Higher-order nonlinearity of electron-acoustic solitary waves with vortex-like electron distribution and electron beam

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The nonlinear wave structure of small-amplitude electron-acoustic solitary waves (EASWs) is investigated in a four-component plasma consisting of cold electron fluid, hot electrons obeying vortex-like distribution traversed by a warm electron beam and stationary ions. The streaming velocity of the beam, u_o, plays the dominant role in determining the roots of the linear dispersion relation associated with the system. Using the reductive perturbation theory, the basic set of equations is reduced to a modified Korteweg–de Vries (mKdV) equation. With the inclusion of higher-order nonlinearity, a linear inhomogeneous mKdV type equation with fifth-order dispersion term is derived and the higher-order solution is obtained using a renormalization method. However, both mKdV and mKdV-type solutions present a positive potential, which corresponds to a hole (hump) in the cold (hot) electron number density. The mKdV-type solution has a smaller energy amplitude and a wider width than that of mKdV solution. The dependence of the energy amplitude, the width, and the velocity on the system parameters is investigated. The findings of this investigation are used to interpret the electrostatic solitary waves observed by the Geotail spacecraft in the plasma sheet boundary layer of the Earth's magnetosphere.
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I. INTRODUCTION

Electron-acoustic waves (EAWs) have been of interest since their first description by Fried and Gould[1] during numerical solutions of the linear electrostatic Vlasov dispersion equation in an unmagnetized and homogenous plasma. They noticed, in addition to the Langmuir and ion-acoustic waves, the existence of a heavily damped acoustic-like solution of the dispersion equation. Later, the EAW was discovered experimentally[2,3] in unmagnetized plasma consisting of two electron populations with different temperature and density. The two populations will be referred as "cold" and "hot" electrons.[4,5] It is found that the EAW is an acoustic (electrostatic) wave in which the cold electrons provide the inertia, and the restoring force comes from the pressure of the hot electrons. The ions play the role of the neutralizing background, i.e., the ion dynamics do not influence the EAWs because the EAW frequency is much larger than the ion plasma frequency. The spectrum of the linear EAWs extends up to the cold electron plasma frequency $\omega_{pc}=(4\pi n_{co}e^2/m_e)^{1/2}$, where n_{co} is the unperturbed cold electron number density, e is the magnitude of the electron charge, and m_e is the mass of the electron.[6]

Study of the EAWs propagation plays an important role not only in laboratory experiment but also in space plasmas. Satellite measurements in the auroral and other regions of the magnetosphere have shown bursts of broadband electrostatic noise (BEN) emissions. The associated electric field intensities of these BEN ranges are from few μV/m to 100 mV/m.[7–13] The observations of solitary waves in the auroral zone suggest that there are two classes of solitary waves: the first kind is associated with electron beams and the other is associated with ion beams.[12] Herein, the first kind will be focused. Solitary waves associated with electron beams were first observed by Geotail,[9–11] then by FAST[12] and later by Polar[13] spacecrafts. The signature of these observations is found to display a nonlinear behavior. In particular, BEN is found to have wave forms of solitary bipolar electric field pulses which are called electrostatic solitary waves (ESWs). The ESW widths in time are in range from a few milliseconds to a few tens of milliseconds.[9,10] This time scale suggests that ESWs are related to electron dynamics rather than ions. The contribution of field-aligned electron beams to the generation of high-frequency spectra was suggested by Parks *et al.*[14] Onsager *et al.*[15] described the correlation of BEN with the high-energy electron component in the absence of ion flows. It is also found that these solitary waves in the plasma sheet boundary layer (PSBL) are either an electron hole (EH) or an EAW propagating with velocities of a few thousand km/s. They have a one-dimensional spatial structure with a very small electric field, about a few μV/m.[9] These ESWs are excited by a bump-on-tail weak electron beam instability that can form relatively small electrostatic potentials moving with the electron beam. These electrostatic potentials are formed by high-energy electrons in the nonlinear stage of electron beam instabilities reproduced in computer simulations.[11,12] On the other hand, these electrostatic

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potentials are close to the Bernstein–Green–Kruskal[16] (BGK) equilibrium, formed the resonant and nonresonant plasma screenings of bunched electrons trapped by a potential pulse moving in a plasma.[17–20]

Several theoretical attempts have been made to explain the observed BENs in different regions of the Earth's magnetosphere.[4–15,21,22] Berthomier *et al.*[4] studied electron-acoustic (EA) solitons in an electron-beam plasma system with isothermal hot electrons. It was found that the introduction of an electron beam in such plasma allows the existence of new EA solitons with velocity related to the beam velocity. Also, the second electron population modifies the topology of the roots of the linear dispersion relation in the phase velocity space. Singh and Lakhina[21] studied the generation of BEN by EAWs in a four-component unmagnetized plasma. They applied the linear theory to study the stability and growth rate of the EAWs in three different regions of the magnetosphere. Their study explained the features of BEN in the dayside auroral zone, PSBL, and polar cusp regions.

