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Kadomtsev-Petviashvili Equation for Dust Acoustic Solitary Waves in a Warm Dusty Plasma with Dust Charge Variation

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Abstract

The properties and stability of nonlinear three-dimensional dust acoustic solitary waves are examined in a dusty plasma, having warm variational charged dust grains, hot ions and electrons, through the derivation of the Kadomtsev-Petviashvili equation using the reductive perturbation method. It is found that only rarefactive solitary waves can propagate in this system. To determine the stability of the waves, we used a method based on energy considerations to obtain a condition for stable solitary waves. It is found that this condition depends on dust charge variation, dust grains temperature, directional cosine of the wave vector k along the x-axis. Also, we study the dependance of the amplitude and the width on various plasma parameters. The findings of this investigation may be useful in understanding laboratory plasma phenomena and astrophysical situations.

1. Introduction

It is well known that dust particles are common in the universe and they represent much of the solid matter in it. Dust particles often contaminate fully ionized or partially ionized gases and form so-called "dusty plasma", which occur frequently in nature. In astrophysics, in the early 1930s, dust was shown to be present in the interstellar clouds where it appears as a selective absorption of stellar radiation (interstellar reddening). Dust particles play a very important role in the solar system, in cometary tails and in planetary rings, and also in the evolution of the solar system from its solar nebula to its present form. Dust particles are also found in environments such as production processes, flames, rocket exhausts and many laboratory experiments [1]. The dust particles are of micrometer or sub-micrometer size, and the masses of dust particles are very large. Due to the large mass, the dust particles sustain low frequency oscillations. Dust acoustic waves (DAWs) is one of the low frequency waves in dusty plasmas, which was reported theoretically first by Rao *et al.* [2] and was conclusively verified in a recent laboratory experiment [3]. Later, many authors have studied the behavior of DAWs in various dusty plasma systems. For example, Mamun *et al.* [4] reported that only negative potential structures associated with nonlinear DAWs can exist in a two component plasma of ions and dust particles. Lakshmi *et al.* [5] carried out the kinetic as well as fluid analysis of nonlinear DAWs in a dusty plasma. Kinetic and fluid models lead to essentially the same results in the limit of dust thermal speed being much smaller than the dust-acoustic speed. Mamun [6] studied the effect of trapped ions on the dust-acoustic solitary waves (DASWs) in unmagnetized three-component dusty plasma which consist of negatively charged dust fluid, free electrons and trapped as well as free ions. It was found that, due to the presence of the trapped ions, the DASWs have larger amplitude, smaller width and higher propagation velocity than those involving adiabatic ions.

It was noted that in the research of collective effects involving charged dust particles in dusty plasmas, in general, the dust charges are usually assumed as constant. However, as it was pointed out by Nejoh [7], the dust charge variation with parameters such as electrostatic potential, electron and ion density would affect the characteristic collective motion of the plasma. Therefore, the effect of dust charge variation should play an important role. Using the reductive perturbation method [8], Xie *et al.* [9] derived small-amplitude DASs with varying dust charge and have shown that only the rarefactive solitary waves exist when the Mach number lies within an appropriate regime depending on the system parameters. DASWs and double layers (DLs) in dusty plasma with variable dust charge and two-temperature ions were studied by Xie *et al.* [10], and it was shown that both compressive and rarefactive solitons as well as DLs exist. Also, the amplitudes of the solitary waves become smaller and the regime of Mach number is extended for the variable dust charge situation compared to the constant dust charge situation. El-Labany *et al.* [11] considered the effect of dust charge variation on the behavior of the nonlinear DAWs with the combined effects of trapped ion distribution, two-ion-temperature, and dust fluid temperature in an unmagnetized dusty plasma. It was found that owing to the departure from the Boltzmann ion distribution to the trapped ion distribution, the dynamics of small but finite amplitude DAWs is governed by a modified Korteweg-de Vries (KdV) equation. The latter admits a stationary DASW solution, which has stronger nonlinearity, smaller amplitude, wider width, and higher propagation velocity than that involving adiabatic ions. The effect of two-ion-temperature is found to provide the possibility for the coexistence of rarefactive and compressive dust acoustic solitary structures and DLs. Although the dust fluid temperature increases the amplitude of the small but finite amplitude solitary waves, the dust charge fluctuation has the opposite effect.

