Effect of two-temperature trapped electrons to nonlinear dust-ion-acoustic solitons

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Propagation of three-dimensional dust-ion-acoustic solitons is investigated in a dusty plasma consisting of positive ions, negatively variable-charged dust particles, and two-temperature trapped electrons. We use the reductive perturbation theory to reduce the basic set of fluid equations to one evolution equation called damped modified Kadontsev-Petviashivili equation. Exact solution of this equation is not possible, so we obtain the time evolution solitary wave form approximate solution. It is found that only compressive soliton can propagate in this system. We develop a theoretical estimate condition under which the solitons can propagate. It is found that this condition is satisfied for Saturn's F ring. It is found also that low electron temperature has a role on the behavior of the soliton width, i.e., for lower (higher) range of low electron temperature the soliton width decreases (increases). However, high electron temperature decreases the width. The trapped electrons have no effect on the soliton width. The ratio of free low (high) to trapped low (high) electron temperatures increases the soliton amplitude. Also, the amplitude increases with free low and free high electron temperatures. To investigate the stabilty of the waves, we used a method based on energy consideration to obtain a condition for stable solitons. It is found that this condition depends on dust charge variation, streaming velocity, directional cosine of the wave vector k along the x axis, and temperatures of dust particles, ions, and free electrons. [DOI: 10.1063/1.2146940]

I. INTRODUCTION

The study of plasmas containing heavy dust particles is very important in understanding many space and astrophysical phenomena as well as for many industrial and physical applications, such as etching experiments and experiments on dust plasma crystals.[1] The presence of such heavy particles could modify the plasma normal modes. In particular, the ion-acoustic waves (IAWs) are one of the modified normal modes, which are called the dust-ion-acoustic waves (DIAWs).

The DIAWs have been of interest since the first theoretical description by Shukla and Silin[2] and experimental observation in laboratory experiment by Barkan *et al.*[3] The linear properties of DIAWs in dusty plasmas were understood from both theoretical and experimental points of view.[1–5] Recently, nonlinear waves associated with DIAWs, particularly the dust-ion-acoustic solitons (DIASs) and shocks, have also received a great deal of interest in understanding the basic properties of localized electrostatic perturbations in space and laboratory dusty plasmas.[6–11] However, all these investigations are limited to constant dust grain charge. In fact, the charges on the dust particles are not constant, because the imbalance of electron current and ion current flowing through the grain surface causes charge fluctuation. On the other hand, one can consider dusty plasma as always an open system because the currents of electrons and ions flowing onto the dust grains (as well as the energy flows) should be maintained by external sources of the plasma particles and the energy. The dissipation rate is high. Therefore, there is a tendency to self-organization and formation of long-living nonlinear dissipative and coherent structures in a plasma such as shock waves, solitons, cavitons, collapsing cavities, etc.[12] Both shocks and solitons in dusty plasmas can be formed by different means. These are not necessarily restricted to the mode excitation due to instabilities or an external forcing, but can also be regular collective process analogous to the shock wave generation in gas dynamics. The anomalous dissipation in dusty plasmas, which originates from the dust particle charging process, makes possible existence of a new kind of shocks related to this dissipation.[13,14] In the absence of dissipation (or if the dissipation is weak at the characteristic dynamical time scales of the system), the balance between nonlinear and dispersion effects can result in the formation of symmetrical solitary waves—a soliton. Investigation of the anomalous dissipation is especially interesting at the ion-acoustic time scales. The charging processes at these time scales are usually not in equilibrium and, hence, the role of anomalous dissipation might be crucial.[13,15] So far, study of nonlinear structures at ion-acoustic time scales (in dusty plasma) was mostly related to shocks.[13,14,16,17]