On the other hand, in practice, the hot electrons may not follow a Maxwellian distribution due to the formation of phase space holes caused by the trapping of hot electrons in a wave potential. Accordingly, in most space plasmas, the hot electrons follow the trapped/vortex-like distribution.[23,24] The electron trapping is observed not only in space plasmas, but also in laboratory experiments.[25,26] Little attention has been paid to study the EAWs via vortex-like electron distribution. Mamun and Shukla[5] made the first attempt to study the nonlinear propagation of one-dimensional EAWs in an unmagnetized plasma composed of a cold electron fluid, hot electrons obeying a vortex-like distribution, and stationary ions. It was shown that the amplitude (width) of the EAWs increases (decreases) with the trapped electron temperature. They applied their theoretical model to interpret the BEN emissions observed in the auroral dayside[8] of the Earth's magnetosphere. However, this region mostly consists of a magnetized plasma with three-dimensional structure solitary waves.[12]

Since EAWs can be excited by electron and laser beams, the electron beams, in addition to the two electron populations are considered to be the main energy source for the excitation of the wave mode.[27] When the beam energy is sufficiently large, the nonlinear and dispersive effects are competing to produce EAWs, which are stationary in their comoving reference frame.[28] However, investigations of small-amplitude EAWs are usually described by Korteweg–de Vries (KdV) or modified KdV (mKdV) equations. These equations contain the lowest-order nonlinearity and dispersion, and consequently can describe only waves of small amplitude. If the wave amplitude or the width deviates significantly from the prediction of the KdV (mKdV) equation, the higher-order nonlinear and dispersion effects must be included to describe such waves accurately. For this end, the higher-order approximation of the reductive perturbation theory has been known to be a powerful tool.[29] Our objective here is to propose a four-component plasma model consisting of cold electron fluid, hot electrons obeying a trapped/vortex-like distribution, warm electron beam, and stationary ions taking into account the effects of higher-order nonlinear and dispersion terms to interpret the observed BENs that formed ESWs in the PSBL of the Earth's magnetosphere.[9–11]

This paper is organized as follows. In Sec. II, we present the basic set of fluid equations governing our plasma model. The nonlinear EAWs are investigated through the derivation of a mKdV equation for the first-order perturbed potential and linear inhomogeneous mKdV type equation for the second-order perturbed potential. In Sec. III, we apply the renormalization method[30] to obtain the stationary solutions of these equations. Section IV is devoted to the discussion and the conclusion.

II. BASIC EQUATIONS

Let us consider an infinite homogeneous, unmagnetized and collisionless plasma consisting of a mixture of cold electron fluid, hot electrons obeying a trapped/vortex-like distribution and stationary ions traversed by a warm electron beam. The normalized one-dimensional basic equations are written as[4]

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}[n_j u_j] = 0, \tag{1}$$

$$\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + \alpha_h \left[3\sigma_j n_j \frac{\partial n_j}{\partial x} - \frac{\partial \phi}{\partial x} \right] = 0. \tag{2}$$

In Eqs. (1) and (2), n_j and u_j ($j=c$ for cold electron and b for electron beam) are the densities and velocities of the two fluids. ϕ is the electric potential, x is the space coordinate, and t is the time variable. n_j, u_j, ϕ, x, and t are normalized to equilibrium densities n_{jo}, to EA speed $C_{ea} = (n_{co} K_B T_h / n_{ho} m_e)^{1/2}$, to $K_B T_h / e$, to the hot electron Debye length $\lambda_{Dh} = (K_B T_h / 4\pi n_{ho} e^2)^{1/2}$, and to ω_{pc}^{-1}, respectively, where K_B is the Boltzmann constant. Also, we introduce the following quantities

$$\alpha_h = \frac{n_{ho}}{n_{co}}, \quad \alpha_b = \frac{n_{bo}}{n_{co}}, \quad \gamma = (1 + \alpha_h + \alpha_b)^{-1},$$

$$\sigma_j = \frac{1}{\theta_j \gamma^2}, \quad \theta_c = \frac{T_h}{T_c}, \quad \theta_b = \frac{T_h}{T_b} \alpha_b^2.$$

