Dust-ion-acoustic solitons with transverse perturbation

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The ionization source model is considered, for the first time, to study the combined effects of trapped electrons, transverse perturbation, ion streaming velocity, and dust charge fluctuations on the propagation of dust-ion-acoustic solitons in dusty plasmas. The solitary waves are investigated through the derivation of the damped modified Kadomtsev–Petviashivili equation using the reductive perturbation method. Conditions for the formation of solitons as well as their properties are clearly explained. The relevance of our investigation to supernovae shells is also discussed.
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I. INTRODUCTION

It is well known that dust particles are common in the universe and they represent much of the solid matter in it. Dust particles often contaminate fully ionized or partially ionized gases and form so-called "dusty plasma," which occur frequently in nature. In astrophysics, in the early 1930s, dust was shown to be present in the interstellar clouds where it appears as a selective absorption of stellar radiation (interstellar reddening). Dust particles play a very important role in the solar system, in cometary tails, in planetary rings, and also in the evolution of the solar system from its solar nebula to its present form. Dust particles are also found in environments such as production processes, flames, rocket exhausts, and many laboratory experiments.[1] The dust particles are of micrometer or submicrometer size, and the mass of the dust particles is very large. Due to the presence of such heavy particles, the plasma normal mode could be modified. In particular, the ion-acoustic waves are one of the modified normal modes, which are called dust-ion-acoustic (DIA) waves. Shukla and Silin[2] were the first to report theoretically the existence of DIA waves in unmagnetized dusty plasmas. The DIA waves have been experimentally observed in laboratory experiment by Barkan *et al.*[3] It was noted that in studying collective effects involving charged dust particles in dusty plasmas, one generally assumes that the dust grains behave like point charges. In fact, the charges on the dust particles are not constant, because the imbalance of electron current and ion current flowing through the grain surface causes charge fluctuation. On the other hand, one can consider dusty plasma is always an open system because the currents of electrons and ions flowing onto the dust grains (as well as the energy flows) should be maintained by external sources of the plasma particles and the energy. The dissipation rate is high. Therefore, there is a tendency to self-organization and to formation of long-living nonlinear dissipative and coherent structures in a plasma such as shock waves, solitons, cavitons, collapsing cavities, etc.[4] Both shocks and solitons in dusty plasmas can be formed by different means. These are not necessarily restricted to the mode excitation due to instabilities, or an external forcing, but can also be a regular collective process analogous to the shock wave generation in gas dynamics. The anomalous dissipation in dusty plasmas, which originates from the dust particles charging process, makes possible existence of a new kind of shocks related to this dissipation.[5,6] In the absence of dissipation (or if the dissipation is weak at the characteristic dynamical time scales of the system) the balance between nonlinear and dispersion effects can result in the formation of symmetrical solitary waves—a soliton. Investigation of the anomalous dissipation is especially interesting at the ion-acoustic time scales. The charging processes at these time scales are usually not in equilibrium and, hence, the role of anomalous dissipation might be crucial.[5,7] So far, study of nonlinear structures at ion-acoustic time scales (in dusty plasma) was mostly related to shocks.[5,6,8,9] There has also been an experimental investigation of DIA solitons.[10] The first theoretical study of DIA solitons in dusty plasmas[11] used an approximation neglecting absorption and scattering of electrons and ions by microparticles. These processes, resulting in the anomalous dissipation, make the existence of "pure" steady-state nonlinear structures impossible.[12] Later, the influence of the anomalous dissipation on DIA solitons was studied by Popel *et al.*[12] On the other hand, they investigated the evolution of the solitonlike perturbations in dusty plasma, taking into account the dissipation processes and trapped electrons. It was found that the properties of the compressive solitons

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with the trapped electrons are very different from those with the Boltzmann electrons. They related the possibility of existence of the solitons to the fact that in case of the presence of trapped electrons the width of the region of the Mach number (for which soliton solution are possible) is much wider than in the case of Boltzmann electrons. During its evolution the soliton is slowed down. Thus in the case of Boltzmann electrons, the soliton leaves the region of the Mach number inherent in solitons (which is rather narrow for the case of Boltzmann electrons) very soon, and the soliton transforms to the shocklike perturbation. Consequently, for the existence of the damped solitons the perturbation should have an initial form, so that it would allow the presence of both free and trapped electrons. Otherwise, there is a possibility of an appearance of DIA shocks in dusty plasmas. El-Labany *et al.*[13] studied the effects of trapped electron temperature, dust charge variation, and grain radius on the nonlinear DIA waves in dusty plasma having trapped electrons. It has been shown that the nonlinear DIA waves damp waves and these waves are governed by a damped modified Korteweg–de Vries equation. It was found that only compressive DIA solitons can propagate in dusty plasmas with trapped electrons. The amplitude and the width of the solitons depend mainly on the trapped electron temperature, dust charge variations, and grain radius. The existence of the solitons is independent of the trapped electron temperature. Finally, it is necessary to mention that the form of the initial perturbation could be important from the viewpoint what we want to observe, shocks or solitons. For example, both DIA shocks[9] and DIA solitons[10] were observed in a double plasma device at the Institute of Space and Astronautical Science (Japan). In both experiments the plasma conditions were (almost) the same but the difference was in initial perturbation.

