Higher-order contribution to obliquely nonlinear electron-acoustic waves with electron beam in a magnetized plasma

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Propagation of electron-acoustic waves in a strongly magnetized four-component plasma consisting of cold and hot electrons, a warm electron beam, and stationary ions is investigated. The present model considered weakly dispersive and strongly magnetized plasma in the limit of long wavelengths. The introduction of an electron beam allows the existence of electron-acoustic solitons with velocity related to the beam velocity. With increasing the beam velocity and the beam temperature, both the soliton amplitude and the width increase. Applying a reductive perturbation theory, a nonlinear Zakharov-Kuznetsov (ZK) equation for the first-order perturbed potential and a linear inhomogeneous Zakharov-Kuznetsov (ZK-type) equation for the second-order perturbed potential are derived. Stationary solutions of these coupled equations are obtained using a renormalization method. These solutions admit either compressive or rarefactive soliton type depending on the electron-beam parameters. Moreover, the dependence of the solution on the beam parameters, obliqueness on the magnetic field, and the magnetic field itself is also investigated. The application of the present investigation to the broadband electrostatic noise in the dayside auroral zone of the Earth's magnetosphere is considered.
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I. INTRODUCTION

The electron-acoustic wave (EAW) is an electrostatic wave which was first discovered experimentally,[1,2] and has been observed when the unmagnetized plasma is composed of two electron populations described by two Maxwellian distribution functions with different temperatures and densities. These two populations will be referred to as "cold" and "hot" electrons,[3,4] where the cold electrons provide the inertia, and the restoring force comes from the pressure of the hot electrons. Here, the ions play the role of the neutralizing background, i.e., the ion dynamics does not influence the EAWs because the EAW frequency is much larger than the ion plasma frequency. The spectrum of the linear EAWs, unlike the well-known Langmuir waves, extends only up to the cold electron plasma frequency $\omega_{pc}=(4\pi n_{oc}e^2/m_e)^{1/2}$, where n_{oc} is the unperturbed cold electron number density, e is the magnitude of the electron charge, and m_e is the mass of the electron.[5]

The study of the EAWs propagation plays an important role not only in laboratory experiment but also in space plasmas.[6] Satellite measurements in the auroral and other regions of the magnetosphere have shown bursts that form broadband electrostatic noise (BEN) emissions. The associated electric-field intensities of these BEN ranges are from few μV/m up to 100 mV/m.[7–14] The observations of solitary waves in the auroral zone suggest that there are two classes of solitary waves: the first kind is associated with electron beams and the other is associated with ion beams. Here we will focus our concern on the first kind. Solitary waves associated with electron beams were first observed by Geotail,[10] then by Fast Auroral Snapshot (FAST) (Refs. 11 and 12), and later by Polar[13] spacecrafts. The signature of these observations is found to display a nonlinear behavior. Several theoretical attempts have been made to explain the observed BENs in different regions of the Earth's magnetosphere.[3–19] Ergun and co-workers[11,12] stated that the observed BEN bursts in the dayside auroral zone must have three-dimensional components with the inclusion of the magnetic-field effects. On the other hand, Berthomier *et al.*[3] studied electron-acoustic (EA) solitons in an electron-beam plasma system with isothermal hot electrons. It was found that the introduction of an electron beam in such plasma allows the existence of new EA solitons with velocity related to the beam velocity. Also, the electron beam modifies the topology of the roots of the linear dispersion relation in the phase velocity space. Mace and Hellberg[17] studied a three-component magnetized plasma system (without an electron-beam component). Considering the case of magnetized ion and nonmagnetized ions, they showed that the case of magnetized ions has a critical behavior and it can describe only the very lowest-frequency waves in the Earth's magnetosphere. Singh and Lakhina[15] studied the generation of BEN by EAWs in a four-component unmagnetized plasma. They applied the linear theory to study the stability and the growth rate of the EAWs for three different regions of the dayside auroral zone, plasma sheet boundary layer (PSBL), and polar cusp regions. Since EAWs can be excited by electron and laser beams, the electron beams in addition to the two electron populations are considered to be the main energy source for the excitation of the electron wave mode.[20] When the

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beam energy is sufficiently large, the nonlinear and dispersive effects are competing to produce EAWs, which are stationary in their comoving reference frame.[21]

On the other hand, in the reductive perturbation theory (RPT),[22] a set of coupled nonlinear partial differential equations is reduced to a single evolution equation such as the Korteweg de Vries (KdV) equation or the nonlinear Schrödinger (NS) equation, depending if the system is weakly dispersive or strongly dispersive.[23,24] For a multidimensional problem, these evolution equations become the Zakharov-Kuznetov[17,25,26] (ZK) equation or the Davey-Stewartson[27,28] (DS) equation. The association between the small-wave-number limit of the NS equation and the oscillatory solution of the KdV equation was satisfied.[29,30]

Investigations of small-amplitude EAWs in a weakly dispersive media are described by the ZK equation.[17] This equation contains the lowest-order nonlinearity and dispersion, and consequently can describe only waves of small amplitude. If the wave amplitude or the width deviates significantly from the prediction of the ZK equation, the higher-order nonlinear and dispersion effects must be included to accurately describe such waves. For this end, the higher-order approximation of the RPT (Ref. 22) is known to be a powerful tool.[31] Our objective here is to propose a four-component magnetized plasma model consisting of cold and hot electrons, a warm electron beam, and stationary ions taking into account the effects of higher-order nonlinear and dispersion terms to interpret the observed BEN in the dayside auroral zone.[8,9] Here, we will focus our interest only on the strongly magnetized and weakly dispersive plasma model only.

This paper is organized as follows: In Sec. II, we present the basic set of fluid equations governing our plasma model. The nonlinear EAWs are investigated through the derivation of a ZK equation for the first-order perturbed potential and a linear inhomogeneous ZK-type equation for the second-order perturbed potential. In Sec. III, the renormalization method[22] is applied to obtain the stationary solutions of the coupled evolution equations. Section IV is devoted to the discussion and the conclusion.

II. BASIC EQUATIONS

Let us consider an infinite, homogenous, collisionless, magnetized plasma consisting of cold and hot electrons, a warm electron beam, and stationary ions. The magnetic field $\mathbf{B}_0$ is uniform and let $\mathbf{b}$ denote the vector $\mathbf{B}_0/|\mathbf{B}_0|$, which for simplicity, we will choose to lie along the z axis of our Cartesian coordinate system,[17,32]

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{u}_j) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j = -\frac{1}{m_j n_j} \nabla p_j + \frac{1}{m_j} \nabla \phi - \Omega_j \mathbf{u}_j \times \mathbf{b}, \tag{2}$$

$$\frac{\partial p_j}{\partial t} + \mathbf{u}_j \cdot \nabla p_j + 3 p_j \nabla \cdot \mathbf{u}_j = 0. \tag{3}$$

In Eqs. (1)–(3), n_j, $\mathbf{u}_j$, and p_j ($j=c$ and b for the cold electron and for the electron beam, respectively) are the densities, velocities with u_j, v_j, and w_j as their cartesian (x,y,z) components, and pressures of the two fluids. ϕ is the electric potential. Ω_j is the j-particle gyrofrequency and equals $eB_0/m_j c$.