In the presence of trapped particles, we employ a vortex-like electron distribution of Schamel[23,24] which solves the electron Vlasov equation, i.e.,

$$f_{hf} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(v^2 - 2\phi) \right] \quad |v| > \sqrt{2\phi},$$

$$f_{ht} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\beta(v^2 - 2\phi) \right] \quad |v| \leq \sqrt{2\phi},$$

where $|\beta| = (T_h / T_{ht})$ (the ratio of the free hot electron temperature T_h to the hot trapped electron temperature T_{ht}). It is the parameter determining the number of trapped electrons. v is the normalized hot electron velocity. The hot electron distribution function, $f_h = f_{hf} + f_{ht}$, is continuous in velocity space and satisfies the regularity requirements for an admissible BGK solution.[16,17] It is obvious from this distribution that $\beta = 1$ ($\beta = 0$) represents a Maxwellian (flat-topped) distribution, whereas $\beta<0$ represents a vortex-like excavated trapped electron distribution. Integrating the distribution function over the velocity space, the hot electron number density, n_h can be expressed for $\beta<0$ as[23,24]

$$n_h = I(\phi) + \frac{2}{\sqrt{\pi|\beta|}} W_D(\sqrt{-\beta\phi}), \tag{3a}$$

where $I(x)=[1-erf(\sqrt{x})]\exp(x)$ and $W_D(x)=\exp(-x^2)\times\int_0^x \exp(y^2)dy$. Equation (3a) can be simplified to the form

$$n_h = \exp(\phi) - G(\phi), \tag{3b}$$

where $G(\phi)=\Sigma_{k=1}^{\infty}[2^{(k+1)}b_k(\phi)^{(2k+1)/2}/\Pi(2k+1)]$, $b_k=(1-\beta^k)/\sqrt{\pi}$. This system of equations is closed by Poissons equation

$$\alpha_h\gamma\frac{\partial^2\phi}{\partial x^2} = n_c + n_h + n_b - 1. \tag{4}$$

In order to study the dynamics of small-amplitude EAWs, we employ the reductive perturbation technique[31] to derive the evolution equation describing the system. First, we introduce the stretched coordinates[23] $\xi=\varepsilon^{1/4}(x-\lambda t)$ and $\tau=\varepsilon^{3/4}t$, where ε is a small parameter and λ is the solitary wave velocity to be determined later. The physical quantities appearing in Eqs (1)–(4), $\Psi\equiv[n_c,n_h,n_b,u_c,u_b,\phi]$, are expanded as power series in ε about their equilibrium values as

$$\Psi = \Psi_{(o)} + \sum_{r=1}^{\infty}\varepsilon^{r+1/2}\Psi_r, \tag{5}$$

where

$$\Psi_r = [n_c,n_h,n_b,u_c,u_b,\phi]^T, \quad \Psi_{(o)} = [n_{co},n_{ho},n_{bo},0,u_o,0]^T.$$

The charge neutrality condition in the plasma is always maintained through the relation

$$n_o = n_{co} + n_{ho} + n_{bo}.$$

Applying the stretched coordinates and the relations (5) to the basic set of Eqs (1)–(4) and following the usual procedure of the reductive perturbation theory,[31] the first-order terms yield

$$n_{c1} = -N_1\phi_1, \quad u_{c1} = -\lambda N_1\phi_1/\gamma,$$
$$n_{b1} = -N_2\phi_1, \quad u_{b1} = -\tilde{\lambda}N_2\phi_1/\alpha_b\gamma, \tag{6}$$

and Poisson's equation gives the linear dispersion relation

$$\alpha_h\gamma = N_1 + N_2, \tag{7}$$

where

$$N_1 = \alpha_h\theta_c\gamma Z_1, \quad N_2 = \alpha_h\alpha_b\theta_b\gamma Z_2,$$

$$Z_1 = (\theta_c\lambda^2 - 3\alpha_h)^{-1}, \quad Z_2 = (\theta_b\tilde{\lambda}^2 - 3\alpha_h\alpha_b^2)^{-1};$$

$$\tilde{\lambda} = \lambda - u_o.$$

Equation (7) is a fourth-order algebraic equation in λ, from which one can get the wave velocity λ. From this equation, one can observe that λ is independent of β and it agrees exactly with that obtained previously by Berthomier *et al.*[4] and Mamun and Shukla.[5] Although Mamun and Shukla[5] considered $\lambda=1$ for describing observed BEN in the auroral dayside[8] of the Earth's magnetosphere, EAWs propagate with a supersonic velocity; i.e., greater than C_{ea}.[9–11] In this model, the introduction of the warm electron beam with the inclusion of the temperature of each species changes the topology of the root of Eq. (7) and overcomes this discrepancy. Figure 1 shows the dependency of the positive roots of Eq. (7) on u_o for two values of θ_b. It is obvious that λ increases as u_o increases. Singh and Lakhina[21] predicted that the maximum growth rate of the EAWs took place at $u_o=2.2$, after which Eq. (7) will admit three different wave velocities. Moreover, λ is affected by θ_b variation but it does not affect significantly by θ_c change. The parameters are chosen in this paper to compare our results with the observed ESW in the PSBL region of the Earth's magnetosphere. This region is selected as the appropriate one because it contains mostly electrostatic waves that has one-dimensional structures.[9–11]

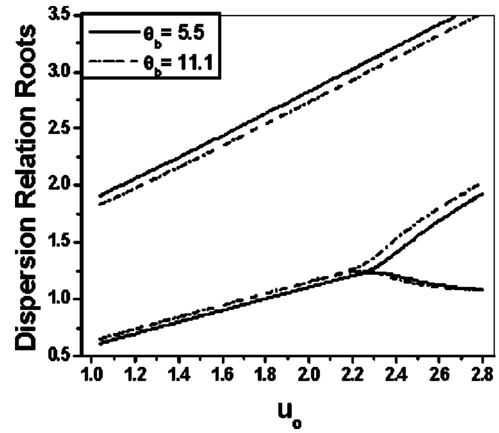

FIG. 1. The dispersion relation roots against u_o for two different values of θ_b, where $\alpha_h=5$, $\alpha_b=0.33$, $\theta_c=500$.

