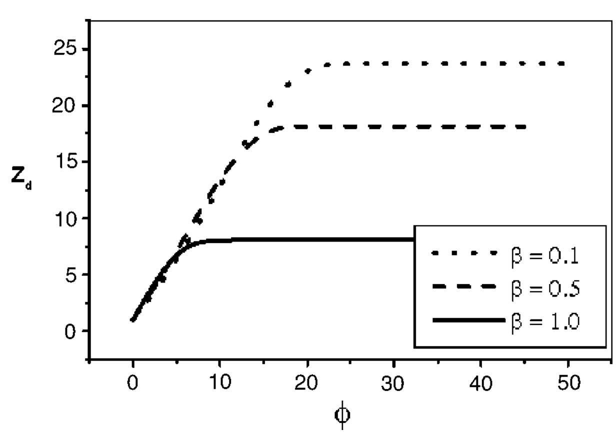

FIG. 2. Z_d vs ϕ for different values of β, where $\delta=10$.

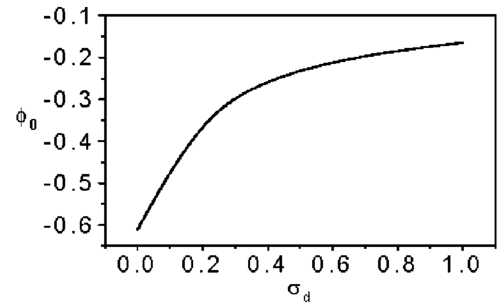

FIG. 3. ϕ_0 vs σ_d for $\delta=30$, and $\beta=0.1$.

$$\phi^{(2)}=\frac{9\vartheta^2}{(AB)^2l^2F}\left\{-\frac{F^2}{2AB}E_1-\frac{F}{6}(E_2-E_3)+\frac{5}{12}ABE_4\right\}\mathrm{sech}^2(\eta/\omega)+\frac{9\vartheta^2}{(AB)^2l^2F}\left[-\frac{F^2}{2AB}E_1+\frac{F}{2}(E_2+E_3)-\frac{5}{4}ABE_4\right]\times\mathrm{sech}^2(\eta/\omega)\tanh^2(\eta/\omega), \tag{25}$$

where

$$E_1=\frac{H_1}{F},\quad E_2=\frac{1}{2F}[H_4l^2+(H_5+H_6)(m^2+n^2)],$$

$$E_3=\frac{1}{F}[H_2l^2+H_3(m^2+n^2)],$$

$$E_4=\frac{1}{F}[H_7l^4+H_8l^2(m^2+n^2)+H_9(m^2+n^2)^2].$$

ϑ is now related to ΔM by

$$\vartheta=lF\left[\left(1+\frac{4E_4}{lF}\Delta M\right)^{1/2}-1\right]\Big/\ 2E_4. \tag{26}$$

A detailed mathematical treatment can be found in Ref. 13 or 24.

Thus, the stationary solution for the DASWs up to the second order of ΔM is given by, with the aid of Eq. (26)

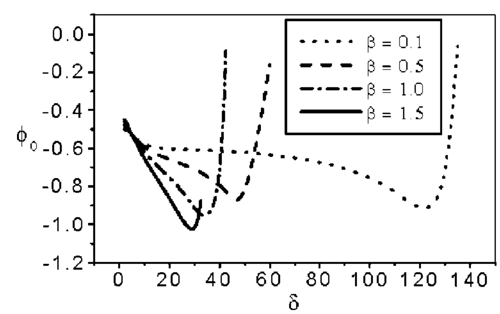

FIG. 4. ϕ_0 vs δ for different values of β, where $\sigma_d=0.001$.

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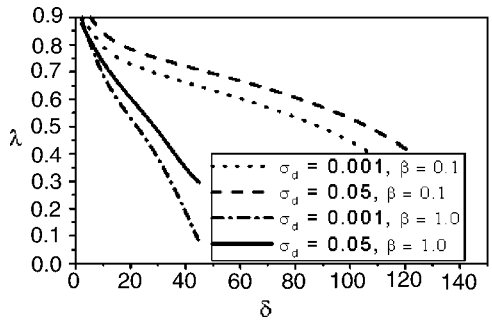

FIG. 5. λ vs δ for different values of β and σ_d.