The system of fluid equations and the Boltzmann-distributed hot electrons are closed by the use of the Poisson equation,

$$\nabla^2 \phi = n_{0h} \exp \phi + n_c + n_b - n_{i0}. \tag{4}$$

In the above equations we have employed the following normalizations:[17] lengths by the modified Debye length $\lambda_{\mathrm{De}} = (K_B T_h / 4\pi n_{0e} e^2)^{1/2}$, time by the inverse plasma frequency $\omega_{\mathrm{pe}}^{-1} = (m_e / 4\pi n_{0e} e^2)^{1/2}$, number densities by the total equilibrium electron density n_{0e}, p_j by $n_{0e} K_B T_h$, temperatures by the hot-electron temperature T_h, u_j by the hot-electron thermal speed $(K_B T_h / m_e)^{1/2}$, ϕ by T_h/e, masses by the electron mass m_e, and Ω_j by $\omega_{\mathrm{pj}} = (4\pi n_{0j} e^2 / m_e)^{1/2}$, where K_B is the Boltzmann constant.

The charge-neutrality condition in such plasma system is always maintained through the relation

$$n_{0e} = n_{0h} + n_{0c} + n_{0b}, \tag{5}$$

where n_{0h} and n_{0b} are the unperturbed hot and beam electron densities, respectively. Furthermore, we assume that the plasma in the equilibrium state obeys the following boundary conditions as $|\mathbf{x}| \to \infty$:

$$\phi \to 0, \quad \nabla \phi \to 0, \quad \nabla^2 \phi \to 0, \quad n_j \to n_j^{(0)},$$

$$p_j \to p_j^{(0)}, \quad u_{0c} \to 0, \quad u_{0b} \to u_0, \tag{6}$$

where $n_j^{(0)} = n_{0j}$ and $p_j^{(0)} = p_{0j}$.

Before addressing the nonlinear evolution, we briefly discuss the dispersion law for the linear EAW modes. Linearizing Eqs. (1)–(4) with the ansatz $\Psi \propto \exp(i\Phi_w)$, where $\Phi_w = \mathbf{k} \cdot \mathbf{r} - \omega t$, we finally obtain the linear dispersion relation[17,32–34]

$$\frac{1}{\lambda_{\mathrm{Dh}}^2} + k^2 = \sum_j \frac{\omega_{\mathrm{pj}}^2 (k^2 \tilde{\omega}_j^2 - k_\parallel^2 \Omega_j^2)}{\tilde{\omega}_j^4 - \tilde{\omega}_j^2 (3k^2 T_j + \Omega_j^2) + 3k_\parallel^2 T_j \Omega_j^2}, \tag{7}$$

where $k^2 = k_\parallel^2 + k_\perp^2$, $k_\parallel^2 = k_z^2$, $k_\perp^2 = k_x^2 + k_y^2$, and $\tilde{\omega}_j = \omega - k_\parallel u_{0j}$.

For parallel propagation, $k_\parallel = k$, and for the strongly magnetized plasma ($\tilde{\omega}_j \ll \Omega_j$), Eq. (7) reduces to

$$\frac{1}{(k\lambda_{\mathrm{Dh}})^2} + 1 = \sum_j \frac{\omega_{\mathrm{pj}}^2}{(\tilde{\omega}_j^2 - 3k^2 T_j)}, \tag{8}$$

where $\lambda_{\mathrm{Dh}} = (K_B T_h / 4\pi n_{0h} e^2)^{1/2}$.

For small wave numbers (long wavelengths) both the dispersion laws (7) and (8) can be approximated to the lowest order as an acoustic-like dispersion-free propagation along the field, with a dispersive correction of order k^3, i.e.,

$$\omega = k_\parallel V - c_1 k_\parallel^3 - c_2 k_\parallel k_\perp^2 + \cdots, \tag{9}$$

where the phase velocity V in the limit of vanishing wave numbers is determined from

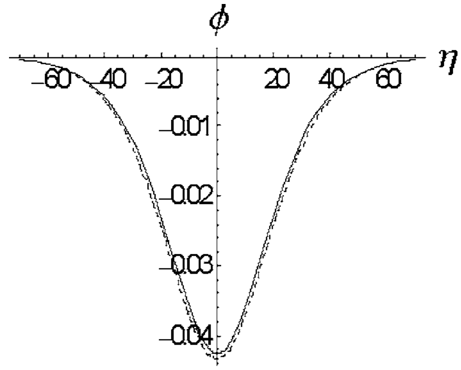

FIG. 1. Plot of $\phi^{(1)}$ (solid curve) and ϕ (dotted curve) against η for the three-component plasma (without an electron beam), where $V=1.5$, $l_z=0.5$, and $\gamma=0.1$.

$$n_{0h}-\sum_j n_{0j}\rho_j=0, \tag{10}$$

where $\rho_j=(\tilde{V}_j^2-3T_j)^{-1}$ and $\tilde{V}_j=V-u_{0j}$ and the coefficients c_1 and c_2 are given by

$$c_1=\frac{1}{A}, \quad c_2=\frac{C}{A},$$

with

$$A=\sum_j 2n_{0j}\tilde{V}_j\rho_j^2 \text{ and } C=1+\sum_j n_{0j}\left(\frac{\rho_j\tilde{V}_j^2}{\Omega_j}\right)^2. \tag{11}$$

For a three-component plasma, i.e., without an electron beam, Eq. (10) will be reduced to $V^2=n_{0c}/n_{0h}+3T_c$ which is identical to that derived by Mace and Hellberg.[17] Also, it agrees exactly with that obtained by Berthomier *et al.*,[3] Mamun and Shukla,[4] and Verheest *et al.*[32] Recently, El-Taibany[18] and El-Taibany and Moslem[19] have derived a similar equation as (10) for a four-component plasma with hot-electron vortices. They showed [Fig. 1] that V increases as u_o increases. Also, V is influenced by T_b variation but it is not affected significantly by T_c change. For a comprehensive study of the electron-acoustic instability in a magnetized plasma with a field-aligned beam, see, e.g., Sooklal and Mace.[35]

On the other hand, for illustrative purpose, if we consider $T_j=0$, $k_\perp\rho_{\rm se}\ll 1$ and $k_\parallel\lambda_{\rm Dh}\ll 1$ in a three-component plasma, Eq. (9) becomes[17,32–34] $\omega=(k_\parallel n_{0c})/n_{0h}\{1-\frac{1}{2}[(k_\parallel\lambda_{\rm Dh})^2-k_\perp^2(\rho_{\rm se}^2+\lambda_{\rm Dh}^2)]\}$, where $\rho_{\rm se}=n_{0c}/(n_{0h}\Omega_c)$ is the Larmor radius of an electron traveling at the linear parallel phase speed. In a weakly dispersive limit, if we define[34] $k^2=\varepsilon K^2$ with K of the order of typical scales of the plasma-like $\rho_{\rm se}$ or $\lambda_{\rm Dh}$, Φ_w can be rewritten as

$$\Phi_w=\mathbf{K}_\perp\cdot(\varepsilon^{1/2}\mathbf{r}_\perp)+\mathbf{K}_\parallel\cdot[\varepsilon^{1/2}(\mathbf{r}_\parallel-Vt)]-\mathbf{K}_\parallel\left(\frac{n_{0c}}{n_{0h}}\right)$$
$$\times(\mathbf{K}^2\lambda_{\rm Dh}^2+\mathbf{K}_\perp^2\rho_{\rm se}^2)(\varepsilon^{3/2}t/2),$$

from which, we conclude that in order to study nonlinear EAWs, we have to use the following stretched variables:[17,32–34]

$$X=\varepsilon^{1/2}x, \quad Y=\varepsilon^{1/2}y, \quad Z=\varepsilon^{1/2}(z-Vt), \quad T=\varepsilon^{3/2}t. \tag{12}$$