If we consider the next order in ε, we obtain a system of equations in the second-order perturbed quantities. Solving this system with the aid of Eqs. (6) and (7), we finally obtain the mKdV equation

$$\tilde{K}(\phi_1)\phi_1 = \frac{\partial\phi_1}{\partial\tau} + \frac{4}{3}b_1A\frac{\partial\phi_1^{3/2}}{\partial\xi} + A\frac{\partial^3\phi_1}{\partial\xi^3} = 0, \tag{8}$$

where

$$A = \alpha_h\gamma[2(\lambda\theta_cZ_1N_1 + \tilde{\lambda}\theta_bZ_2N_2)]^{-1}.$$

The second-order perturbed quantities can be calculated, with the aid of Eq. (8), as

$$n_{c2} = -N_1\left[\phi_2 - 2\lambda\theta_cZ_1A\left(\frac{4b_1}{3}\phi_1^{3/2} + \frac{\partial^2\phi_1}{\partial\xi^2}\right)\right], \tag{9a}$$

$$u_{c2} = -(N_1/\gamma)\left[\lambda\phi_2 + (1-2\lambda^2\theta_cZ_1)\times A\left(\frac{4b_1}{3}\phi_1^{3/2} + \frac{\partial^2\phi_1}{\partial\xi^2}\right)\right], \tag{9b}$$

$$n_{b2} = -N_2\left[\phi_2 - 2\tilde{\lambda}\theta_b Z_2 A\left(\frac{4b_1}{3}\phi_1^{3/2} + \frac{\partial^2\phi_1}{\partial\xi^2}\right)\right], \qquad (9c)$$

$$u_{b2} = -(N_2/\gamma\alpha_b)\left[\tilde{\lambda}\phi_2 + (1-2\tilde{\lambda}^2\theta_b Z_2) \times A\left(\frac{4b_1}{3}\phi_1^{3/2} + \frac{\partial^2\phi_1}{\partial\xi^2}\right)\right]. \qquad (9d)$$

For $O(\varepsilon^2)$, we obtain a system of equations in the third-order perturbed quantities. Solving this system with the aid of Eqs. (6), (7), and (9), we finally obtain the linear inhomogeneous mKdV (mKdV-type) equation for the second-order perturbed potential ϕ_2:

$$\tilde{L}(\phi_1)\phi_2 = \frac{\partial\phi_2}{\partial\tau} + 2b_1 A\frac{\partial}{\partial\xi}(\phi_1^{1/2}\phi_2) + A\frac{\partial^3\phi_2}{\partial\xi^3} = (4b_1^2A^2B + C)\phi_1\frac{\partial\phi_1}{\partial\xi} + A^2B\left\{\frac{\partial^5\phi_1}{\partial\xi^5} + b_1\left[\frac{3}{\phi_1^{1/2}}\frac{\partial\phi_1}{\partial\xi}\frac{\partial^2\phi_1}{\partial\xi^2} - \frac{1}{2\phi_1^{3/2}}\left(\frac{\partial\phi_1}{\partial\xi}\right)^3 + 4\phi_1^{1/2}\left(\frac{\partial^3\phi_1}{\partial\xi^3}\right)\right]\right\}, \qquad (10)$$

where

$$B = (A/\alpha_h\gamma)[N_1\theta_c Z_1(1-4\lambda^2\theta_c Z_1) + N_2\theta_b Z_2(1-4\tilde{\lambda}^2\theta_b Z_2)],$$

$$C = (A/\alpha_h\gamma)\{N_1[1+3\theta_c\alpha_h Z_1^2(\alpha_h+\lambda^2\theta_c)] + N_2[1+3\theta_b\alpha_h Z_2^2(\alpha_h\alpha_b^2+\tilde{\lambda}^2\theta_b)]\}.$$

Thus we have reduced the basic Eqs. (1)–(4) to a nonlinear mKdV equation for ϕ_1, Eq. (8), and a linear inhomogeneous differential equation for ϕ_2, Eq. (10), for which the source term, the right-hand side of Eq. (10), is described by a known function ϕ_1.