The aims of this paper are the following: (i) determine the condition when the existence of (quasi) steady-state solitons is possible in case of a weak dissipation, (ii) investigate the combined effects of trapped electrons, transverse perturbation, ion streaming velocity, dust charge fluctuations, variation of the ion density, ion momentum dissipation, and the source of plasma particles on the propagation characteristics of the DIA solitons, and (iii) describe the DIA solitons that may appear in supernovae shells.

This paper is organized as follows: The basic equations governing the dynamics of the nonlinear DIA solitons are presented in Sec. II. In Sec. III, the condition under which the solitons can be formed is obtained. The evolution of the nonlinear DIA solitons is described through the derivation of the damped modified Kadomtsev–Petviashivili equation and its approximate solution is obtained. In Sec. IV, the relevance of our investigation to supernovae shell is discussed. Sec. V is devoted to the conclusions.

II. MODEL

We consider fully ionized, collisionless, unmagnetized dusty plasmas consisting of a mixture of warm positive ions, warm negatively charged dust grains, and nonisothermal electrons. In two dimensions, the basic equations describing the propagation of the DIA solitons are given in dimensionless variables as follows:

for positive ions,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_{ix}) + \frac{\partial}{\partial y}(n_i u_{iy}) = -\nu_{ch} n_i + \nu_i n_e, \tag{1a}$$

$$\frac{\partial}{\partial t}(n_i u_{ix}) + \frac{\partial}{\partial x}(n_i u_{ix}^2) + \frac{\partial}{\partial y}(n_i u_{ix} u_{iy}) + 2\sigma_i n_i \frac{\partial n_i}{\partial x} + n_i \frac{\partial \phi}{\partial x} = -\tilde{\nu} n_i u_{ix}, \tag{1b}$$

$$\frac{\partial}{\partial t}(n_i u_{iy}) + \frac{\partial}{\partial x}(n_i u_{ix} u_{iy}) + \frac{\partial}{\partial y}(n_i u_{iy}^2) + 2\sigma_i n_i \frac{\partial n_i}{\partial y} + n_i \frac{\partial \phi}{\partial y} = -\tilde{\nu} n_i u_{iy}, \tag{1c}$$

for dust grains,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_{dx}) + \frac{\partial}{\partial y}(n_d u_{dy}) = 0, \tag{2a}$$

$$\frac{\partial u_{dx}}{\partial t} + u_{dx}\frac{\partial u_{dx}}{\partial x} + u_{dy}\frac{\partial u_{dx}}{\partial y} + \frac{2\sigma_d}{\mu_d}\frac{\partial n_d}{\partial x} - \frac{Z_d^{(0)} Z_d}{\mu_d}\frac{\partial \phi}{\partial x} = 0, \tag{2b}$$

$$\frac{\partial u_{dy}}{\partial t} + u_{dx}\frac{\partial u_{dy}}{\partial x} + u_{dy}\frac{\partial u_{dy}}{\partial y} + \frac{2\sigma_d}{\mu_d}\frac{\partial n_d}{\partial y} - \frac{Z_d^{(0)} Z_d}{\mu_d}\frac{\partial \phi}{\partial y} = 0. \tag{2c}$$

The positive ions, dust grains, and electrons are coupled through Poisson's equation,

$$\alpha\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + n_i - Z_d^{(0)} Z_d n_d - n_e = 0. \tag{3}$$

In the dynamical system, some of the electrons are attached to the dust grains to form the charged dust grains, while some of the remaining electrons are bounced back and forth in the potential well, lose energy continuously and, as a result, become ultimately trapped electrons. However, to consider the effect of trapped electrons the following inequality must be satisfied:[12]

$$t_{\mathrm{sol}} \geqslant L_{\mathrm{sol}}/v_{Te},$$

where t_{sol} is the characteristic time of the soliton formation and L_{sol} is the soliton width. The magnitude of t_{sol} is of the order of a few ω_{pi}^{-1} (where $\omega_{pi,e} = \sqrt{4\pi e^2 n_{i,e}/m_{i,e}}$ is the ion and electron plasma frequency), the spatial scale L_{sol} is about several λ_{De}. Thus, $L_{\mathrm{sol}}/v_{Te} \sim \omega_{pe}^{-1}$, and therefore the last inequality normally holds.[12] In this case, the electron density is defined from the Vlasov equation consisting of free and trapped electrons. Following Schamel,[14] the nonisothermality of the plasma is introduced through the electron densities that have the normalized form

$$n_e = 1 + \phi - \frac{4}{3} b \phi^{3/2} + \frac{1}{2}\phi^2 + \cdots. \tag{4}$$