V is the EA linear phase velocity (parallel to the magnetic-field direction) when nonlinearities and dispersion coefficients are omitted and it basically corresponds to the linear phase velocity in the limit $k\to 0$, and obeys Eq. (10). ε measures the size of the perturbation amplitude. Let us expand the variables n_j, p_j, ϕ, u_j, v_j, and w_j in powers of ε,[17,32–34]

$$n_j=n_{0j}+\varepsilon n_j^{(1)}+\varepsilon^2 n_j^{(2)}+\varepsilon^3 n_j^{(3)}+\cdots, \tag{13a}$$

$$p_j=p_{0j}+\varepsilon p_j^{(1)}+\varepsilon^2 p_j^{(2)}+\varepsilon^3 p_j^{(3)}+\cdots, \tag{13b}$$

$$\phi=\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+\varepsilon^3\phi^{(3)}+\varepsilon^4\phi^{(4)}+\cdots, \tag{13c}$$

$$u_j=\varepsilon^{3/2}u_j^{(1)}+\varepsilon^2 u_j^{(2)}+\varepsilon^{5/2}u_j^{(3)}+\varepsilon^3 u_j^{(4)}+\cdots, \tag{13d}$$

$$v_j=\varepsilon^{3/2}v_j^{(1)}+\varepsilon^2 v_j^{(2)}+\varepsilon^{5/2}v_j^{(3)}+\varepsilon^3 v_j^{(4)}+\cdots, \tag{13e}$$

$$w_j=u_{0j}+\varepsilon w_j^{(1)}+\varepsilon^2 w_j^{(2)}+\varepsilon^3 w_j^{(3)}+\varepsilon^4 w_j^{(4)}+\cdots. \tag{13f}$$

Using a RPT (Ref. 36) and substituting Eqs. (12) and (13) into the basic set of equations (1)–(4), the lowest orders in ε give the following relations:

$$n_j^{(1)}=-n_{0j}\rho_j\phi^{(1)}, \quad p_j^{(1)}=-3p_{0j}\rho_j\phi^{(1)},$$

$$w_j^{(1)}=-\tilde{V}_j\rho_j\phi^{(1)},$$

$$u_j^{(1)}=\frac{\tilde{V}_j^2\rho_j}{\Omega_j}\frac{\partial\phi^{(1)}}{\partial Y}, \quad v_j^{(1)}=-\frac{\tilde{V}_j^2\rho_j}{\Omega_j}\frac{\partial\phi^{(1)}}{\partial X}. \tag{14}$$

The next order of the perturbation gives

$$\frac{\partial n_j^{(1)}}{\partial T}-\tilde{V}_j\frac{\partial n_j^{(2)}}{\partial Z}+n_{0j}\frac{\partial u_j^{(2)}}{\partial X}+n_{0j}\frac{\partial v_j^{(2)}}{\partial Y}+n_{0j}\frac{\partial w_j^{(2)}}{\partial Z}+\frac{\partial}{\partial Z}[n_j^{(1)}w_j^{(1)}]$$
$$=0, \tag{15a}$$

$$\frac{\partial w_j^{(1)}}{\partial T}-\tilde{V}_j\frac{\partial w_j^{(2)}}{\partial Z}+w_j^{(1)}\frac{\partial w_j^{(1)}}{\partial Z}+\frac{1}{n_{0j}}\frac{\partial p_j^{(2)}}{\partial Z}-\frac{n_j^{(1)}}{n_{0j}^2}\frac{\partial p_j^{(1)}}{\partial Z}-\frac{\partial\phi^{(2)}}{\partial Z}$$
$$=0, \tag{15b}$$

$$\frac{\partial p_j^{(1)}}{\partial T}-\tilde{V}_j\frac{\partial p_j^{(2)}}{\partial Z}+w_j^{(1)}\frac{\partial p_j^{(1)}}{\partial Z}+3p_{0j}\left[\frac{\partial u_j^{(2)}}{\partial X}+\frac{\partial v_j^{(2)}}{\partial Y}+\frac{\partial w_j^{(2)}}{\partial Z}\right]$$
$$+3p_j^{(1)}\frac{\partial w_j^{(1)}}{\partial Z}=0, \tag{15c}$$

$$\left[\frac{\partial^2\phi^{(1)}}{\partial X^2}+\frac{\partial^2\phi^{(1)}}{\partial Y^2}+\frac{\partial^2\phi^{(1)}}{\partial Z^2}\right]-n_{0h}\phi^{(2)}-\frac{n_{0h}}{2}{\phi^{(1)}}^2-n_j^{(2)}=0, \tag{15d}$$

$$u_j^{(2)}=-\frac{\tilde{V}_j^3\rho_j}{\Omega_j^2}\frac{\partial^2\phi^{(1)}}{\partial Z\partial X}, \tag{15e}$$

$$v_j^{(2)} = -\frac{\widetilde{V}_j^3 \rho_j}{\Omega_j^2} \frac{\partial^2 \phi^{(1)}}{\partial Z \partial Y}. \tag{15f}$$

Eliminating the second-order perturbed quantities and using Eq. (14), one can obtain the ZK equation[17,34] for EAWs,

$$\Psi_1(\phi^{(1)})\phi^{(1)} = \frac{\partial \phi^{(1)}}{\partial T} + c_1 \frac{\partial^3 \phi^{(1)}}{\partial Z^3} + c_2 \left[\frac{\partial^3 \phi^{(1)}}{\partial Z \partial X^2} + \frac{\partial^3 \phi^{(1)}}{\partial Z \partial Y^2} \right] + c_3 \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial Z} = 0, \tag{16}$$

where

$$c_3 = \frac{B}{A}, \quad B = -\sum_j \frac{3 n_{0j}(\widetilde{V}_j^2 + T_j)}{(\widetilde{V}_j^2 - 3T_j)^3} - n_{0h},$$

and A is still given by (11). The coefficients c_1, c_2, and c_3 can be easily reduced to those obtained by Mace and Hellberg[17] and Verheest *et al.*[32] for the three-component magnetized plasma (without a beam electron component). A similar equation to Eq. (16) was first derived by Zakharov and Kuznetsov[37] for weakly nonlinear ion acoustic waves in magnetized plasma. They showed that the dispersion arising from a combination of charge separation and finite Larmor radius effects can balance nonlinearity.[17,37]