Kadomtsev and Petviashvili [12] made the first attempt to model a soliton in a two dimensional system. For cold plasma, they obtained a two dimensional differential equation which is known as the famous Kadomtsev-Petviashvili (KP) equation. The KP equation has been studied by many authors [13 15] using a reductive perturbation method. To our knowledge, no attempt has been made so far to study the propagation characteristics of nonlinear three dimensional DASWs in collisionless, unmagnetized dusty plasmas

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with finite dust temperature and variational dust charge. So, it is the first time to obtain a qualitative study and stability conditions of the propagation characteristics of DASWs in collisionless, unmagnetized three component dusty plasmas; consisting of warm variational charged dust particles, hot ions and electrons. In section 2, we write down the basic set of fluid equations and use the reductive perturbation method to derive the KP equation which describes the evolution of the system. Section 3 contains a detailed discussion of our paper. Section 4 is devoted to the conclusions.

2. Basic Equations and Formulation of the Problem

We consider a fully ionized three component dusty plasma consisting of warm variational charged dust grains, hot ions and electrons. The basic normalized equations governing the dusty plasma dynamics are written in the following form:

$$\frac{\partial \bar{n}_d}{\partial \bar{t}} + \bar{\nabla} \cdot (\bar{n}_d \bar{\boldsymbol{u}}_d) = 0, \tag{1}$$

$$\frac{\partial \bar{\boldsymbol{u}}_d}{\partial \bar{t}} + \bar{\boldsymbol{u}}_d \cdot \bar{\nabla} \bar{\boldsymbol{u}}_d - \bar{Z}_d \bar{\nabla} \bar{\phi} + \frac{5\sigma_d}{3\bar{n}_d^{1/3}} \bar{\nabla} \bar{n}_d = 0, \tag{2}$$

$$\bar{\nabla}^2 \bar{\phi} = \bar{Z}_d \bar{n}_d + \bar{n}_e - \bar{n}_i. \tag{3}$$

The dimensionless number densities of ions and electrons are expressed as follows

$$\bar{n}_i = \mu \exp(-S\bar{\phi}), \tag{4}$$

$$\bar{n}_e = \nu \exp(S\beta\bar{\phi}). \tag{5}$$

The normalized plasma parameters (indicated by an overbar) are defined as

$$\bar{n}_d = \frac{n_d}{n_o}, \quad \bar{n}_{i,e} = \frac{n_{i,e}}{Z_{do} n_o}, \quad \bar{Z}_d = \frac{Z_d}{Z_{do}},$$

$$\bar{\boldsymbol{u}}_d = \boldsymbol{u}_d \left[\frac{Z_{do} T_{eff}}{m_d}\right]^{1/2}, \quad \bar{\phi} = \phi \left[\frac{T_{eff}}{e}\right]^{-1},$$

$$\bar{t} = t \left[\frac{4\pi e^2 Z_{do}^2 n_o}{m_d}\right]^{1/2}, \quad \bar{\nabla} = \nabla \left[\frac{T_{eff}}{4\pi e^2 Z_{do} n_o}\right]^{-1/2},$$

with

$$\beta = \frac{T_i}{T_e}, \quad S = \frac{1}{\mu + \beta\nu}, \quad T_{eff} = \frac{T_i}{\mu + \beta\nu} \quad \text{and} \quad \sigma_d = \frac{T_d}{Z_{do} T_{eff}}.$$

Here n_d is the density of dust grains of mass m_d moving with velocity $\boldsymbol{u}_d$. n_i and n_e is the density of ions and electrons, respectively. ∇ is the space coordinate, t is the time variable, ϕ is the electrostatic potential, T_{eff} is the effective temperature, μ and ν are the initial densities of the ions and electrons, respectively. T_e, T_i and T_d and the electron, ion and dust temperatures.