An experimental investigation of DIASs was done by Nakamura and Sarma.[9] The first theoretical study of DIASs in dusty plasmas[7] neglected absorption and scattering of electrons and ions by microparticles. These processes, resulting in the anomalous dissipation, make the existence of "pure" steady-state nonlinear structures impossible.[18] Later, the influence of the anomalous dissipation on DIASs was studied by Popel *et al.*[18] They investigated the evolution of the soliton-like perturbations in dusty plasma, taking into account the dissipation processes and trapped electrons. They found that the properties of compressive solitons with trapped electrons are very different from those with Boltzmann electrons. They related the possibility of existence of the solitons to the fact that in case of the presence of trapped electrons, the width of the region of the Mach number (for which soliton solution are possible) is much wider than in the case of Boltzmann electrons. During its evolution a soliton is slowed down. Thus in the case of Boltzmann electrons, a soliton leaves the region of the Mach number inherent in solitons (which is rather narrow for the case of Boltzmann electrons) very soon, and a soliton transforms into the shock-like perturbation. Consequently, for the existence of the damped solitons, the perturbation should have an initial form, so that it would allow the presence of both free and trapped electrons. Otherwise, there is a possibility of an appearance of DIA shocks in dusty plasmas. El-Labany *et al.*[19] studied the effects of trapped electron temperature, dust charge variation, and grain radius on the nonlinear DIASs in dusty plasma with trapped electrons. They showed that the nonlinear DIASs are damped waves and these waves are governed by a damped modified Korteweg-de Vries equation. Also, they found that only compressive DIASs can propagate in dusty plasmas with trapped electrons. The amplitude and the width of the solitons depend mainly on the trapped electron temperature, dust charge variations, and grain radius. The existence of the solitons is independent of the trapped electron temperature. Moslem *et al.*[20] used the ionization source model to investigate the effects of ion streaming velocity, dust charge fluctuations, and trapped electrons on the propagation of two-dimensional DIASs in dusty plasmas. They showed that this model could be applicable to describe the solitons that may appear in supernova shells. The soliton amplitude and width depend mainly on trapped electrons, transverse perturbation, ion streaming velocity, dust charge fluctuations, the frequency of ion recombination on dust particles, the plasma ionization frequency, and the frequency characterizing a loss in ion momentum due to recombination on dust particles and Coulomb elastic collisions between ions and dusts. Finally, it is necessary to mention that the form of the initial perturbation could be important from the viewpoint that we want to observe, shocks or solitons. For example, both DIA shocks[17] and DIA solitons[9] were observed in a double plasma device at the Institute of Space and Astronautical Science (Japan). In both experiments the plasma conditions were (almost) the same but the difference was in initial perturbation.

Many laboratory as well as space plasma observation showed that the IAWs may be observed in the presence of two-electron temperatures. On the other hand, plasmas with two-temperature electrons may occur in edge plasmas of fusion devices consisting of energetic (hot) electrons due to plasma heating by strong rf fields including lower hybrid waves, Alfven waves, ion Bernstein waves, etc. Two-temperature Maxwellian electron distributions may also occur in low-temperature plasmas due to inelastic electron collisions with exited atoms, ions, and molecules as well as in expanding corona of plasma heated by a laser and in negative-ion sources.[21] For example, Jones *et al.*,[22] in order to model their experimental observations on IAWs, they treated such a two-electron temperature plasma as two fluids. Goswami and Buti[23] studied the solution of a two-electron temperature plasma using a Korteweg-de Vries (KdV) equation, while Shukla and Tagare[24] obtained the shock solution for a collisional two-electron temperature plasma using KdV-Burgers equation. Buti[25] obtained the soliton solution by using the quasipotential analysis of a two-electron temperature plasma. Nishihara and Tajiri[26] studied the condition for getting compressive and rarefactive ion-acoustic solitons for both small and large amplitude cases. Tagare[27] used the reductive perturbation method and quasipotential analysis to reinvestigate the model of Nishihara and Tajiri.[26] He found that a two-electron temperature plasma with isothermal electrons and cold ions admits both compressive and rarefactive solitons as well as compressive and rarefactive double layers (depending on the concentration of low-temperature isothermal electrons). These investigations were considered in an ordinary plasma, however, this subject (two-temperature electrons) has little attention in plasma containing heavy dust particles. For example, Chutov *et al.*[21] studied the parameters of self-consistent dusty sheaths using computer simulation of the temporal evolution of one-dimensional slab plasma with two-temperature electrons and dust particles.

The goals of this paper are the following.

(i) Investigate the effect of two-temperature trapped electrons to the propagation characteristics of the DIASs.
(ii) Determine the condition when the existence of (quasi) steady-state solitons is possible in case of a weak dissipative.
(iii) Investigate the stability of the DIASs.
(iv) Describe the DIASs that may appear in Saturn's F ring.