Here $b[=(1-\beta)/\sqrt{\pi}]$ is a constant depending on the temperature parameters of resonant electrons (both free and trapped), and $\beta[=T_{ef}/T_{et}]$ represents the ratio of the free electron temperature T_{ef} to the trapped electron temperature T_{et}.

Equations (1)–(4) are completed by the normalized dust grain charging equation,[15]

$$\frac{dZ_d}{dt} = - L_1 n_e \exp(L_3 Z_d) + L_2 n_i (F_1 - F_2 L_4 Z_d), \quad (5)$$

where

$$L_1 = (n_e^{(0)} r_d^2/\omega_{pi} Z_d^{(0)}) \sqrt{8\pi T_{ef}/m_e},$$

$$L_2 = (n_e^{(0)} r_d^2/\omega_{pi} Z_d^{(0)}) \sqrt{8\pi T_i/m_i},$$

$$L_3 = -(e^2 Z_d^{(0)}/C_p T_{ef}),$$

$$L_4 = -(e^2 Z_d^{(0)}/C_p T_i),$$

$$F_1(u_o) = \frac{\sqrt{\pi}}{4u_o}(1 + 2u_o^2)\mathrm{erf}(u_o) + \frac{1}{2}\exp(-u_o^2),$$

$$F_2(u_o) = \frac{\sqrt{\pi}}{2u_o}\mathrm{erf}(u_o).$$

In Eqs. (1)–(5), n_i, n_d, and n_e are the densities of positive ions, dust grains, and electrons, respectively. u_q $[q=i$ and $d]$ are the velocities of positive ions and dust grains, respectively. ϕ is the electrostatic potential, x and y are the space coordinates, and t is the time variable. ν_{ch} is the frequency of ion recombination on dust particles, ν_i is the plasma ionization frequency, and $\tilde{\nu}$ is the frequency characterizing a loss in ion momentum due to recombination on dust particles and Coulomb elastic collisions between ions and dusts. $\sigma_i[=T_i/T_{ef}]$ and $\sigma_d[=T_d/T_{ef}]$ are the ratios of the temperatures of positive ions T_i and dust grains T_d to the free electron temperature T_{ef}. Z_d denotes to the dust grain charge number. $\mu_d[=m_d/m_i]$ is the ratio of the dust grain mass m_d to the ion mass m_i. $C_p[=r_d \exp(-r_d/\lambda_D)]$ is the capacitance of the spherical dust grains. r_d is the radius of the dust grains. $u_o[=v_o/V_{Ti}]$ is the ion streaming velocity, v_o is the unnormalized ion streaming velocity, and $V_{Ti}[=(T_i/m_i)^{1/2}]$ is the ion thermal velocity. We normalized all physical quantities as follows: The background electron density $n_e^{(0)}$ normalizes the densities, u_q by the ion-acoustic speed $C_s[=(T_{ef}/m_i)^{1/2}]$, ϕ by T_{ef}/e, t by the inverse of the plasma frequency ω_{pi}^{-1}, x and y by the electron Debye length $\lambda_{\mathrm{De}}=(T_{ef}/4\pi e^2 n_i^{(0)})^{1/2}$, ν_{ch}, ν_i, and $\tilde{\nu}$ by the plasma frequency ω_{pi}. Z_d by the unperturbed number of charges residing on the dust grains $Z_d^{(0)}$. The charge neutrality at equilibrium requires that $n_i^{(0)}=Z_d^{(0)} n_d^{(0)} + n_e^{(0)}$, where $n_i^{(0)}$ and $n_d^{(0)}$ are the unperturbed ion and dust number densities, respectively.

III. SOLITON EXISTENCE CONDITION AND NONLINEAR ANALYSIS

Before going to nonlinear development, it is necessary to clarify the condition under which the solitons can propagate in dusty plasma. This condition could be derived from Poisson's equation and the equation describing the dust particle charging but in dimensional forms. When the soliton wave structure has formed, the soliton width $\Delta\zeta$ is described by the following theoretical estimate:

$$\frac{\Delta\zeta}{\rho} \ll 1, \quad (6)$$

where $\rho=(M|\phi_0|/4\pi n_d \nu_d q_d)^{1/3}$, $\zeta=x-Mt$, $M[=V/C_s]$ is the Mach number, V is the soliton speed, ϕ_0 is the soliton amplitude, ν_d is the grain charging rate, and it was given in Ref. 4. When one uses the inequality (6) it is important to determine which terms are more important, i.e., if one considers some nonlinear structure and its characteristic width $\Delta\zeta \ll \rho$, then this nonlinear structure is soliton. Otherwise, if the characteristic scale of the change of the parameters of the structure satisfies the inequality $\Delta\zeta \gg \rho$ then this nonlinear structure is expected to be shock wave.