We assume that the charging of the dust grains arises from plasma currents coming from the electrons and the ions reaching the grain surface. In this case, the dust charge variable Q_d is determined by the charge current balance equation [9]

$$\frac{\partial Q_d}{\partial t} + u_d \cdot \nabla Q_d = I_e + I_i.$$

We notice that the characteristic time for dust motion is of the order of tens of milliseconds for micrometersized grains, while the dust charging time is typically of order of 10^{-8} s. Therefore, on the hydrodynamic time scale, the dust charge can quickly reach local equilibrium, at which the currents from the electrons and ions to the dust are balanced. It follows that $dQ_d/dt \ll I_e$ and I_i, and the current balance equation reads [9]

$$I_e + I_i \approx 0. \tag{6}$$

Assuming that the streaming velocities of the electrons and ions are much smaller than their thermal velocities, we have the electron and ion currents for spherical grains of radius r [9]:

$$I_e = -e\pi r^2 (8T_e/\pi m_e)^{1/2} n_e \exp\left(\frac{e\Phi}{T_e}\right),$$

$$I_i = e\pi r^2 (8T_i/\pi m_i)^{1/2} n_i \left(1 - \frac{e\Phi}{T_i}\right),$$

where Φ denotes the dust grain surface potential relative to the plasma potential ϕ. Using the expressions of I_e and I_i in Eq. (6), we get

$$\alpha\delta(1 - S\Psi)\exp(-S\phi) - \exp(S\beta[\Psi + \phi]) = 0, \tag{7}$$

where $\Psi = e\Phi/T_{eff}$, $\alpha = \sqrt{\beta/\mu_i}$, $\mu_i = m_i/m_e \approx 1840$, $\delta = \mu/\nu = n_{io}/n_{eo}$, $\mu = \delta/(\delta - 1)$, $\nu = 1/(\delta - 1)$ and $S = (\delta - 1)/(\delta + \beta)$.

Eq. (7) is important for determining the dust charges due to the relation $Q_d = C\Phi$, where C is the capacitance of dust grains ($C = r$), i.e., $-eZ_d = rT_{eff}\Psi/e$. We have the normalized dust charge $Z_d = \Psi/\Psi_0$, where $\Psi_0 = \Psi(\phi = 0)$ is the dust surface floating potential with respect to the unperturbed plasma potential at infinity. Ψ_0 can be determined by the following relation:

$$\alpha\delta(1 - S\Psi_0) - \exp(S\beta\Psi_0) = 0. \tag{8}$$

As can be seen, the dust charge is very sensitive to the small disturbance of ϕ around the unperturbed state. This point is very important for explaining how a variable dust charge influences the shape of solitary waves [16].

In order to study the dynamics of small- but finite- amplitude DASWs in the presence of adiabatic variation of dust charges, we derive an evolution equation from Eqs. (1) (5) by employing the reductive perturbation method [8]. According to this method, the independent [x, y, z, and t] and dependent [n_d, u_d, ϕ, and Z_d] variables can be easily obtained by using simple and straight forward algebra which are available in Ref. [14]. The stretched space-time coordinates are found to be

$$X = \varepsilon^{1/2}(x - \lambda t), \quad Y = \varepsilon y, \quad Z = \varepsilon z \quad \text{and} \quad T = \varepsilon^{3/2} t, \tag{9}$$

where λ is the phase velocity of the DASWs to be determined later, ε measures the size of the perturbation amplitude. Applying the obtained stretching (9) one can get the perturbation quantities in x-, y- and z-directions as (omitting overbars from now on):

$$n_d = 1 + \varepsilon n_d^{(1)} + \varepsilon^2 n_d^{(2)} + \varepsilon^3 n_d^{(3)} + \dots, \tag{10}$$