III. THE STATIONARY SOLUTION

In the preceding section, it has been shown that the higher-order approximations are given by mKdV, Eq. (8), and mKdV type, Eq. (10), but these equations contain resonant terms that give rise to secular solutions.[29–33] So, to eliminate this secular behavior, we adopt the renormalization method.[31–33] According to this method, Eq. (8) is added to Eq. (10) to yield

$$\tilde{K}(\phi_1)\phi_1 + \sum_{n\geq 2}\varepsilon^n\tilde{L}(\phi_1)\phi_n = \sum_{n\geq 2}\varepsilon^n S_n, \qquad (11)$$

where S_2 represents the right-hand side of Eq. (10). We add

$$\sum_{n\geq 1}\varepsilon^n\delta\nu\frac{\partial\phi_1}{\partial\xi}$$

to both sides of Eq. (11), where $\delta\nu$ is given by a power series in ε, $\delta\nu = \varepsilon\nu_1 + \varepsilon_2^2\nu + \cdots$, with coefficients to be determined later. The crucial point in this procedure is that on the left-hand side $\delta\nu$ is not expanded, while on the right-hand side it is expanded, so that the ν_n are determined successively to cancel out the resonant term in S_n. Then Eqs. (8) and (10) are transformed to

$$\frac{\partial\tilde{\phi}_1}{\partial\tau} + \frac{4b_1}{3}A\frac{\partial\tilde{\phi}_1^{3/2}}{\partial\xi} + A\frac{\partial^3\tilde{\phi}_1}{\partial\xi^3} + \delta\nu\frac{\partial\tilde{\phi}_1}{\partial\xi} = 0 \qquad (12)$$

and

$$\frac{\partial\tilde{\phi}_2}{\partial\tau} + 2Ab_1\frac{\partial(\tilde{\phi}_1^{1/2}\tilde{\phi}_2)}{\partial\xi} + A\frac{\partial^3\tilde{\phi}_2}{\partial\xi^3} + \delta\nu\frac{\partial\tilde{\phi}_2}{\partial\xi} = S_2(\tilde{\phi}_1) + \nu_1\frac{\partial\tilde{\phi}_1}{\partial\xi}. \qquad (13)$$

The upper sign on ϕ_1 and ϕ_2 indicates the renormalization variables.

Let us introduce the variable

$$\eta = \xi - (\nu + \delta\nu)\tau, \qquad (14)$$

where the parameter ν is related to the Mach number $M = V/C_{ea}$ by[32–34]

$$\nu + \delta\nu = M - 1 = \Delta M. \qquad (15)$$

Here V is the soliton velocity.

Equations (12) and (13) can be integrated with respect to the variable η and using the vanishing boundary conditions for $\tilde{\phi}_1(\eta)$ and $\tilde{\phi}_2(\eta)$ and their derivatives up to second order for $|\eta|\to\infty$, yield

$$\frac{d^2\tilde{\phi}_1}{d\eta^2} + \left(\frac{4b_1}{3}\tilde{\phi}_1^{1/2} - \frac{\nu}{A}\right)\tilde{\phi}_1 = 0, \qquad (16)$$

$$\frac{d^2\tilde{\phi}_2}{d\eta^2} + \left(2b_1\tilde{\phi}_1^{1/2} - \frac{\nu}{A}\right)\tilde{\phi}_2 = \frac{1}{A}\int_{-\infty}^{\eta}\left[S_2(\tilde{\phi}_1) + \nu_1\frac{d\tilde{\phi}_1}{d\eta}\right]d\eta. \qquad (17)$$

The one-soliton solution of Eq. (16) is given by

$$\tilde{\phi}_1(\eta) = \phi_{1m}\,\mathrm{sech}^4(\eta w), \qquad (18)$$

where the amplitude $\phi_{1m} = (15\nu/16b_1A)^2$ and the width $w^{-1} = 4\sqrt{A/\nu}$. Equation (18) permits a compressive soliton only. This corresponds to a hole (hump) in the cold (hot) electron number density that agrees with Mamun and Shukla.[5]

Using the expression (18) for $\tilde{\phi}_1(\eta)$, the source term of Eq. (17) becomes

$$\frac{1}{A}\int_{-\infty}^{\eta}\left[S_2(\tilde{\phi}_1) + \nu_1\frac{d\tilde{\phi}_1}{d\eta}\right]d\eta = \frac{\phi_{1m}}{2A}\{2[\nu_1 + \nu^2B]\mathrm{sech}^4(\eta w) + \phi_{1m}C\,\mathrm{sech}^8(\eta w)\}.$$

Thus, to cancel out the resonant terms in $S_2(\tilde{\phi}_1)$, we have to put

$$\nu_1 = -\nu^2 B. \qquad (19)$$

To solve Eq. (17), we define a new independent variable

$$\mu = \tanh(\eta w). \tag{20}$$

Equation (17) thereby becomes

$$\frac{d}{d\mu}\left\{(1-\mu^2)\frac{d}{d\mu}\tilde{\phi}_2\right\} + \left\{5(5+1) - \frac{4^2}{1-\mu^2}\right\}\tilde{\phi}_2 = \frac{(15)^4 C\nu^3}{2(8b_1A)^4}(1-\mu^2)^3. \tag{21}$$