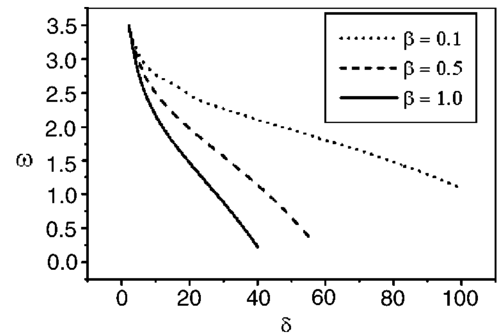

FIG. 7. ω vs δ for different values of β, where λ=0.9, $\sigma_d=0.1$, and $\Omega=27$.

$$\phi=\phi^{(1)}+\phi^{(2)}=\frac{3\Delta M}{ABl}\operatorname{sech}^2(\eta/\omega)+\frac{9\Delta M^2}{(AB)^2l^2F}\times\left[-\frac{F^2}{2AB}E_1-\frac{F}{6}(E_2-E_3)+\frac{1}{12}ABE_4\right]\operatorname{sech}^2(\eta/\omega)+\frac{9\Delta M^2}{(AB)^2l^2F}\left[-\frac{F^2}{2AB}E_1+\frac{F}{2}(E_2+E_3)-\frac{5}{4}ABE_4\right]\times\operatorname{sech}^2(\eta/\omega)\tanh^2(\eta/\omega), \tag{27}$$

where ω is given by

$$\left[\frac{4lF}{\Delta M}\right]^{1/2}\left[1+\frac{E_4}{2lF}\,\Delta M\right].$$

III. DISCUSSION AND CONCLUSIONS

In this paper, we have analyzed the properties of DASWs in a magnetized three-component dusty plasma, comprising warm variational charged dust grains, isothermal ions, and electrons. The reductive perturbation theory has been used to derive ZK [Eq. (16)] for $\phi^{(1)}$ and LIZKT [Eq. (18)] for $\phi^{(2)}$. To examine the effects of various plasma parameters as well as the contribution of higher-order nonlinearity on the nature of the DASWs, we numerically analyze the dust charge variation, the phase velocity, the amplitude, and the width of the solitary waves. Figure 1 illustrates the dependence of Z_d on δ and β. It shows that Z_d has a nearly fixed value for a variation in δ but it increases rapidly at certain value of δ (that we call $\delta_{\max}$) for different values of β. Figure 2 illustrates the dependence of Z_d on the plasma potential ϕ. It is clear that Z_d increases with ϕ, but it reaches a constant value when the potential takes certain value. It is also noticed that Z_d decreases with β. Figure 3 shows that the amplitude ϕ_0 decreases with σ_d. Figure 4 shows that ϕ_0 increases slowly as δ increases, however near to $\delta_{\max}$, it decreases rapidly. Figure 4 shows also that ϕ_0 increases with β. From Figs. 3 and 4, it is clear that the amplitude has negative sign; thus, only rarefactive solitons can propagate in this system. Figure 5 shows the variation of λ with δ, β, and σ_d. It shows that λ decreases as δ and β increases, while it increases with σ_d. Figure 6 shows the variation of the width ω with Ω. It is clear that ω decreases with Ω. Figure 7 shows that the variation of ω with δ and β. It is seen that ω decreases with δ and β. Figure 8 shows that the variation of ω with λ and σ_d. It is clear that ω increases (decreases) with $\lambda(\sigma_d)$.