$$u_{dx} = \varepsilon u_{dx}^{(1)} + \varepsilon^2 u_{dx}^{(2)} + \varepsilon^3 u_{dx}^{(3)} + \ldots\ldots\ldots, \tag{11}$$

$$u_{dy} = \varepsilon^{3/2} u_{dy}^{(1)} + \varepsilon^2 u_{dy}^{(2)} + \varepsilon^{5/2} u_{dy}^{(3)} + \ldots, \tag{12}$$

$$u_{dz} = \varepsilon^{3/2} u_{dz}^{(1)} + \varepsilon^2 u_{dz}^{(2)} + \varepsilon^{5/2} u_{dz}^{(3)} + \ldots, \tag{13}$$

$$\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \varepsilon^3\phi^{(3)} + \ldots\ldots\ldots\ldots, \tag{14}$$

$$Z_d = 1 + \varepsilon Z_d^{(1)} + \varepsilon^2 Z_d^{(2)} + \varepsilon^3 Z_d^{(3)} + \ldots, \tag{15}$$

where u_{dx}, u_{dy} and u_{dz} are the component of dust velocity in the direction of x, y and z, respectively. The charge neutrality condition is given by

$$\mu - \nu = 1. \tag{16}$$

Substituting (10) (15) into the basic set of Eqs. (1) (5) and using $Z_d = \Psi/\Psi_0$ in Eq. (7), then collecting terms of different powers of ε, in the lowest-order we obtain

$$\phi^{(1)} = \Delta n_d^{(1)} = \frac{\Delta}{\lambda} u_{dx}^{(1)} = \frac{1}{\gamma_1} Z_d^{(1)}, \tag{17}$$

and Poisson's equation gives the linear dispersion relation

$$\lambda^2 = \tfrac{5}{3}\sigma_d + (1+\gamma_1)^{-1}, \tag{18}$$

where

$$\gamma_1 = \frac{-(1+\beta)(1-S\Psi_0)}{\Psi_0[1+\beta(1-S\Psi_0)]} \text{ and } \Delta = \tfrac{5}{3}\sigma_d - \lambda^2.$$

For the next-order of ε, we obtain

$$-\lambda\frac{\partial n_d^{(2)}}{\partial X} + \frac{\partial n_d^{(1)}}{\partial T} + \frac{\partial u_{dx}^{(2)}}{\partial X} + \frac{\partial}{\partial X}(n_d^{(1)} u_{dx}^{(1)}) + \frac{\partial u_{dy}^{(1)}}{\partial Y} + \frac{\partial u_{dz}^{(1)}}{\partial Z} = 0, \tag{19}$$

$$-\lambda\frac{\partial u_{dx}^{(2)}}{\partial X} + \frac{\partial u_{dx}^{(1)}}{\partial T} + u_{dx}^{(1)}\frac{\partial u_{dx}^{(1)}}{\partial X} - \frac{\partial\phi^{(2)}}{\partial X} - Z_d^{(1)}\frac{\partial\phi^{(1)}}{\partial X} + \frac{5}{3}\sigma_d\frac{\partial n_d^{(2)}}{\partial X} - \frac{5}{9}\sigma_d n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial X} = 0, \tag{20}$$

$$\frac{\partial^2\phi^{(1)}}{\partial X^2} - S(\mu+\nu\beta)\phi^{(2)} - \tfrac{1}{2}S^2(\mu+\nu\beta^2)\phi^{(1)2} - n_d^{(2)} - Z_d^{(1)} n_d^{(1)} - Z_d^{(2)} = 0, \tag{21}$$

$$\frac{\partial}{\partial X}\left(\frac{\partial u_{dy}^{(1)}}{\partial Y}\right) = \frac{\lambda}{\Delta}\frac{\partial^2\phi^{(1)}}{\partial Y^2}, \tag{22}$$

$$\frac{\partial}{\partial X}\left(\frac{\partial u_{dz}^{(1)}}{\partial Z}\right) = \frac{\lambda}{\Delta}\frac{\partial^2\phi^{(1)}}{\partial Z^2}, \tag{23}$$