On the other hand, the perturbed quantities $n_j^{(2)}$, $w_j^{(2)}$, $p_j^{(2)}$, $u_j^{(3)}$, and $v_j^{(3)}$ can be obtained in terms of $\phi^{(1)}$ and $\phi^{(2)}$ as follows:

$$n_j^{(2)} = D_1 \phi^{(1)^2} + D_2 \frac{\partial^2 \phi^{(1)}}{\partial Z^2} + D_3 \left[\frac{\partial^2 \phi^{(1)}}{\partial X^2} + \frac{\partial^2 \phi^{(1)}}{\partial Y^2} \right] + D_4 \phi^{(2)}, \tag{17a}$$

$$w_j^{(2)} = E_1 \phi^{(1)^2} + E_2 \frac{\partial^2 \phi^{(1)}}{\partial Z^2} + E_3 \left[\frac{\partial^2 \phi^{(1)}}{\partial X^2} + \frac{\partial^2 \phi^{(1)}}{\partial Y^2} \right] + E_4 \phi^{(2)}, \tag{17b}$$

$$p_j^{(2)} = F_1 \phi^{(1)^2} + F_2 \frac{\partial^2 \phi^{(1)}}{\partial Z^2} + F_3 \left[\frac{\partial^2 \phi^{(1)}}{\partial X^2} + \frac{\partial^2 \phi^{(1)}}{\partial Y^2} \right] + F_4 \phi^{(2)}, \tag{17c}$$

$$u_j^{(3)} = \frac{1}{\Omega_j} \frac{\partial}{\partial Y} \left\{ G_1 \phi^{(1)^2} + G_2 \frac{\partial^2 \phi^{(1)}}{\partial Z^2} + G_3 \left[\frac{\partial^2 \phi^{(1)}}{\partial X^2} + \frac{\partial^2 \phi^{(1)}}{\partial Y^2} \right] + G_4 \phi^{(2)} \right\}, \tag{17d}$$

$$v_j^{(3)} = \frac{-1}{\Omega_j} \frac{\partial}{\partial X} \left\{ G_1 \phi^{(1)^2} + G_2 \frac{\partial^2 \phi^{(1)}}{\partial Z^2} + G_3 \left[\frac{\partial^2 \phi^{(1)}}{\partial X^2} + \frac{\partial^2 \phi^{(1)}}{\partial Y^2} \right] + G_4 \phi^{(2)} \right\}, \tag{17e}$$

where D_h, E_h, F_h, and G_h $(h=1,2,3,4)$ are given in the Appendix.

From $O(\varepsilon^{5/2})$, and using Eqs. (17d) and (17e), we have

$$u_j^{(4)} = -\frac{\widetilde{V}_j}{\Omega_j^2} \left\{ H_1 \left[\frac{\partial \phi^{(1)}}{\partial X} \frac{\partial \phi^{(1)}}{\partial Z} \right] + H_2 \left[\phi^{(1)} \frac{\partial^2 \phi^{(1)}}{\partial Z \partial X} \right] + H_3 \frac{\partial^4 \phi^{(1)}}{\partial X \partial Z^3} + H_4 \left[\frac{\partial^4 \phi^{(1)}}{\partial Z \partial X^3} + \frac{\partial^4 \phi^{(1)}}{\partial Z \partial X \partial Y^2} \right] + G_4 \frac{\partial^2 \phi^{(2)}}{\partial Z \partial X} \right\}, \tag{18a}$$

$$v_j^{(4)} = -\frac{\widetilde{V}_j}{\Omega_j^2} \left\{ H_1 \left[\frac{\partial \phi^{(1)}}{\partial Y} \frac{\partial \phi^{(1)}}{\partial Z} \right] + H_2 \left[\phi^{(1)} \frac{\partial^2 \phi^{(1)}}{\partial Z \partial Y} \right] + H_3 \frac{\partial^4 \phi^{(1)}}{\partial Y \partial Z^3} + H_4 \left[\frac{\partial^4 \phi^{(1)}}{\partial Z \partial Y \partial X^2} + \frac{\partial^4 \phi^{(1)}}{\partial Z \partial Y^3} \right] + G_4 \frac{\partial^2 \phi^{(2)}}{\partial Z \partial Y} \right\}, \tag{18b}$$

where H_h $(h=1,2,3,4)$ is given in the Appendix.

For $O(\varepsilon^3)$, and following the usual procedure of the RPT (Ref. 36), we can obtain a set of nonlinear equations in the third perturbed quantities. Eliminating $n_j^{(3)}$, $w_z^{(3)}$, $p_j^{(3)}$, $u_j^{(4)}$, and $v_j^{(4)}$ from this set of equations, we finally obtain a linear inhomogeneous ZK-type equation for the second-order perturbed potential $\phi^{(2)}$,

$$\begin{aligned}
\Psi_2(\phi^{(1)})\phi^{(2)} &= \frac{\partial \phi^{(2)}}{\partial T} + c_1 \frac{\partial^3 \phi^{(2)}}{\partial Z^3} + c_2 \left[\frac{\partial^3 \phi^{(2)}}{\partial Z \partial X^2} + \frac{\partial^3 \phi^{(2)}}{\partial Z \partial Y^2} \right] + c_3 \frac{\partial \phi^{(1)} \phi^{(2)}}{\partial Z} \\
&= -J_1 \phi^{(1)^2} \frac{\partial \phi^{(1)}}{\partial Z} - J_2 \phi^{(1)} \frac{\partial^3 \phi^{(1)}}{\partial Z^3} - J_3 \left[\phi^{(1)} \frac{\partial^3 \phi^{(1)}}{\partial Z \partial X^2} + \phi^{(1)} \frac{\partial^3 \phi^{(1)}}{\partial Z \partial Y^2} \right] - J_4 \frac{\partial \phi^{(1)}}{\partial Z} \frac{\partial^2 \phi^{(1)}}{\partial Z^2} \\
&\quad - J_5 \left[\frac{\partial \phi^{(1)}}{\partial Z} \frac{\partial^2 \phi^{(1)}}{\partial X^2} + \frac{\partial \phi^{(1)}}{\partial Z} \frac{\partial^2 \phi^{(1)}}{\partial Y^2} \right] - J_6 \frac{\partial^5 \phi^{(1)}}{\partial Z^5} - J_7 \left[\frac{\partial \phi^{(1)}}{\partial X} \frac{\partial^2 \phi^{(1)}}{\partial Z \partial X} + \frac{\partial \phi^{(1)}}{\partial Y} \frac{\partial^2 \phi^{(1)}}{\partial Z \partial Y} \right] \\
&\quad - J_8 \left[\frac{\partial^5 \phi^{(1)}}{\partial Z^3 \partial X^2} + \frac{\partial^5 \phi^{(1)}}{\partial Z^3 \partial Y^2} \right] - J_9 \left[\frac{\partial^5 \phi^{(1)}}{\partial Z \partial X^4} + 2 \frac{\partial^5 \phi^{(1)}}{\partial Z \partial X^2 \partial Y^2} + \frac{\partial^5 \phi^{(1)}}{\partial Z \partial Y^4} \right],
\end{aligned} \tag{19}$$

where $J_q(q=1,2,3,\ldots,9)$ is given in the Appendix.

Thus, we conclude that the basic set of Eqs. (1)–(4) is reduced to a ZK equation, Eq. (16), for the first-order perturbed potential $\phi^{(1)}$, and a ZK-type equation, Eq. (19) for the second-order perturbed potential $\phi^{(2)}$, with source term, the right-hand side of Eq. (19).