It is well known that the perturbation analysis is only valid for small but finite amplitudes. However, the values of γ_1 and γ_2 can increase the wave amplitude enough to break down the perturbation theory. Therefore, the range of γ_1 and γ_2 to be applied can be written as

$$\frac{3\gamma_1}{\Delta}+2\gamma_2<\left[-\frac{6\vartheta\in\lambda}{\Delta^2 l}+\frac{3\lambda^2-\frac{5}{9}\sigma_d}{\Delta^3}+\frac{(\delta-1)(\delta-\beta^2)}{(\delta+\beta^2)^2}\right]. \tag{28}$$

Now, we investigate the effect of higher-order corrections on the features of the DASWs. In Figs. 9 and 10, the

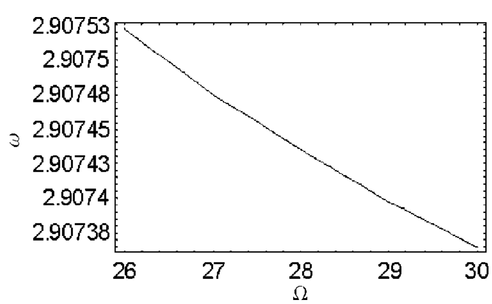

FIG. 6. ω vs Ω, for λ=0.9, λ=0.9.

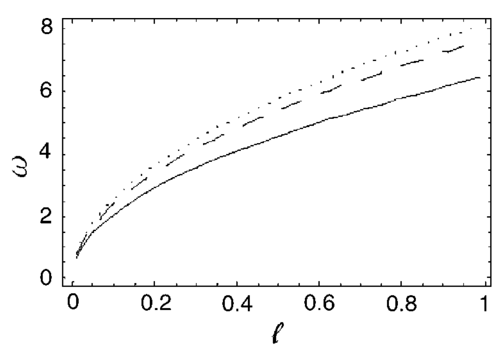

FIG. 8. ω vs λ for λ=0.9, $\Omega=27$, $\sigma_d=0.01$ (dotted line), $\sigma_d=0.1$ (dashed line), and $\sigma_d=0.3$ (solid line).

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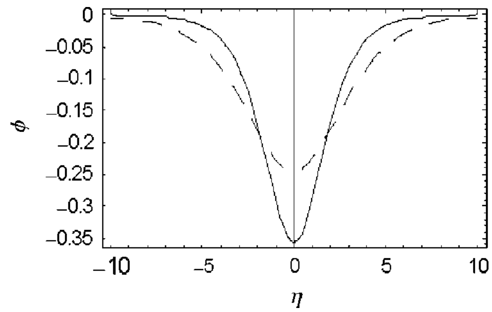

FIG. 9. ϕ vs η for $\delta=2$, $\beta=0.1$, $\lambda=0.9$, $\Omega=27$, and $\sigma_d=0.1$.

dashed line stands for $\phi^{(1)}$, while the solid line represents $\phi(=\phi^{(1)}+\phi^{(2)})$. From Fig. 9, for small value of δ, the higher-order correction increases the amplitude but decreases the width. Figure 10 shows that, for a large value of δ, the amplitude and the width decrease by introducing the higher-order correction. We believe that the model and results presented here should be applicable to dusty plasma devices as well as specific space plasma systems.

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APPENDIX

Coefficients of Eq. (18):

$$H_1=\frac{1}{2}A\left[\frac{3(9I_1\lambda^2+18J_1\lambda-5I_1\sigma_d)}{(3\lambda^2-5\sigma_d)^2}+\frac{12\gamma_2}{(3\lambda^2-5\sigma_d)}+\frac{6AB(J_1+\lambda I_1)}{(3\lambda^2-5\sigma_d)}+\frac{30\sigma_d}{(3\lambda^2-5\sigma_d)^4}-\frac{\nu S^3\beta^3}{2}-\frac{\mu S^3}{2}-2I_1\gamma_1-3\gamma_3\right],$$

$$H_2=\frac{1}{2}A\left[\frac{9\lambda^2I_2+18\lambda J_2-5\sigma_dI_2}{(3\lambda^2-5\sigma_d)^2}+\frac{3A(J_1+\lambda I_1)}{(3\lambda^2-5\sigma_d)}+\frac{3AB(J_2+\lambda I_2)}{(3\lambda^2-5\sigma_d)}-\gamma_1I_2\right],$$