To investigate the behavior of the small, but finite, amplitude DIA solitons in dusty plasma, we employ the standard reductive perturbation method.[16] According to this method, the independent variables are stretched as[14,17]

$$\xi = \varepsilon^{1/4}(x - \lambda t), \quad \eta = \varepsilon^{1/2} y, \quad \tau = \varepsilon^{3/4}\lambda t, \quad (7a)$$

where ε is a smallness parameter measuring the weakness of the nonlinearity and λ is the wave speed normalized by C_s. The dependent variables are expanded as

$$\Psi = \Psi^{(0)} + \sum_{n=1}^{\infty} \varepsilon^{(n+1)/2}\Psi^{(n)}, \quad (7b)$$

where

$$\Psi = [n_i, n_d, u_{ix}, u_{dx}, \phi, Z_d]^T, \quad (7c)$$

$$\Psi^{(0)} = [\alpha, \delta, u_{ixo}, 0, 0, 1]^T. \quad (7d)$$

where $u_{i,dy}$ are given as

$$u_{i,dy} = \varepsilon^{5/4} u_{i,dy}^{(1)} + \varepsilon^{7/4} u_{i,dy}^{(2)} + \cdots. \quad (7e)$$

We assume that the ion streaming velocity is along the x axis only. Applying the relations (7a)–(7e) to the basic equations (1)–(5) and following the usual procedure of the reductive perturbation method, the lowest-order terms yield (we have assumed that $\nu_{ch} \sim \varepsilon^{3/4}\nu_{cho}$, $\nu_i \sim \varepsilon^{3/4}\nu_{io}$ and $\tilde{\nu} \sim \varepsilon^{3/4}\nu_o$),

$$\frac{S}{\alpha} n_i^{(1)} = \frac{S}{\lambda_1} u_{ix}^{(1)} = \frac{-G}{\delta Z_d^{(0)}} n_d^{(1)} = \frac{-G}{\lambda Z_d^{(0)}} u_{dx}^{(1)} = \frac{-F}{R + (\alpha Q/S)} Z_d^{(1)} = \phi^{(1)}, \quad (8a)$$

$$\frac{S}{\lambda_1}\frac{\partial u_{iy}^{(1)}}{\partial \xi} = \frac{-G}{\lambda Z_d^{(0)}}\frac{\partial u_{dy}^{(1)}}{\partial \xi} = \frac{\partial \phi^{(1)}}{\partial \eta}, \quad (8b)$$

and Poisson's equation gives the following dispersion relation:

$$\frac{\alpha}{S} + \frac{\delta Z_d^{(0)2}}{G} + \frac{\delta R Z_d^{(0)}}{F} + \frac{\alpha\delta Q Z_d^{(0)}}{FS} = 1, \quad (8c)$$

where

$$R = -L_1 - L_1L_3 - \tfrac{1}{2}L_1L_3^2, \quad Q = F_1L_2 - F_2L_2L_4,$$

$$F = -L_1L_3 - L_1L_3^2 - F_2L_2L_4\alpha, \quad S = \lambda_1^2 - 2\sigma_i\alpha,$$

$$G = \lambda^2\mu_d - 2\sigma_d\delta, \quad \lambda_1 = \lambda - u_{ixo}.$$

If we consider the next order in ε, we obtain a system of equations in the second-order perturbed quantities as

$$-\lambda_1\frac{\partial}{\partial\xi}n_i^{(2)} + \lambda\frac{\partial}{\partial\tau}n_i^{(1)} + \alpha\frac{\partial}{\partial\xi}u_{ix}^{(2)} + \alpha\frac{\partial}{\partial\eta}u_{iy}^{(1)} = -\nu_{cho}n_i^{(1)} + \nu_{io}n_e^{(1)}, \tag{9a}$$

$$-\lambda_2\alpha\frac{\partial}{\partial\xi}u_{ix}^{(2)} - u_{ixo}\lambda_1\frac{\partial}{\partial\xi}n_i^{(2)} + \frac{\lambda^2\alpha}{\lambda_1}\frac{\partial}{\partial\tau}u_{ix}^{(1)} + \alpha u_{ixo}\frac{\partial}{\partial\eta}u_{iy}^{(1)} + 2\sigma_i\alpha\frac{\partial}{\partial\xi}n_i^{(2)} + \alpha\frac{\partial}{\partial\xi}\phi^{(2)} = -\frac{\nu_o\lambda\alpha}{\lambda_1}u_{ix}^{(1)}, \tag{9b}$$