III. THE STATIONARY SOLUTIONS

In Sec. II, it has been shown that the higher-order nonlinearities are given by the ZK equation, Eq. (16), and ZK-type equation, Eq. (19), with an inhomogeneous term (a source term). These equations have resonant terms that give rise to secular behavior.[19,22,38] To eliminate this nonfavorable behavior, we adopt the renormalization method introduced by Kodama and Taniuti.[22] According to this method, Eq. (16) is added to Eq. (19) to yield

$$\Psi_1(\phi^{(1)})\phi^{(1)}+\sum_{n\geqslant 2}\varepsilon^n\Psi_2(\phi^{(1)})\phi^{(n)}=\sum_{n\geqslant 2}\varepsilon^n S^{(n)}, \tag{20}$$

where $S^{(2)}$ represents the right-hand side of (19). We add

$$\sum_{n\geqslant 1}\varepsilon^n\frac{\delta\gamma}{l_z}\frac{\partial\phi^{(n)}}{\partial Z}$$

to both sides of (20), where $\delta\gamma$ is given by a power series in $\varepsilon, \delta\gamma=\varepsilon\gamma^{(1)}+\varepsilon^2\gamma^{(2)}+\ldots$, with coefficients to be determined later. The crucial point in this procedure is that we expand $\delta\gamma$ in the right-hand side, while in the left-hand side, it is not expanded. Thereafter, the $\gamma^{(n)}$ are determined successively to cancel out the resonant term in $S^{(n)}$.

Thus Eqs. (16) and (19) become

$$\frac{\partial\tilde{\phi}^{(1)}}{\partial T}+c_1\frac{\partial^3\tilde{\phi}^{(1)}}{\partial Z^3}+c_2\left[\frac{\partial^3\tilde{\phi}^{(1)}}{\partial Z\partial X^2}+\frac{\partial^3\tilde{\phi}^{(1)}}{\partial Z\partial Y^2}\right]+c_3\tilde{\phi}^{(1)}\frac{\partial\tilde{\phi}^{(1)}}{\partial Z}+\frac{\delta\gamma}{l_z}\frac{\partial\tilde{\phi}^{(1)}}{\partial Z}=0, \tag{21}$$

$$\frac{\partial\tilde{\phi}^{(1)}}{\partial T}+c_1\frac{\partial^3\tilde{\phi}^{(2)}}{\partial Z^3}+c_2\left[\frac{\partial^3\tilde{\phi}^{(2)}}{\partial Z\partial X^2}+\frac{\partial^3\tilde{\phi}^{(2)}}{\partial Z\partial Y^2}\right]+c_3\frac{\partial\tilde{\phi}^{(1)}\tilde{\phi}^{(2)}}{\partial Z}+\frac{\delta\gamma}{l_z}\frac{\partial\tilde{\phi}^{(2)}}{\partial Z}=S^{(2)}(\tilde{\phi}^{(1)})+\frac{\gamma^{(1)}}{l_z}\frac{\partial\tilde{\phi}^{(1)}}{\partial Z}. \tag{22}$$

The upper sign on $\phi^{(1)}$ and $\phi^{(2)}$ indicates the renormalization potentials. Let us introduce the variable

$$\eta=l_xX+l_yY+l_zZ-(\gamma+\delta\gamma)T, \tag{23}$$

where l_x, l_y, and l_z are the directional cosines on the magnetic-field direction $(l_x^2+l_y^2+l_z^2=1)$ and the parameter γ is related to the Mach number $M=V/C_{\mathrm{ea}}$ by[19,38]

$$\gamma+\delta\gamma=M-1=\Delta M. \tag{24}$$

Using the vanishing boundary condition for $\tilde{\phi}^{(1)}(\eta)$ and $\tilde{\phi}^{(2)}(\eta)$ and their derivatives up to second order for $|\eta|\to\infty$, Eqs. (21) and (22) can be integrated with respect to the variable η to yield

$$\frac{d^2\tilde{\phi}^{(1)}}{d\eta^2}+\frac{1}{f}\left[\frac{c_3}{2}\phi^{(1)}-\frac{\gamma}{l_z}\right]\tilde{\phi}^{(1)}=0, \tag{25}$$

$$\frac{d^2\tilde{\phi}^{(2)}}{d\eta^2}+\frac{1}{f}\left[c_3\tilde{\phi}^{(1)}-\frac{\gamma}{l_z}\right]\tilde{\phi}^{(2)}=\frac{1}{l_zf}\int_{-\infty}^{\eta}\left[S^{(2)}(\tilde{\phi}^{(1)})+\gamma^{(1)}\frac{d\tilde{\phi}^{(1)}}{d\eta}\right]d\eta, \tag{26}$$

where $f=c_2+(c_1-c_2)l_z^2$. The one-soliton solution of Eq. (25) is given by

$$\tilde{\phi}^{(1)}(\eta)=\phi_0\,\mathrm{sech}^2(\eta\Lambda^{-1}), \tag{27}$$

where the amplitude ϕ_0 and the width Λ are given by $3\gamma/c_3l_z$ and $2\sqrt{l_zf/\gamma}$, respectively. Using Eq. (27), the source term of Eq. (26) becomes

$$\begin{aligned}\frac{1}{l_zf}\int_{-\infty}^{\eta}&\left[S^{(2)}(\tilde{\phi}^{(1)})+\gamma^{(1)}\frac{d\tilde{\phi}^{(1)}}{d\eta}\right]d\eta\\ &=-(2K_2\phi_0^2\Lambda^{-2}+2K_3\phi_0^2\Lambda^{-2}-120K_4\phi_0\Lambda^{-4})\mathrm{sech}^4(\eta\Lambda^{-1})\\ &\quad-\left(\frac{K_1}{3}\phi_0^3-2K_2\phi_0^2\Lambda^{-2}-4K_3\phi_0^2\Lambda^{-2}+120K_4\phi_0\Lambda^{-4}\right)\mathrm{sech}^6(\eta\Lambda^{-1}),\end{aligned} \tag{28}$$

where

$$K_1=\frac{J_1}{f},\quad K_2=\frac{[(J_4-J_5-J_6)l_z^2+J_5+J_6]}{f},$$

$$K_3=\frac{[(J_2-J_3)l_z^2+J_3]}{f},$$

$$K_4=\frac{[(J_6-J_8+J_9)l_z^4+(J_8-2J_9)l_z^2+J_9]}{f}.$$

The resonant term in $S^{(2)}(\phi^{(1)})$ is canceled out if we put

$$\gamma^{(1)}=16l_zfK_4\Lambda^{-4}. \tag{29}$$

To solve Eq. (26), we define the new independent variable

$$\mu=\tanh(\eta\Lambda^{-1}), \tag{30}$$

thereby Eq. (26) becomes

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{d\tilde{\phi}^{(2)}}{d\mu}\right]+\left[3(3+1)-\frac{2^2}{(1-\mu^2)}\right]\tilde{\phi}^{(2)}=T(\mu), \tag{31}$$

with

$$T(\mu)=K_5(1-\mu^2)+K_6(1-\mu^2)^2,$$

where

$$K_5=-[2\phi_0^2(K_2+K_3)-120K_4\phi_0\Lambda^{-2}],$$

$$K_6 = -\left[\frac{K_1\phi_0^3\Lambda^2}{3} - 2\phi_0^2(K_2+2K_3) + 120K_4\phi_0\Lambda^{-2}\right].$$

The two independent solutions of the homogeneous part of Eq. (31) are given by the associated Legendre functions of the first and the second kinds,

$$P_3^2(\mu) = 15\mu(1-\mu^2), \tag{32}$$

$$Q_3^2(\mu) = \frac{15}{2}\mu(1-\mu^2)\ln\left(\frac{1+\mu}{1-\mu}\right) + \frac{\mu^2(5\mu^2-3)}{(1-\mu^2)} + 20\mu^2 - 8. \tag{33}$$

The complementary solution of Eq. (31) is given by

$$\tilde{\phi}_c^{(2)} = C_1P_3^2(\mu) + C_2Q_3^2(\mu). \tag{34}$$

Here the first term is the secular one, which can be eliminated by renormalizing the amplitude. Also, $C_2=0$ as a result of the vanishing boundary condition for $\tilde{\phi}^{(2)}(\eta)$ as $|\eta|\to\infty$.