II. THEORY

Let us consider a fully ionized, collisionless, unmagnetized dusty plasma consisting of a mixture of warm positive ions, warm negatively charged dust particles, and two-temperature trapped electrons. The nonlinear dynamics of three-dimensional DIASs is governed as follows:

for positive ions,

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_iu_{ix})+\frac{\partial}{\partial y}(n_iu_{iy})+\frac{\partial}{\partial z}(n_iu_{iz}) =-\nu_{ch}n_i+\nu_i(n_{e\ell}+n_{\mathrm{eh}}), \tag{1}$$

$$\frac{\partial}{\partial t}(n_iu_{ix})+\frac{\partial}{\partial x}(n_iu_{ix}^2)+\frac{\partial}{\partial y}(n_iu_{ix}u_{iy})+\frac{\partial}{\partial z}(n_iu_{ix}u_{iz}) +\frac{5}{3}\sigma_in_i^{2/3}\frac{\partial n_i}{\partial x}+n_i\frac{\partial\phi}{\partial x}=-\tilde{\nu}n_iu_{ix}, \tag{2}$$

$$\frac{\partial}{\partial t}(n_i u_{iy}) + \frac{\partial}{\partial x}(n_i u_{iy} u_{ix}) + \frac{\partial}{\partial y}(n_i u_{iy}^2) + \frac{\partial}{\partial z}(n_i u_{iy} u_{iz}) + \frac{5}{3}\sigma_i n_i^{2/3}\frac{\partial n_i}{\partial y} + n_i \frac{\partial \phi}{\partial y} = -\tilde{\nu} n_i u_{iy}, \tag{3}$$

$$\frac{\partial}{\partial t}(n_i u_{iz}) + \frac{\partial}{\partial x}(n_i u_{iz} u_{ix}) + \frac{\partial}{\partial y}(n_i u_{iz} u_{iy}) + \frac{\partial}{\partial z}(n_i u_{iz}^2) + \frac{5}{3}\sigma_i n_i^{2/3}\frac{\partial n_i}{\partial z} + n_i \frac{\partial \phi}{\partial z} = -\tilde{\nu} n_i u_{iz}, \tag{4}$$

for negative dust particles,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_{dx}) + \frac{\partial}{\partial y}(n_d u_{dy}) + \frac{\partial}{\partial z}(n_d u_{dz}) = 0, \tag{5}$$

$$\frac{\partial u_{dx}}{\partial t} + \left(u_{dx}\frac{\partial}{\partial x} + u_{dy}\frac{\partial}{\partial y} + u_{dz}\frac{\partial}{\partial z}\right)u_{dx} + \frac{5\sigma_d}{3\mu_d}n_d^{-1/3}\frac{\partial n_d}{\partial x} - \frac{Z_d^{(0)}Z_d}{\mu_d}\frac{\partial \phi}{\partial x} = 0, \tag{6}$$

$$\frac{\partial u_{dy}}{\partial t} + \left(u_{dx}\frac{\partial}{\partial x} + u_{dy}\frac{\partial}{\partial y} + u_{dz}\frac{\partial}{\partial z}\right)u_{dy} + \frac{5\sigma_d}{3\mu_d}n_d^{-1/3}\frac{\partial n_d}{\partial y} - \frac{Z_d^{(0)}Z_d}{\mu_d}\frac{\partial \phi}{\partial y} = 0, \tag{7}$$

$$\frac{\partial u_{dz}}{\partial t} + \left(u_{dx}\frac{\partial}{\partial x} + u_{dy}\frac{\partial}{\partial y} + u_{dz}\frac{\partial}{\partial z}\right)u_{dz} + \frac{5\sigma_d}{3\mu_d}n_d^{-1/3}\frac{\partial n_d}{\partial z} - \frac{Z_d^{(0)}Z_d}{\mu_d}\frac{\partial \phi}{\partial z} = 0, \tag{8}$$

for two-temperature trapped electrons,

$$n_{e\ell} = \mu_{e\ell}\left[\exp(\beta_{e\ell}\phi) - \frac{4}{3}b_\ell(\beta_{e\ell}\phi)^{3/2}\right], \tag{9}$$

$$n_{eh} = \mu_{eh}\left[\exp(\beta_{eh}\phi) - \frac{4}{3}b_h(\beta_{eh}\phi)^{3/2}\right]. \tag{10}$$

These equations are coupled through Poisson's equation as

$$\alpha\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi = -n_i + Z_d^{(0)}Z_d n_d + n_{e\ell} + n_{eh}. \tag{11}$$