$$Z_d^{(2)} = \gamma_1\phi^{(1)} + \gamma_2\phi^{(1)2}, \tag{24}$$

where

$$\gamma_2 = \frac{-S(1+\beta)^2(1-S\Psi_0)}{2\Psi_0[1+\beta(1-S\Psi_0)]^3}.$$

Eliminating the second-order perturbed quantities and making use of Eqs. (17), (18), (22) (24), we derive the KP equation

$$\frac{\partial}{\partial X}\left\{\frac{\partial\phi^{(1)}}{\partial T} + AB\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial X} + \frac{1}{2}A\frac{\partial^3\phi^{(1)}}{\partial X^3}\right\} + D\left\{\frac{\partial^2\phi^{(1)}}{\partial Y^2} + \frac{\partial^2\phi^{(1)}}{\partial Z^2}\right\} = 0, \tag{25}$$

where

$$A = \frac{\Delta^2}{\lambda},$$

$$B = \frac{1}{2}\left\{\frac{3\lambda^2 - \frac{5}{9}\sigma_d}{\Delta^3} + \frac{(\delta-\beta^2)(\delta-1)}{(\delta+\beta)^2} - \frac{3\gamma_1}{\Delta} - 2\gamma_2\right\},$$

$$D = \frac{\lambda}{2}.$$

Now, the DASWs can be described by the KP Eq. (25). However, this equation contains the lowest-order nonlinearity and dispersion, and consequently can describe a wave of only small amplitude. As the wave amplitude increases, the width and velocity of a soliton deviate from the prediction of the KP equation: the breakdown of the KP equation. To describe the soliton of larger amplitude, the higher-order nonlinear and dispersive effects have to be taken into account. For this end, the higher-order approximation of the reductive perturbation method is a powerful tool. However, this is out of the scope of this paper.

To get the soliton solution of Eq. (25), we introduce the variable

$$\eta = \ell X + mY + nZ - UT, \tag{26}$$

where η is the transformed coordinates with respect to a frame moving with velocity U. ℓ, m, and n are the directional cosines of the wave vector k along the X, Y and Z axes, respectively, so that $\ell^2 + m^2 + n^2 = 1$. Eq. (25) can be integrated, with respect to the variable η and using the vanishing boundary condition for $\phi^{(1)}$ and their derivatives up to second-order for $|\eta| \to \infty$, to yield

$$\frac{d^2\phi^{(1)}}{d\eta^2} = \frac{2h}{A\ell^4}\phi^{(1)} - \frac{B}{\ell^2}\phi^{(1)2}. \tag{27}$$

The one-soliton solution of Eq. (27) is given by

$$\phi^{(1)} = \phi_0 \sec h^2(\eta/\omega), \tag{28}$$

where $\phi_0 = 3h/AB\ell^2$ is the amplitude of the soliton, $\omega = 1/\sqrt{2h/A\ell^4}$ is the width of the soliton and $h = U\ell - D(1-\ell^2)$.

3. Discussion

To determine the stability or the properties of the instability associated with a given plasma equilibrium; we will use a method based on energy considerations. According to this method it is necessary to calculate the change in potential energy of the plasma as a result of a given perturbation [17]. In order to obtain the potential energy (Sagdeev potential)

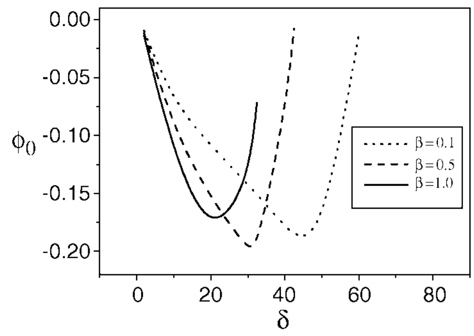

Fig. 1. ϕ_0 vs. δ for $U=0.1$, $\ell=0.9$, $\sigma_d=0.01$.