Using the method of variation of parameters,[19,36,38] the particular solution of Eq. (31) can be written as

$$\tilde{\phi}_p^{(2)} = L_1(\mu)P_3^2(\mu) + L_2(\mu)Q_3^2(\mu), \tag{35}$$

where $L_1(\mu)$ and $L_2(\mu)$ are given by

$$L_1(\mu) = -\int\frac{T(\mu)Q_3^2(\mu)}{(1-\mu^2)W(P_3^2,Q_3^2)}d\mu \quad \text{and}$$

$$L_2(\mu) = \int\frac{T(\mu)P_3^2(\mu)}{(1-\mu^2)W(P_3^2,Q_3^2)}d\mu, \tag{36}$$

and the Wronskian $W(P_3^2,Q_3^2)$ is given by

$$W(P_3^2,Q_3^2) = P_3^2\frac{dQ_3^2}{d\mu} - Q_3^2\frac{dP_3^2}{d\mu} = 120(1-\mu^2)^{-1}. \tag{37}$$

Finally, $\tilde{\phi}_p^{(2)}$ is given by

$$\begin{aligned}\tilde{\phi}_p^{(2)} = {}& \frac{9\gamma^2}{c_3^2l_z^2f}\left[-\frac{f^2}{2c_3}K_1 - \frac{f}{12}(K_2-2K_3)\right.\\ &\left.+\frac{5}{12}c_3K_4\right]\mathrm{sech}^2(\eta\Lambda^{-1}) + \frac{9\gamma^2}{c_3^2l_z^2f}\left[-\frac{f^2}{2c_3}K_1 + \frac{f}{4}(K_2\right.\\ &\left.+2K_3) - \frac{5}{4}c_3K_4\right]\mathrm{sech}^2(\eta\Lambda^{-1})\tanh^2(\eta\Lambda^{-1}),\end{aligned} \tag{38}$$

where

$$\gamma = \Delta M\left(1 - \frac{K_4}{l_zf}\Delta M\right), \tag{39}$$

and the modified width is given by

$$\Lambda = \left(\frac{4l_zf}{\Delta M}\right)^{1/2}\left(1 + \frac{K_4}{2l_zf}\Delta M\right). \tag{40}$$

Using Eqs. (27) and (38) and keeping only terms of order $(\Delta M)^2$, the stationary solution for nonlinear EAWs, with the aid of Eq. (39), is given by

$$\tilde{\phi} = \tilde{\phi}^{(1)} + \tilde{\phi}_p^{(2)}$$

$$\begin{aligned}\tilde{\phi} = {}& \frac{3\Delta M}{c_3l_z}\mathrm{sech}^2(\eta\Lambda^{-1}) + \frac{9(\Delta M)^2}{c_3^2l_z^2f}\left[-\frac{f^2}{2c_3}K_1 - \frac{f}{12}(K_2\right.\\ &\left.-2K_3) + \frac{1}{12}c_3K_4\right]\mathrm{sech}^2(\eta\Lambda^{-1})\\ &+\frac{9(\Delta M)^2}{c_3^2l_z^2f}\left[-\frac{f^2}{2c_3}K_1 + \frac{f}{4}(K_2+2K_3) - \frac{5}{4}c_3K_4\right]\\ &\times\mathrm{sech}^2(\eta\Lambda^{-1})\tanh^2(\eta\Lambda^{-1}).\end{aligned} \tag{41}$$

Tiwari and Sharma[39] studied ion acoustic waves in a plasma with two temperature ions using the renormalization procedure. Later, Yashvir *et al.*[40] investigated the same wave in an ion-beam plasma system. Their evolution equations are the standard KdV and KdV-type equations for the first- and second-order perturbed potentials, respectively. They obtained a solution similar to Eq. (41) that nominated as "dressed soliton." It was shown that the dressed soliton has a very good agreement with the predicted exact solution than the solution of the KdV did.

IV. DISCUSSION AND CONCLUSION

We considered a four-component magnetized plasma consisting of cold and hot electrons, a warm electron beam, and stationary ions. Here we restrict our study to a strongly magnetized and weakly dispersive plasma in the limit of long wavelenghts. Using a RPT, the basic set of fluid equations describing the system leads, at the lowest order of perturbation theory, to a ZK equation, Eq. (16). For a better accuracy, the higher-order nonlinear and dispersion terms have to be included. In this case, a linear inhomogeneous ZK-type equation, Eq. (19), is derived. Using the renormalization method, the solutions of these coupled evolution equations are obtained: (27) and (41).

To examine the effect of the beam parameters, the obliqueness of the wave on the magnetic fields and the magnetic field itself, we analyze numerically both the amplitude and the width of the nonlinear EAWs. Recently, Berthomier *et al.*[34] presented a scaling of three-dimensional (3D) solitary waves observed by FAST and Polar spacecrafts using a nonlinear fluid model of 3D EAWs. Here, we will apply our results to the observed BEN emissions that were recorded as two main bursts (burst a and b) in the auroral dayside region of the Earth's magnetosphere by the Viking satellite.[7–9] Because burst b is more intense than burst a, we will focus our attention on burst b. Figure 1 shows that the three-component magnetized plasma model (without the electron-beam component) permits a rarefactive soliton only and this agrees exactly with the limit of our work (Mace and Hellberg).[17] Also, it is shown that the effect of higher-order nonlinearity is unnecessary in a three-component magnetized plasma. Thus, it is sufficient to describe EAW with a ZK evolution equation in such systems.[17] Figure 2 proves that the introduction of the warm electron beam permits the existence of either rarefactive or compressive soliton depending on the electron-beam streaming velocity. We will con-

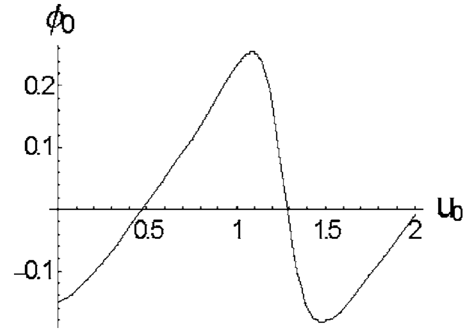

FIG. 2. Plot of the amplitude ϕ_0 against u_0, for V=1.25, l_z=0.75, and γ =0.1.

sider the parametric study of compressive solitons. Figure 3 shows the dependence of both the higher nonlinear solutions $\phi^{(1)}$ and ϕ on the beam parameters and the magnetic field associated with the electron beam. For simplicity, we omit the upper sign on both ϕ. Figure 3 illustrates that, for $\phi^{(1)}$ and ϕ, amplitudes/widths increase as u_0 or T_b increases/decreases. However, their amplitudes are not affected by Ω_b. Their associated widths decrease by increasing Ω_b. Although, ϕ_0 is directly proportional to the reciprocal of l_z, the soliton width is a nonlinear function of l_z. Figure 4 shows the variation of Λ with l_z. It is obvious that for $l_z<0.6$ (>0.6), Λ