The two independent solutions of the homogeneous part of Eq. (21) are given by the associated Legendre functions of the first and second kind:

$$P_5^4(\mu) = 945\mu(1-\mu^2)^2,$$

$$Q_5^4(\mu) = \frac{945}{2}\mu\big(1-\mu^2\big)^2 \ln\Big(\frac{1+\mu}{1-\mu}\Big) - 63(8 - 25\mu^2 + 15\mu^4) + \frac{96}{(1-\mu^2)} + \frac{24(1+\mu^2)}{(1-\mu^2)^2},$$

and thus the complementary solution of Eq. (21) is given by

$$\tilde{\phi}_{2c} = C_1 P_5^4(\mu) + C_2 Q_5^4(\mu). \tag{22}$$

Here the first term is the secular one, which can be eliminated by renormalizing the amplitude. Also, the constant $C_2=0$ as a result of the vanishing boundary condition for $\tilde{\phi}_2(\eta)$. as $|\eta| \to \infty$

Using the method of variation of parameters, the particular solution of Eq. (21) can be written as

$$\tilde{\phi}_{2p} = L_1(\mu)P_5^4(\mu) + L_2(\mu)Q_5^4(\mu), \tag{23}$$

where L_1 and L_2 are given by

$$L_1(\mu) = -\int \frac{T(\mu)Q_5^4(\mu)}{(1-\mu^2)W(P_5^4,Q_5^4)}d\mu,$$
$$L_2(\mu) = \int \frac{T(\mu)P_5^4(\mu)}{(1-\mu^2)W(P_5^4,Q_5^4)}d\mu, \tag{24}$$

where

$$T(\mu) = \frac{(15)^4 C\nu^3}{2(8b_1A)^4}(1-\mu^2)^3 = D(1-\mu^2)^3$$

and the Wronskian W is given by

$$W(P_5^4,Q_5^4) = P_5^4\frac{dQ_5^4}{d\mu} - Q_5^4\frac{dP_5^4}{d\mu} = \frac{945\times 348}{(1-\mu^2)}. \tag{25}$$

Substituting for P_5^4 and Q_5^4 into Eq. (24) and carrying out the integrations, we obtain

$$L_1(\mu) = \frac{-D}{945\times 348}\left[\frac{-315}{8}(1-\mu^2)^6 \ln\frac{1+\mu}{1-\mu} - \frac{1221}{4}\mu + \frac{3335}{4}\mu^3 - \frac{2529}{2}\mu^5 + \frac{2079}{2}\mu^7 - \frac{1785}{4}\mu^9 + \frac{315}{4}\mu^{11}\right],$$

$$L_2(\mu) = \frac{-D}{12\times 348}(1-\mu^2)^6.$$

Finally, we can obtain the solution of Eq. (21)

$$\tilde{\phi}_2(\mu) = \tilde{\phi}_{2p}(\mu) = \frac{D}{6}(1-\mu^2)^2\left[1 - \frac{1}{2}(1-\mu^2)\right]. \tag{26}$$

In terms of the old variable η, the stationary solution for the EAW is given by

$$\tilde{\phi}(\eta) = \tilde{\phi}_1 + \tilde{\phi}_2 = \left(\frac{15\nu}{16b_1A}\right)^2 \operatorname{sech}^4(\eta w) + \frac{D}{6}\operatorname{sech}^4(\eta w) \times\left[1 - \frac{1}{2}\operatorname{sech}^2(\eta w)\right], \tag{27}$$

where ν and the modified width are given by

$$\nu = \Delta M[1 + B\Delta M] \text{ and } w^{-1} = 4\sqrt{\frac{A}{\Delta M}}\left[1 - \frac{1}{2}B\Delta M\right].$$

Using the renormalization procedure, Tiwari and Sharma[35] studied ion acoustic waves in a plasma with two-temperature ions, and later Yashvir *et al.*[36] investigated the same wave in an ion-beam plasma system. Because both of the two systems contain isothermal electrons, the evaluation equations are the standard KdV and KdV-type equations for the first and second perturbed potential respectively. They obtained a solution for the KdV-type equation similar to Eq. (27). that nominated as "dressed soliton." It was shown that the dressed soliton has a very good agreement with the predicted exact solution than the solution of the KdV does.