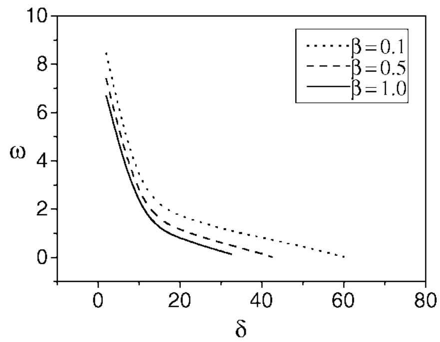

Fig. 3. ω vs. δ for $U=0.1$, $\ell=0.9$, $\sigma_d=0.01$.

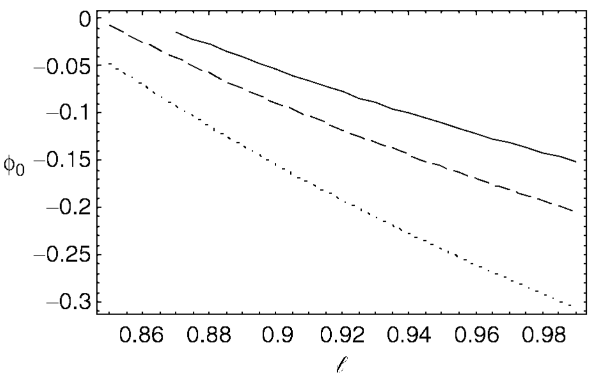

Fig. 2. ϕ_0 vs. ℓ for $U=0.1$, $\delta=15$, $\beta=0.5$, $\sigma_d=0.01$. (dotted line), $\sigma_d=0.05$ (dashed line) and $\sigma_d=0.01$ (solid line).

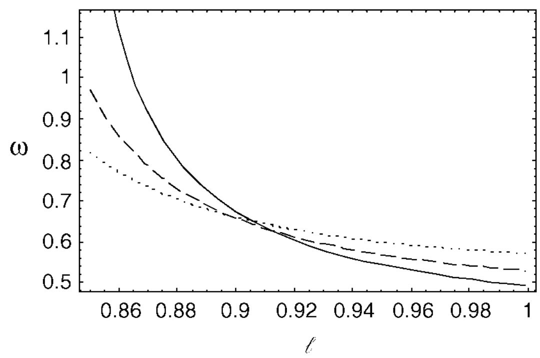

Fig. 4. ω vs. ℓ for $U=0.1$, $\delta=80$, $\beta=0.1$, $\sigma_d=0.01$. (dotted line), $\sigma_d=0.05$ (dashed line) and $\sigma_d=0.1$ (solid line).

integrate Eq. (27) to yield the nonlinear equation of motion as

$$\frac{1}{2}\left\{\frac{\mathrm{d}\phi^{(1)}}{\mathrm{d}\eta}\right\}^2 + V(\phi^{(1)}) = 0, \tag{29}$$

where the Sagdeev potential $V(\phi^{(1)})$ is given by

$$V(\phi^{(1)}) = -\frac{h}{A\ell^4}\phi^{(1)2} + \frac{B}{3\ell^2}\phi^{(1)3}. \tag{30}$$

A necessary condition for the existence of solitary waves is

$$\mathrm{d}^2V(\phi^{(1)})/\mathrm{d}\phi^{(1)2} < 0 \quad \text{for} \quad \phi^{(1)} = 0. \tag{31}$$

A value of $\mathrm{d}^2V(\phi^{(1)})/\mathrm{d}\phi^{(1)2}$ greater than zero predicts the formation of a shock in the plasma. From (30) and (31) we have

$$\mathrm{d}^2V(\phi^{(1)})/\mathrm{d}\phi^{(1)2} = -\frac{2h}{A\ell^4}. \tag{32}$$

Eq. (32) shows that stable solitons will exist when $2h/A\ell^4 > 0$; otherwise stable solitons do not exist in the plasma. It is clear that A and ℓ are greater than zero but h may be less than zero. To be $h > 0$, this condition must satisfy

$$\ell^2 + \frac{2U\ell}{\sqrt{\frac{5}{3}\sigma_d + (1+\gamma_1)^{-1}}} - 1 > 0. \tag{33}$$

From Eq. (33), it is clear that the existence of solitary waves requires a necessary condition depending on ℓ, σ_d and γ_1.