$$-\lambda\frac{\partial}{\partial\xi}n_d^{(2)} + \lambda\frac{\partial}{\partial\tau}n_d^{(1)} + \delta\frac{\partial}{\partial\xi}u_{dx}^{(2)} + \delta\frac{\partial}{\partial\eta}u_{dy}^{(1)} = 0, \tag{9c}$$

$$-\lambda\frac{\partial}{\partial\xi}u_{dx}^{(2)} + \lambda\frac{\partial}{\partial\tau}u_{dx}^{(1)} + 2\frac{\sigma_d}{\mu_d}\frac{\partial}{\partial\xi}n_d^{(2)} - \frac{Z_d^{(0)}}{\mu_d}\frac{\partial}{\partial\xi}\phi^{(2)} = 0, \tag{9d}$$

$$Z_d^{(2)} = \frac{1}{F}\left\{\frac{4}{3}bR\phi^{(1)3/2} - R\phi^{(2)} - Qn_i^{(2)}\right\}, \tag{9e}$$

$$\alpha\frac{\partial^2}{\partial\xi^2}\phi^{(1)} = Z_d^{(0)}n_d^{(2)} + \delta Z_d^{(0)}Z_d^{(2)} - n_i^{(2)} + n_e^{(2)}. \tag{9f}$$

Differentiating Eq. (9f) with respect to ξ and inserting Eqs. (8) and (9a)–(9e), we obtain the following damped modified Kadomtsev–Petviashivili (DMKP) equation as

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi^{(1)}}{\partial\tau} + \frac{2}{3}AB\frac{\partial\phi^{(1)3/2}}{\partial\xi} + \frac{1}{2}\alpha A\frac{\partial^3\phi^{(1)}}{\partial\xi^3} + \frac{1}{2}AC\phi^{(1)}\right] + \frac{1}{2}AD\frac{\partial^2\phi^{(1)}}{\partial\eta^2} = 0, \tag{10}$$

where

$$A = \left[\frac{\lambda^2\delta\mu_dZ_d^{(0)2}}{G^2} + \frac{\lambda\lambda_1\alpha\delta QZ_d^{(0)}}{FS^2} + \frac{\lambda\lambda_1\alpha}{S^2}\right]^{-1},$$

$$B = b\left[1 - \frac{\delta RZ_d^{(0)}}{F}\right],$$

$$C = \left[1 + \frac{\delta QZ_d^{(0)}}{F}\right]\left[\frac{\lambda_2\alpha\nu_{cho}}{S^2} - \frac{\lambda_2\nu_{io}}{S} + \frac{\lambda\alpha\nu_o}{S^2}\right],$$

$$D = \left[\frac{\lambda^2\delta\mu_dZ_d^{(0)2}}{G^2} + \frac{\lambda_1^2\alpha}{S^2}\left(1 + \frac{\delta QZ_d^{(0)}}{F}\right)\right],$$

$$\lambda_2 = \lambda - 2u_{ixo}.$$

To obtain the solution of Eq. (10) we introduce the variable

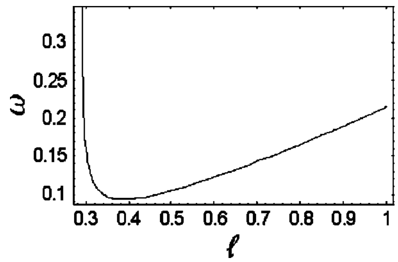

FIG. 1. Graph of ω vs ℓ for α=1.2, δ=0.01, $n_e=10^3$, Z_d=20, λ=1.35, $r_d=1.2\times10^{-13}$, T_i=0.02, T_{ef}=0.2, T_{et}=0.4, T_d=0.01, u_{ixo}=0.5, ν_{cho}=0.000 12, ν_{io}=0.000 09, ν_o=0.000 16, and τ=3.