Moreover, the present method can be extended to include the N-soliton case. The N-soliton solution of the renormalized mKdV equation (8) and the renormalized linear inhomogeneous equation (10) can be written as $\tau \to \infty$ as[30]

$$\tilde{\phi}_1 \to \sum_{i=1}^{N} \frac{225}{256(b_1A)^2}\Delta M_i^2[1 + 2B\Delta M_i] \times \operatorname{sech}^4(\eta w_i),$$

$$\tilde{\phi}_2 \to \sum_{i=1}^{N} \frac{50625C}{49152(b_1A)^4}\Delta M_i^3[1 + 3B\Delta M_i] \times \operatorname{sech}^4(\eta w_i)\left[1 - \frac{1}{2}\operatorname{sech}^2(\eta w_i)\right],$$

where w_i^{-1} is related to the velocity V_i of the ith soliton by

$$w_i^{-1} = 4\sqrt{\frac{A}{\Delta M_i}}\left[1 - \frac{1}{2}B\Delta M_i\right]; \quad \Delta M_i = \frac{V_i}{C_{ea}} - 1.$$

IV. DISCUSSION AND CONCLUSION

We have considered a four-component plasma model consisting of a cold electron fluid, a warm electron beam, hot electrons obeying a trapped/vortex-like distribution, and stationary ions. Using the reductive perturbation theory, we studied the combined effects of electron beam and higher-order nonlinearity on the nonlinear EAWs. The basic set of fluid equations describing the system leads, at the lowest order of perturbation theory, to a mKdV equation, Eq. (8). The mKdV equation permits a compressive soliton only which corresponds to a hole (hump) in the cold (hot) electron number density that agrees with Mamun and Shukla.[5] For a better accuracy, the higher-order nonlinear and dispersion terms have been included and the linear inhomogeneous mKdV-type equation, Eq. (10), with fifth-order dispersion term is employed. The higher-order solution (27) is obtained using the renormalization method. Though it is predicted that the wave amplitude may increase and consequently the width decreases with the inclusion of higher-order nonlinearity,[32,33] Fig. 2 contradicts this. It shows the variation of the predicted energy E_1 (corresponding to $\tilde{\phi}_1$) and E (corresponding to $\tilde{\phi} = \tilde{\phi}_1 + \tilde{\phi}_2$) against η for different system parameter changes. As mentioned in Sec. II, the parameters chosen are corresponding to ESW observed in the PSBL of the Earth's magnetosphere.[9–11,14,15,21,22] They lead to $\lambda_{Dh} \simeq 192$ m. Using the formula[5,8] $E_1 = \phi_{1m}(K_B T_h / e\lambda_{Dh}) \simeq \phi_{1m}(1000/192)$ V/m, the solution (18) can be transformed into the energy wave form. A similar transformation is used to transform $\tilde{\phi}$ to E. Figure 2 shows that as u_o increases, the amplitude E_1 decreases but its width increases whatever $u_o >$ or $< \lambda$. Also, the same effect can be observed if $|\beta|$, α_b, or θ_b increases. The case of three-component plasma, i.e., without electron beam, corresponds to the higher amplitude and the smaller width (thick curve) in Fig. 2(e). However, E_1's are always compressive solitons that correspond to a hole (hump) in the cold (hot) electron number density E represent compressive solitons also but with negative energy tails. In Fig. 2, two conditions must be fulfilled. According to the principal rule of the reductive perturbation theory,[31] the following condition must be satisfied:

$$\frac{|\tilde{\phi}_2|}{|\tilde{\phi}_1|} \leqslant 1. \qquad (28)$$

Thus, the negative energy values appearing in Fig. 2(d) are corresponding to higher negative contribution of $\tilde{\phi}_2$ in the total $\tilde{\phi}$. According to condition (28), these values are forbidden. Moreover, the second condition is that the width of both E_1 and E must be non-negative for the soliton existence. However, Omura *et al.*[10,11] considered the case that λ is slower than u_o in their numerical study of ESW in the PSBL. Here both $\lambda >$ and $< u_o$ are considered here to cover a wide range of u_o. For, $\lambda < u_o$, the amplitude E decreases but its width increases by increasing u_o with the allowance of compressive soliton only. Though, for $\lambda > u_o$, E increases and its width decreases by increasing u_o, the forbidden negative values in the energy tail Fig. 2(d) will disappear when u_o comes closer to λ. Moreover, as $|\beta|$, α_b, or θ_b increases, E decreases but its width increases as in the case of E_1. On the other hand, the contribution of $\tilde{\phi}_2$ to $\tilde{\phi}$ is usually appeared as a decrease in $\tilde{\phi}_1$ for any system parameter changes except for $\alpha_b < 0.03171$ where it increases $\tilde{\phi}_1$. Thus, it is obvious that the beam parameter changes significantly the ESW features. Also, the inclusion of higher-order nonlinear and dispersion terms modifies the EAWs structures. The previous study of