Dust particles are charged due to a variety of processes including the bombardment of the dust grain surface by background plasma electrons, ions and incident ion beams, photoelectron emission by ultraviolet (UV) radiation, ion sputtering, secondary electron production, etc. Dust particles are mainly negatively charged because any plasma electrons hitting the surface of the dust grains are attached to it and simply lost from the background plasma.[28] In general, the dust charge variable Q_d is determined by the charge current balance equation[29,30]

$$\frac{dQ_d}{dt} = I_i + I_{e\ell} + I_{eh}. \tag{12}$$

Now, we consider a more general dusty plasma situation in which the ions have some finite streaming speed. For such a situation, the ion current has the following expression:[1]

$$I_i = e\pi r^2\left(\frac{8T_i}{\pi m_i}\right)^{1/2} n_i\left[F_1 - F_2\frac{e\Phi}{T_i}\right], \tag{13}$$

and the electron currents can be written as[1]

$$I_{e\ell} = -e\pi r^2\left(\frac{8T_{e\ell}}{\pi m_e}\right)^{1/2} n_{e\ell}\exp\left(\frac{e\Phi}{T_{e\ell}}\right), \tag{14}$$

$$I_{eh} = -e\pi r^2\left(\frac{8T_{eh}}{\pi m_e}\right)^{1/2} n_{eh}\exp\left(\frac{e\Phi}{T_{eh}}\right), \tag{15}$$

where Φ denotes the dust grain surface potential which is related to the plasma potential ϕ. Using Eqs. (13)–(15) into Eq. (12) and normalizing the final equation, we obtain the normalized charging equation in the form

$$-\frac{dZ_d}{dt} = -L_{1\ell}n_{e\ell}\exp(L_{3\ell}Z_d) - L_{1h}n_{eh}\exp(L_{3h}Z_d) + L_2 n_i(F_1 - F_2 L_4 Z_d), \tag{16}$$

where

$$\mu_d = \frac{m_d}{m_i}, \quad \sigma_{i,d} = \frac{T_{i,d}}{T_{eff}}, \quad \mu_{e\ell,h} = \frac{n_{e\ell,h}^{(0)}}{n_0}, \quad \beta_{e\ell,h} = \frac{T_{eff}}{T_{efl,h}},$$

$$b_{\ell,h} = \frac{1}{\sqrt{\pi}}(1 - \beta_{\ell,h}), \quad \beta_{\ell,h} = \frac{T_{efl,h}}{T_{etl,h}},$$

$$L_{1\ell,h} = \frac{\pi r_d^2 n_0}{\omega_{pi}Z_d^{(0)}}\sqrt{\frac{8T_{efl,h}}{\pi m_e}}, \quad L_2 = \frac{\pi r_d^2 n_0}{\omega_{pi}Z_d^{(0)}}\sqrt{\frac{8T_i}{\pi m_i}},$$

$$L_{3\ell,h} = -\frac{e^2 Z_d^{(0)}}{CT_{efl,h}}, \quad L_4 = -\frac{e^2 Z_d^{(0)}}{CT_i},$$

$$F_1(u_o) = \frac{\sqrt{\pi}}{4u_o}(1 + 2u_o^2)\mathrm{erf}(u_o) + \frac{1}{2}\exp(-u_o^2),$$

$$F_2(u_o) = \frac{\sqrt{\pi}}{2u_o}\mathrm{erf}(u_o), \quad T_{eff} = \frac{T_{efl}T_{efh}}{\mu_{e\ell}T_{efh} + \mu_{eh}T_{ef\ell}}.$$

Such modification in the electron current modifies the corresponding frequencies (ν_{ch}, ν_i, and $\tilde{\nu}$) in the ion continuity equation and ion momentum equation. These new frequencies can be calculated with the aid of Refs. 12 and 18. Thus, we will use the ionization source model with these new frequencies.

In Eqs. (1)–(16), n_i, n_d, $n_{e\ell}$, and n_{eh} are the densities of positive ions, dust grains, and low- and high-temperature electrons, respectively. u_q ($q=i$ and d) are the velocities of positive ions and dust grains, respectively. ϕ is the electrostatic potential. x, y, and z are the space coordinate, and t is the time variable. ν_{ch} is the frequency of ion recombination on dust particles, ν_i is the plasma ionization frequency, and $\tilde{\nu}$ is the frequency characterizing a loss in ion momentum due to recombination on dust particles and Coulomb elastic collisions between ions and dusts. T_i, T_d, $T_{ef\ell}$, T_{efh}, $T_{et\ell}$, and T_{eth}