$$\chi = \ell\xi + m\eta - U\tau, \tag{11}$$

where χ is the transformed coordinates with respect to a frame moving with velocity U. ℓ and m are the directional cosine of the wave vector k along the ξ and η axes, respectively, so that $\ell^2+m^2=1$. Equation (10) can be integrated, with respect to the variable χ and using the vanishing boundary condition for $\phi^{(1)}$ and their derivatives up to second order for $|\chi|\rightarrow\infty$, we obtain the time evolution solitary wave form approximate solution as

$$\phi = He^{-(1/2)AC\tau}\,\mathrm{sech}^4\sqrt{\frac{B(He^{-(1/2)AC\tau})^{1/2}}{15}}\chi, \tag{12}$$

where $\phi\equiv\phi^{(1)}$. To obtain the value of H, let $C=0$ in Eq. (10) and then its solution is given by

$$\phi = \left(\frac{15\bar{h}}{8AB\ell^2}\right)^2\mathrm{sech}^4\sqrt{\frac{\bar{h}}{8\alpha A\ell^4}}\bar{\chi}, \tag{13}$$

where $\bar{\chi}$ is the transformed coordinates with respect to a frame moving with velocity $\bar{U}$ at $C=0$ (i.e., for $C=0$; $\chi\rightarrow\bar{\chi}$, and $U\rightarrow\bar{U}$). $\bar{h}=\bar{U}\ell-\frac{1}{2}AD(1-\ell^2)$. From Eqs. (12) and (13) it is clear that $H=(15\bar{h}/8AB\ell^2)^2$. Therefore, Eq. (13) can be rewritten as

$$\phi = \phi_o\,\mathrm{sech}^4(\chi/\omega), \tag{14}$$

where the amplitude ϕ_o and the width ω are given by $(15\bar{h}/8AB\ell^2)^2e^{(-1/2)AC\tau}$ and $\sqrt{(8\alpha A\ell^4/\bar{h})\sqrt{e^{(1/2)AC\tau}}}$, respectively. From Eq. (14), it is clear that stable solitary waves exist only when ω is real. A, C, and τ are always positive but

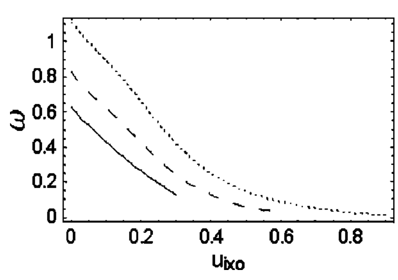

FIG. 2. Graph of ω vs u_{ixo}, α=1.2, δ=0.01, $n_e=10^3$, Z_d=20, ℓ=0.5, $r_d=1.2\times10^{-13}$, T_i=0.02, T_{ef}=0.2, T_{et}=0.4, T_d=0.01, ν_{cho}=0.000 12, ν_{io}=0.000 09, ν_o=0.000 16, and τ=3; (—) for λ=1, (---) for λ=1.2, and (⋯⋯) for λ=1.5.

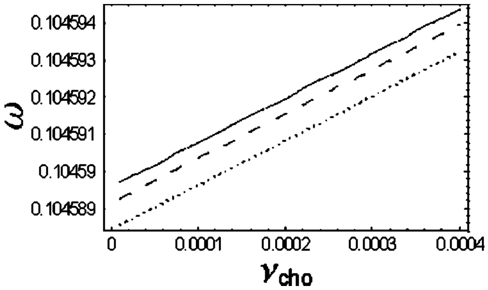

FIG. 3. Graph of ω vs ν_{cho}, $\alpha=1.2$, $\delta=0.01$, $n_e=10^3$, $Z_d=20$, $\ell=0.5$, $r_d=1.2\times10^{-13}$, $T_i=0.02$, $T_{ef}=0.2$, $T_{et}=0.4$, $T_d=0.05$, $u_{ixo}=0.5$, $\lambda=1.35$, $\nu_o=0.000\,16$, and $\tau=3$; (—) for $\nu_{io}=0.000\,01$, (----) for $\nu_{io}=0.0001$, and (⋯⋯) for $\nu_{io}=0.0025$.

$\bar{h}$ may be negative. To be $\bar{h}>0$ the following condition must satisfy:

$$\frac{\ell}{1-\ell^2}>\frac{AD}{2\bar{U}}. \tag{15}$$

From the last condition, it is clear that the existence of solitary waves requires a necessary condition depending on ℓ, A, and D. Also, one can notice that for $A=0$ the soliton cannot exist. On the other hand, when $\lambda=\lambda_c=(2\alpha T_i/T_{ef})^{1/2}$ or $(2\delta T_d/T_{ef}\mu_d)^{1/2}$ the value of S or G equals zero and then $A=0$.