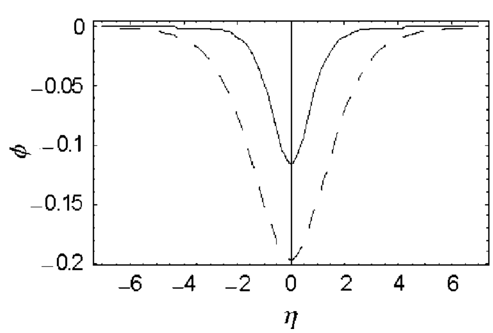

FIG. 10. ϕ vs η for $\delta=60$, $\beta=0.1$, $\lambda=0.9$, $\Omega=27$, and $\sigma_d=0.1$.

$$H_3=\frac{1}{2}A\left[\frac{9AB\lambda^3}{\Omega^2(3\lambda^2-5\sigma_d)^2}-\frac{10\lambda^2I_1\sigma_d}{\Omega^2(3\lambda^2-5\sigma_d)}+\frac{15\lambda^2\sigma_d}{\Omega^2(3\lambda^2-5\sigma_d)^3}-\frac{3\gamma_1(3\lambda^2+5\sigma_d)\lambda^2}{\Omega^2(3\lambda^2-5\sigma_d)^2}+\frac{9\lambda^2I_3+18\lambda J_3-5\sigma_dI_3}{(3\lambda^2-5\sigma_d)^2}+\frac{3AD(J_1+\lambda I_1)}{(3\lambda^2-5\sigma_d)}+\frac{3AB(J_3+\lambda I_3)}{(3\lambda^2-5\sigma_d)}-\gamma_1I_3\right],$$

$$H_4=\frac{1}{2}A\left[\frac{9AB(J_2+\lambda I_2)}{(3\lambda^2-5\sigma_d)}+\frac{9\lambda^2I_2+18\lambda J_2-5\sigma_dI_2}{(3\lambda^2-5\sigma_d)^2}-\gamma_1I_2\right],$$

$$H_5=\frac{1}{2}A\left[\frac{9AB\lambda^3}{\Omega^2(3\lambda^2-5\sigma_d)^2}+\frac{10\lambda^2\sigma_dI_1}{\Omega^2(3\lambda^2-5\sigma_d)^2}-\frac{15\gamma_1\sigma_d\lambda^2}{\Omega^2(3\lambda^2-5\sigma_d)^2}+\frac{405\lambda^2\sigma_d}{\Omega^2(9\lambda^2-5\sigma_d)^3}+\frac{9\lambda^2I_3+18\lambda J_3-5\sigma_dI_3}{(3\lambda^2-5\sigma_d)^2}+\frac{3AB(J_3+\lambda I_3)}{(3\lambda^2-5\sigma_d)}-\gamma_1I_3\right],$$

$$H_6=\frac{A}{18\Delta^3}\left[\frac{18AB\Delta\lambda^3}{\Omega^2}+6AB(J_3+\lambda I_3)\Delta^2+\frac{180\sigma_dI_1\lambda^2\Delta^2}{3\Omega^2}-\frac{9\gamma_1\Delta\lambda^2(\lambda^2+10\sigma_d/3)}{\Omega^2}+\frac{9\lambda^4-10\lambda^2\sigma_d}{\Omega^2}\right],$$

$$H_7=-\frac{A^2(\lambda I_2+J_2)}{4\Delta},$$

$$H_8=\frac{A}{36\Omega^4\Delta^2}\{-18\lambda^6-30\Omega^2\sigma_d\lambda^4I_2+9A\Omega^2(\Omega^2I_3+\Omega^2DI_2+1)\lambda^3+\lambda^2[9A(J_3+DJ_2)\Omega^4+50\Omega^2\sigma_d^2I_2]-15A(I_3-DI_2)\lambda\sigma_d\Omega^4-15A(J_3+DJ_2)\sigma_d\Omega^4\},$$

$$H_9=\frac{1}{2}A\left[\frac{-AD(\lambda I_3+J_3)}{2\Delta}+\frac{5\lambda^2\sigma_dI_3}{3\Omega^2\Delta}+\frac{AD\lambda^3}{2\Omega^2\Delta^2}\right].$$

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