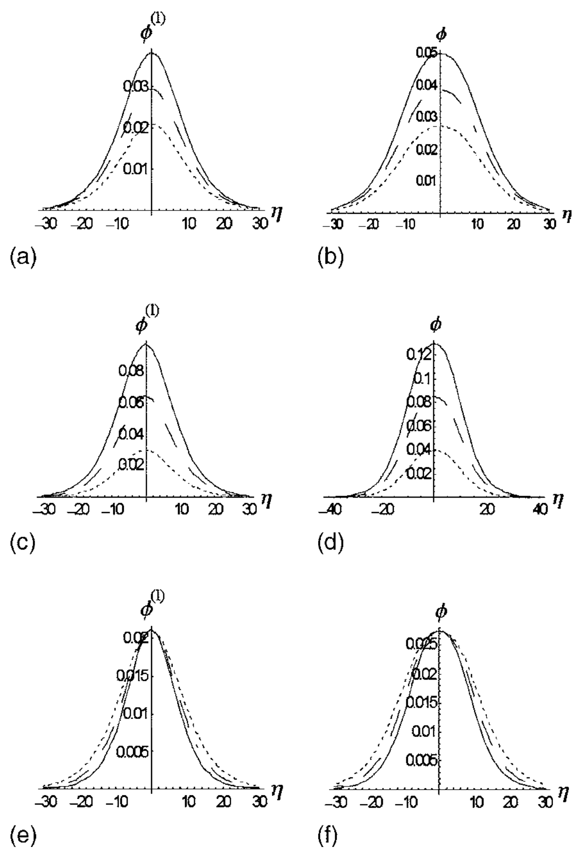

FIG. 3. Plot of $\phi^{(1)}$ and ϕ against η where in (a, b), u_0=0.5 (dotted line), u_0=0.51 (dashed line), and u_0=0.52 (solid line); in (c, d) u_0=0.55, T_b =0.18 (dotted line), T_b=0.2 (dashed line), and T_b=0.22 (solid line); and in (e, f) u_0=0.5, Ω_b=0.14 (dotted line), Ω_b=0.16 (dashed line), and Ω_b=0.18 (solid line) with V=1.25, γ=0.1, and l_z=0.35.

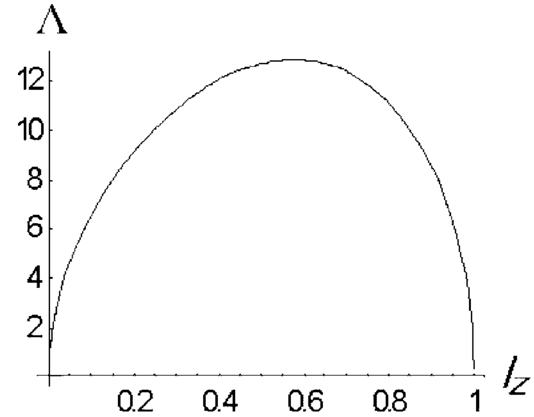

FIG. 4. Plot of soliton width Λ against l_z for V=1.25, u_0=0.5, and γ=0.1.

increases (decreases). This situation is very complicated if we try to study the variation of ϕ with l_z. Moreover, according to the principal rule of the reductive perturbation theory,[19,22,33] the following condition must be satisfied:

$$\frac{|\tilde{\phi}^{(2)}|}{|\tilde{\phi}^{(1)}|} \leqslant 1. \tag{42}$$

Figures 5(a), 5(c), and 5(d) show that, for $0.7<l_z<0.3$, the higher-order nonlinear solution $\tilde{\phi}$ is valid around η=0 only, otherwise $|\tilde{\phi}^{(2)}|>|\tilde{\phi}^{(1)}|$ which leads to forbidden regions. Vice versa, Fig. 5(b) shows that, for $0.7>l_z>0.3$, $\tilde{\phi}$ presents a higher-order nonlinear EAWs for all η values with the transformation of the single hump soliton, $\tilde{\phi}^{(1)}$, to be a new

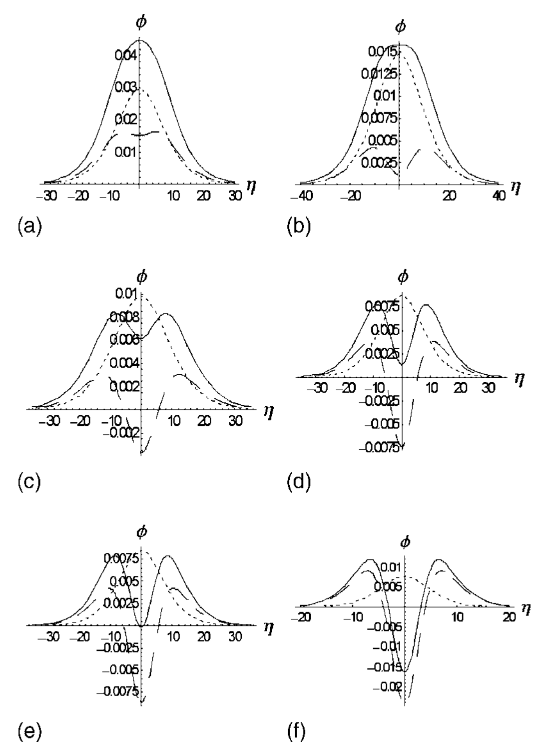

FIG. 5. Plot of $\phi^{(1)}$ (dotted line), $\phi^{(2)}$ (dashed line), and ϕ (solid line) against η with V=1.5, u_0=0.75, l_z=0.25, and γ=0.1 where (a) l_z=0.25, (b) l_z=0.5, (c) l_z=0.75, (d) l_z=0.85, (e) l_z=0.87, and (f) l_z=0.95.

soliton with two humps until $l_z = 0.87$. If the wave obliqueness to the magnetic-field direction, l_z, is > 0.87, no solution is allowable for all η because $|\tilde{\phi}^{(2)}| > |\tilde{\phi}^{(1)}|$ [Figs. 5(e) and 5(f)].

On the other hand, by using the parameters of Dubouloz *et al.*[8,9] (Table 2) for the burst b, $n_{0c} = 0.2\ \mathrm{cm}^{-3}$, $n_{0h} = 1.5\ \mathrm{cm}^{-3}$, $n_{0b} = 1\ \mathrm{cm}^{-3}$, $T_c = 2$ eV, $T_h = 250$ eV, $T_b = 50$ eV, $\Omega_c = 0.3$, and $\Omega_b = 0.14$ will lead to $\lambda_{\mathrm{Dh}} \simeq 71.5$ m which differs than that obtained by Mamun and Shukla[4] due to introduction of new effects here. Using the formula[4,8,9,18,19] $E_0 = \phi_0(K_B T_h / e\lambda_{\mathrm{Dh}}) \simeq \phi_0(250/71.5)\ \mathrm{V/m} \sim 110\ \mathrm{mV/m}$ (Refs. 9 and 15) for $\phi_0 \sim 0.032$, the solution (27) can be transformed into the energy wave form. A similar transformation can be used to transform $\tilde{\phi}$ to E. The attached figures show a good agreement between the present model and the recorded data[7–9] to interpret the BEN emissions observed in the dayside auroral zone of the Earth's magnetosphere.

It is noted here that Ghosh *et al.*[28] obtained recently a dromion solution for obliquely nonlinear EAWs in strongly dispersive space plasmas through derivation DS equation. However, our model is focused on the weakly dispersive ones. At the end, it is quite clear that the higher the nonlinearity, then the wave obliqueness on the magnetic-field direction must be involved to model the observed BEN in the auroral dayside region of the Earth's magnetosphere accurately.