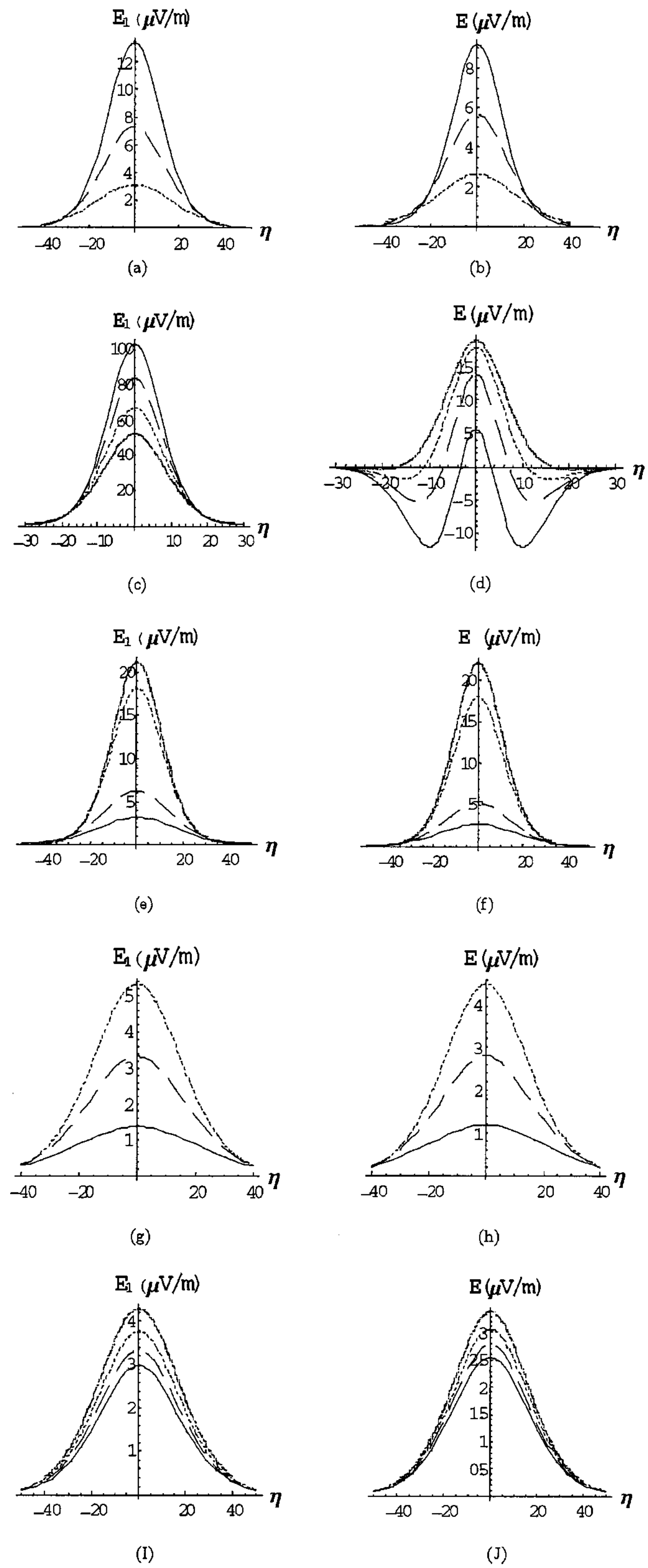

FIG. 2. Plot of E_1 (a, c, e, g, I) and E (b, d, f, h, J) against η. In (a) $\alpha_b = 0.33$, $\theta_b = 5.5$, $\lambda = 1.77$, $\beta = -0.75$, $u_o = 1.78$ (solid), 1.79 (dashed), 1.8 (dotted); (c) $\lambda = 1.8$, $u_o = 1.74$ (solid), 1.75 (dashed), 1.76 (dotted), 1.77 (thick curve); (e) $\theta_b = 50\,\alpha_b^2$, $\lambda = 1.77$, $u_o = 1.8$, $\alpha_b = 0.33$ (solid), 0.25 (dashed), 0.04 (dotted), no beam (thick curve); (g) $\alpha_b = 0.33$, $\lambda = 1.77$, $u_o = 1.8$, $\theta_b = 6.06$ (solid), 5.45 (dashed), 4.95 (dotted); (I) $\alpha_b = 0.33$, $\theta_b = 5.5$, $\lambda = 1.77$, $u_o = 1.8$, $\beta = -0.8$ (solid), -0.7(dashed), -0.6 (dotted), -0.5 (thick curve), the remainder parameters in each graph are chosen as in (a) with $\alpha_c = 5$, $\theta_c = 500$, $\nu = 0.1$. Figures (b, d, f, h, J) show the variation of E with the same parameter choice in (a, c, e, g, I), respectively.

the ESWs observed in the PSBL showed that the associated electric field is ranged from few microvolts per meter to 100 μV/m.[9–11] Thus, our present model has a good agreement with the Geotail registered data of the PSBL. Moreover, this model can be considered as a generalization of the simple three-component plasma studied by Mamun and Shukla[5] by inclusion of the fluid temperatures, electron beam, and the higher-order nonlinearity.

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