IV. DISCUSSION

Now, one may ask to what extent the fluid equations that were used are applicable to experimental situations or space plasma observations? At the beginning, we have assumed that the system under investigation is a fully ionized, weakly coupled, three-component dusty plasma consisting of warm variational charged dust grains, warm positive ions, and nonisothermal electrons. Also, we have neglected the effect of gravity. Fully ionized means that there are no neutrals in the plasma. The term dusty plasma means that $d/\lambda_D<1$, where d is the intergrain distance between dust particles and λ_D is the dust plasma Debye radius.[15] Weakly coupled means that the coupling parameter $\Gamma\ll1$. To neglect the effect of gravity the dust particle sizes (r_d) should not be more than 1 μm. Actually, we have an example that achieves these four conditions: (i) there are no neutrals, (ii) $d/\lambda_D<1$, (iii) $\Gamma\ll1$, and (iv) $r_d<1$ μm: Mendis[18] cleared that the supernovae shells are one of the space plasma observations which satisfy the last four conditions. The typical plasma parameter values are $n_e=10^3$ cm^{-3}, $T_{ef}=0.2$ eV, $n_d=10$ cm^{-3}, $r_d=0.01$ μm, $Z_d=20$. However, the other parameters were not given in Ref. 18. So, these values are supposed to investigate their effects on the behavior of the solitons. From the data we can calculate the value of $\alpha=1.2$ and $\delta=0.01$.

Before going to investigate the soliton behavior that may appear in the supernovae shells, it is important to check the validity of the inequality (6) for the supernovae shells plasma parameters. Actually, it is found that the ratio $\Delta\zeta/\rho$ is of the order 10^{-2}. Therefore, the inequality (6) is well satisfied and the nonlinear structure in the supernovae shells is expected to be soliton.

The behavior of the soliton width ω is displayed in Figs. 1–4. Figure 1 shows the dependence of ω on ℓ. It is clear that ω decreases with ℓ; from $\ell=0.289$ to 0.4 but it increases from $\ell=0.4$ to 1, while the values of $\ell<0.289$ gives unstable solitons due to the condition (15). From Fig. 2, it is clear that ω decreases with u_{ixo} at different values of λ. The lower limit of λ can be calculated from Eq. (8c). For certain values of λ there are only some specific values of u_{ixo} that cannot be exceeded. It is obvious also that when u_{ixo} increases the width does not decrease rapidly. Figure 3 clears the relation between ω and ν_{cho} (ν_{io}). It is seen that ω increases (decreases) with ν_{cho} (ν_{io}). From Fig. 4, it is noticed that ω increases with ν_o and τ.

The behavior of the soliton amplitude ϕ_o is displayed in Figs. 5–9. It is obvious from Fig. 5 that ϕ_o increases with ℓ; from $\ell=0.289$ to 0.6 but it decreases from $\ell=0.6$ to 1. From Fig. 6, we can see that ϕ_o increases very slowly with u_{ixo} but at certain values of u_{ixo} it goes up suddenly. Figure 7 clears that ϕ_o increases with β. The amplitude increases slowly for

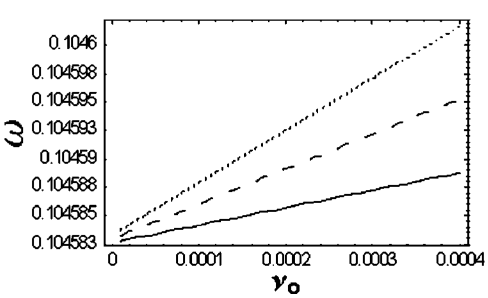

FIG. 4. Graph of ω vs ν_o, $\alpha=1.2$, $\delta=0.01$, $n_e=10^3$, $Z_d=20$, $\ell=0.5$, $r_d=1.2\times10^{-13}$, $T_i=0.02$, $T_{ef}=0.2$, $T_{et}=0.4$, $T_d=0.05$, $u_{ixo}=0.5$, $\lambda=1.35$, $\nu_{io}=0.000\,12$, and $\nu_o=0.000\,09$; (—) for $\tau=1$, (----) for $\tau=2$, and (⋯⋯) for $\tau=3$.

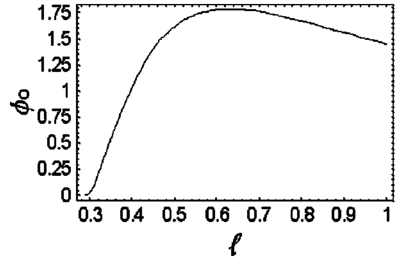

FIG. 5. Graph of ϕ_o vs ℓ. The parameters are the same as those of Fig. 1.

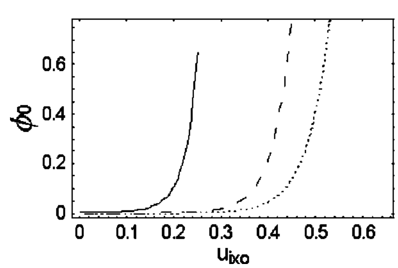

FIG. 6. Graph of ϕ_o vs u_{ixo}. (—) for $\lambda=0.9$, (---) for $\lambda=1.3$, and (⋯⋯) for $\lambda=1.5$. The parameters are the same as those of Fig. 2.