APPENDIX

The coefficients presented in this paper are expressed as follows (we omit Σ_j for simplicity):

$$E_1 = \frac{\rho_j^2}{2}(c_3\tilde{V}_j^2 + 3T_j c_3 + \rho_j\tilde{V}_j^3 + 9T_j\rho_j\tilde{V}_j),$$

$$E_2 = c_1\rho_j^2(\tilde{V}_j^2 + 3T_j),$$

$$E_3 = c_2\rho_j^2(\tilde{V}_j^2 + 3T_j) - \frac{3T_j\rho_j^2\tilde{V}_j^3}{\Omega_j^2}, \quad E_4 = -\rho_j\tilde{V}_j, \tag{A1}$$

$$D_1 = \frac{n_{0j}}{2\tilde{V}_j}(c_3\rho_j + 2\rho_j^2\tilde{V}_j + 2E_1), \quad D_2 = \frac{n_{0j}}{\tilde{V}_j}(c_1\rho_j + E_2),$$

$$D_3 = \frac{n_{0j}}{\tilde{V}_j\Omega_j^2}(c_2\rho_j\Omega_j^2 - \rho_j\tilde{V}_j^3 + E_3\Omega_j^2), \quad D_4 = -n_{0j}\rho_j, \tag{A2}$$

$$F_1 = \frac{3p_{0j}}{2\tilde{V}_j}(c_3\rho_j + 4\rho_j^2\tilde{V}_j + 2E_1), \quad F_2 = \frac{3p_{0j}}{\tilde{V}_j}(c_1\rho_j + E_2),$$

$$F_3 = \frac{3p_{0j}}{\tilde{V}_j\Omega_j^2}(c_2\rho_j\Omega_j^2 - \tilde{V}_j^3\rho_j + E_3\Omega_j^2), \quad F_4 = -3p_{0j}\rho_j, \tag{A3}$$

$$G_1 = \frac{3p_{0j}\rho_j^2}{2n_{0j}} - \frac{F_1}{n_{0j}}, \quad G_2 = -\frac{\tilde{V}_j^4\rho_j}{\Omega_j^2} - \frac{F_2}{n_{0j}},$$

$$G_3 = -\frac{F_3}{n_{0j}}, \quad G_4 = 1 - \frac{F_4}{n_{0j}}, \tag{A4}$$

$$H_1 = c_3\rho_j\tilde{V}_j + 2G_1, \quad H_2 = H_1 + \tilde{V}_j^2\rho_j^2,$$

$$H_3 = c_1\tilde{V}_j\rho_j + G_2, \quad H_4 = c_2\tilde{V}_j\rho_j + G_3, \tag{A5}$$

$$J_1 = \frac{-n_{0h}}{2A} + \frac{D_1}{A\tilde{V}_j}(2c_3 - 3T_j\rho_j^2\tilde{V}_j + 3\rho_j\tilde{V}_j) + \frac{E_1 n_{0j}}{A\tilde{V}_j}(2c_3\rho_j\tilde{V}_j + 3\rho_j^2\tilde{V}_j^2 + 21T_j\rho_j^2 + 3\rho_j) + \frac{F_1}{A\tilde{V}_j}(2c_3\rho_j + 3\rho_j^2\tilde{V}_j) + \frac{3p_{0j}\rho_j^4}{A},$$

$$J_2 = \frac{2c_1}{A\tilde{V}_j}(D_1 + F_1\rho_j + E_1\rho_j\tilde{V}_j n_{0j}) + \frac{D_2}{A\tilde{V}_j}(c_3 + \rho_j\tilde{V}_j) + \frac{c_3F_2\rho_j}{A\tilde{V}_j} + \frac{E_2 n_{0j}}{A\tilde{V}_j}(c_3\rho_j\tilde{V}_j + \rho_j^2\tilde{V}_j^2 + 9T_j\rho_j^2 + \rho_j\tilde{V}_j),$$

$$J_3 = \frac{2c_2}{A\tilde{V}_j}(D_1 + F_1\rho_j + E_1\rho_j\tilde{V}_j n_{0j}) + \frac{D_3}{A\tilde{V}_j}(c_3 + \rho_j\tilde{V}_j) + \frac{F_3c_3\rho_j}{A\tilde{V}_j} + \frac{E_3 n_{0j}}{A\tilde{V}_j}(c_3\rho_j\tilde{V}_j + \rho_j^2\tilde{V}_j^2 + 9T_j\rho_j^2 + \rho_j) + \frac{H_2 n_{0j}}{A\Omega_j^2}(3T_j\rho_j + 1) - \frac{n_{0j}}{A\Omega_j^2}(9T_j\rho_j^3\tilde{V}_j^2 + \rho_j^2\tilde{V}_j^2),$$

$$J_4 = \frac{D_2}{A\tilde{V}_j}(3c_3 - 3T_j\rho_j^2\tilde{V}_j + \rho_j\tilde{V}_j) + \frac{3F_2}{A\tilde{V}_j}(c_3\rho_j + \rho_j^2\tilde{V}_j) + \frac{E_2 n_{0j}}{A\tilde{V}_j}(3c_3\rho_j\tilde{V}_j + \rho_j^2\tilde{V}_j^2 + 3T_j\rho_j^2 + \rho_j),$$

$$J_5 = \frac{D_3}{A\tilde{V}_j}(c_3 - 3T_j\rho_j^2\tilde{V}_j + \rho_j\tilde{V}_j) + \frac{F_3\rho_j}{A\tilde{V}_j}(c_3 + 3\rho_j\tilde{V}_j) + \frac{E_3 n_{0j}}{A\tilde{V}_j}(c_3\rho_j\tilde{V}_j + \rho_j^2\tilde{V}_j^2 + 3T_j\rho_j^2 + \rho_j) + \frac{H_1 n_{0j}}{A\Omega_j^2}(3T_j\rho_j + 1),$$

$$J_6 = \frac{c_1}{A\tilde{V}_j}(D_2 + F_2\rho_j + E_2\rho_j\tilde{V}_j n_{0j}),$$

$$J_7 = \frac{2c_3}{A\tilde{V}_j}(D_3 + F_3\rho_j + E_3\rho_j\tilde{V}_j n_{0j}) + \frac{(H_1 + H_2)n_{0j}}{A\Omega_j^2}(3T_j\rho_j + 1) - \frac{n_{0j}}{A\Omega_j^2}(\rho_j^3\tilde{V}_j^4 + 3T_j\rho_j^3\tilde{V}_j^2 + \rho_j^2\tilde{V}_j^2)$$

$$J_8 = \frac{c_2}{A\widetilde{V}_j}(D_2 + F_2\rho_j + E_2\rho_j\widetilde{V}_j n_{0j}) + \frac{c_1}{A\widetilde{V}_j}(D_3 + F_3\rho_j + E_3\widetilde{V}_j\rho_j n_{0j}) + \frac{H_3 n_{0j}}{A\Omega_j^2}(3T_j\rho_j + 1),$$

$$J_9 = \frac{c_2}{A\widetilde{V}_j}(D_3 + F_3\rho_j + E_3\rho_j\widetilde{V}_j n_{0j}) + \frac{H_4 n_{0j}}{A\Omega_j^2}(3T_j\rho_j + 1). \quad \text{(A6)}$$

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