One may ask to what extent the fluid equations used in section 2 are applicable to experimental situations or space plasma observations. At the beginning, we have assumed that the system under investigation is a fully ionized three component dusty plasma consisting of warm variational charged dust grains, hot ions and electrons. Fully ionized means there are no neutrals in the plasma and the term dusty plasma means $d/\lambda_D < 1$, where d is the intergrain distance between dust particles and λ_D is the Debye length. In addition, the plasma is weakly coupled, i.e., the coupling parameter $\Gamma \ll 1$. Mendis [18] cleared that the Saturn's F-ring and supernovae shells satisfy these three conditions; (*i*) there are no neutrals, (*ii*) $d/\lambda_D < 1$ and (*iii*) $\Gamma \ll 1$. Thus our results could be applicable to such space plasma observations. Now, we investigate the solitons in Saturn's F-ring, which have typical values $n_e = 10\,\mathrm{cm}^{-3}$, $T_e = 10 - 100\,\mathrm{eV}$, $n_d < 10\,\mathrm{cm}^{-3}$ and $Z_d = 10 - 100$. However, the values of T_i, T_d, and ℓ were not given in Ref. [18]. So, these values are supposed to have wide ranges to confirm that the behavior of the soliton can be described for possible changes of T_i, T_d, and ℓ. From the given data, we can calculate the value of $\delta = 2 - 90$. We assume the values of $\beta = 0.1$, 0.5, and 1, $\sigma_d = 0.01$, 0.05, and 0.1. The behaviors of the amplitude and the width are displayed in Figs. 1 4. Figure 1 illustrates the dependence of the amplitude ϕ_0 on δ and β. It shows that ϕ_0 increases slowly as δ increases, however nearest a certain value of δ [that we call $\delta_{\max}$], it decreases rapidly. Figure 1 shows also that ϕ_0 increases with β. Figure 2 shows that ϕ_0 increases (decreases) with ℓ (σ_d). From Figures 1 and 2, it is clear that the amplitude has negative sign, thus only the rarefactive solitons can propagate in this system. Figure 3 illustrates the dependence of the width ω on δ and β. It shows that as δ increases ω decreases at different values of β. Figure 4 shows that both of ℓ and σ_d have significant roles in the behavior of the width, i.e., ω decreases with ℓ. It is also seen that ω increases with σ_d in the lower range of ℓ (from 0.85 to ~ 0.91), but ω decreases with σ_d in the higher range of ℓ (from 0.91 to ~ 0.99). The results obtained here agree exactly with those obtained by Xie *et al.* [9] (by neglecting the transverse perturbations and the effect of dust temperature), and that obtained by Duan [19] (by neglecting the effect of dust temperature). Also, it is the first time both the effects of dust temperature and variation of dust charge on the behavior of the nonlinear three-dimensional DASWs in dusty plasma are examined together. Thus, we can consider this study as a modification and a generalization of previous works.

4. Conclusions

In this paper, we have analysed the properties of DASWs in unmagnetized three component dusty plasmas, comprising warm variational charged dust grains, hot ions and electrons. The reductive perturbation method has been used to derive the KP equation (25). It is found that the rarefactive amplitude increases with β and ℓ but it decreases with σ_d. The rarefactive amplitude increases with δ, however, nearest to $\delta_{\max}$ it decreases rapidly. The width decreases with δ, ℓ and σ_d. The conditions for stable soliton are obtained, which must satisfy Eq. (33). On the other hand, the stability of the soliton depends mainly on ℓ, σ_d and γ_1.

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