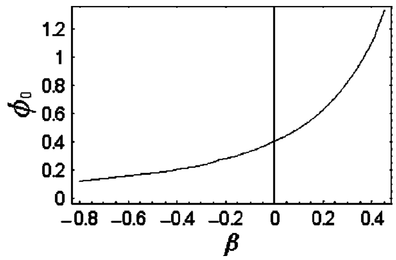

FIG. 7. Graph of ϕ_o vs β for $\alpha=1.2$, $\delta=0.01$, $n_e=10^3$, $Z_d=20$, $\lambda=1.35$, $r_d=1.2\times10^{-13}$, $T_i=0.02$, $T_{ef}=0.2$, $T_d=0.01$, $\ell=0.5$, $u_{ixo}=0.5$, $\nu_{cho}=0.000\,12$, $\nu_{io}=0.000\,09$, $\nu_o=0.000\,16$, and $\tau=3$.

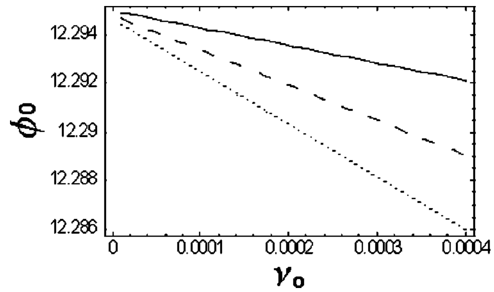

FIG. 9. Graph of ϕ_o vs ν_o. (—) for $\tau=1$, (----) for $\tau=2$, and (⋯⋯) for $\tau=3$. The parameters are the same as those of Fig. 4.

negative β but it increases rapidly for positive β. From Figs. 8 and 9, it is clear that ϕ_o decreases with ν_{cho}, ν_o, and τ but it increases with ν_{io}.

Now, it is important to clarify that we have studied the soliton which has a following characteristic property; it is a coherent pulse whose shape and speed are not altered by a collision with other solitons.[19] This property is inherent in all solitons.

It is interesting also to compare the one-dimensional (1D) case results of Popel *et al.*[12] with 2D case results presents here. For 1D case, Popel *et al.*[12] cleared that the perturbation amplitude decreases with time. This result agrees with our investigation (see Fig. 9). However, for 1D case the value of ℓ equals unity while for 2D case, ℓ varies from 0.289 to 1. On the other hand, our model clarifies that the wave cannot propagate for any directions but in the directions that satisfy Eq. (15) only. This result could not be obtained for 1D case. Thus, we can consider this study as a modification and generalization of previous work.

V. CONCLUSIONS

In this paper, we have investigated two-dimensional DIA solitons in collisionless, unmagnetized three-component dusty plasmas, consisting of negatively charged dust grains, positive ions, and trapped electrons. The reductive perturbation method was used to reduce the basic set of fluid equations to DMKP, Eq. (10). The exact solution of this equation is not possible so we obtain its time evolution solitary wave form approximate solution (14). We can conclude that the existence of solitons requires a necessary condition depending on ℓ, A, D, as well as λ. We have referred to the supernovae shells as an application to our study and it is found that the soliton width (amplitude) decreases (increases) for lower values of ℓ and increases (decreases) for higher values of ℓ. The soliton width (amplitude) decreases (increases) with u_{ixo}. The soliton width increases with ν_{cho}, ν_o, and τ but it decreases with ν_{io}. The soliton amplitude decreases (increases) with ν_{cho}, ν_o, and τ (β and ν_{io}).

Although we have referred to the supernovae shells as an application to our study, the present analysis is applicable to other experimental situations or space plasma observations that achieve the four conditions (i) there are no neutrals, (ii) $d/\lambda_D<1$, (iii) $\Gamma\ll1$, and (iv) $r_d<1$ μm and also satisfy the inequality (6).

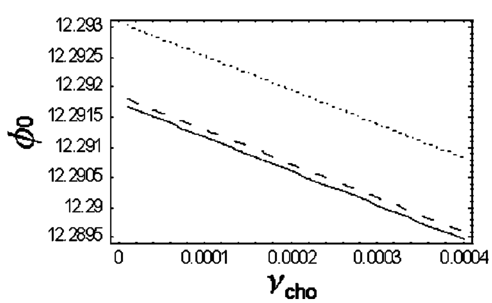

FIG. 8. Graph of ϕ_o vs ν_{cho}. (—) for $\nu_{io}=0.000\,01$, (----) for $\nu_{io}=0.0001$, and (⋯⋯) for $\nu_{io}=0.001$. The parameters are the same as those of Fig. 3.

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