are the temperatures of positive ions, dust grains, free low electrons, free high electrons, trapped low electrons, and trapped high electrons, respectively. Z_d denotes to the dust grain charge number. m_d and m_i are the dust grain mass and the ion mass, respectively. $C[=r_d\exp(-r_d/\lambda_D)]$ is the capacitance of the spherical dust grains. r_d is the radius of the dust grains. $u_o[=v_o/V_{Ti}]$ is the ion streaming velocity, v_o is the unnormalized ion streaming velocity, and $V_{Ti}[=(T_i/m_i)^{1/2}]$ is the ion thermal velocity. We normalized all physical quantities as follows: n_i, n_d, $n_{e\ell}$, and n_{eh} are normalized by background total electron density n_0, u_q by the ion-acoustic speed $C_s[=(T_{eff}/m_i)^{1/2}]$, ϕ by T_{eff}/e, t by the inverse of the plasma frequency $\omega_{pi}^{-1}[=(m_i/4\pi e^2 n_i^{(0)})^{1/2}]$, x, y, and z by the modified electron Debye length $\lambda_D=(T_{eff}/4\pi e^2 n_i^{(0)})^{1/2}$, ν_{ch}, ν_i, and $\tilde{\nu}$ by the plasma frequency ω_{pi} and Z_d by the unperturbed number of charges residing on the dust grains $Z_d^{(0)}$. The charge neutrality at equilibrium requires that $n_i^{(0)}=Z_d^{(0)}n_d^{(0)}+n_{e\ell}^{(0)}+n_{eh}^{(0)}$, where $n_i^{(0)}$ and $n_d^{(0)}$ are the unperturbed ion and dust number densities, respectively.

To derive the nonlinear dynamical equation for the DIASs from Eqs. (1)–(16), we employ a reductive perturbation theory.[31] According to this theory, we introduce the stretched space-time coordinates[31,32]

$$X=\varepsilon^{1/4}(x-\lambda t),\quad Y=\varepsilon^{1/2}y,\quad Z=\varepsilon^{1/2}z,\quad \text{and } T=\varepsilon^{3/4}\lambda t, \tag{17}$$

where ε is a smallness parameter measuring the weakness of the nonlinearity, and λ is the wave speed normalized by C_s. The dependent variables are expanded as

$$\Psi=\Psi^{(0)}+\sum_{n=1}^{\infty}\varepsilon^{(n+1)/2}\Psi^{(n)}, \tag{18}$$

where

$$\Psi=[n_i,n_d,u_{ix},u_{dx},\phi,Z_d]^T, \tag{19}$$

$$\Psi^{(0)}=[\alpha,\delta,u_{ixo},0,0,1]^T, \tag{20}$$

while $u_{iy,z}$ and $u_{dy,z}$ are given by

$$u_{iy,z}=\varepsilon^{5/4}u_{iy,z}^{(1)}+\varepsilon^{7/4}u_{iy,z}^{(2)}+\cdots, \tag{21}$$

$$u_{dy,z}=\varepsilon^{5/4}u_{dy,z}^{(1)}+\varepsilon^{7/4}u_{dy,z}^{(2)}+\cdots. \tag{22}$$

Assuming that the ion streaming velocity is along x axis only. Applying the relations (17)–(22) to the basic equations (1)–(11) and (16) and following the usual procedure of the reductive perturbation theory, the lowest-order terms yield (we have assumed that $\nu_{ch}\sim\varepsilon^{3/4}\nu_{cho}$, $\nu_i\sim\varepsilon^{3/4}\nu_{io}$, and $\tilde{\nu}\sim\varepsilon^{3/4}\nu_o$) (Ref. 19)

$$\frac{S}{\alpha}n_i^{(1)}=\frac{S}{\lambda_1}u_{ix}^{(1)}=\frac{-G}{\delta Z_d^{(0)}}n_d^{(1)}=\frac{-G}{\lambda Z_d^{(0)}}u_{dx}^{(1)}=\frac{-F}{R_1+(\alpha Q/S)}Z_d^{(1)}=\phi^{(1)}, \tag{23}$$

$$\frac{S}{\lambda_1}\frac{\partial u_{iy}^{(1)}}{\partial X}=\frac{-G}{\lambda Z_d^{(0)}}\frac{\partial u_{dy}^{(1)}}{\partial X}=\frac{\partial\phi^{(1)}}{\partial Y}, \tag{24}$$

$$\frac{S}{\lambda_1}\frac{\partial u_{iz}^{(1)}}{\partial X}=\frac{-G}{\lambda Z_d^{(0)}}\frac{\partial u_{dz}^{(1)}}{\partial X}=\frac{\partial\phi^{(1)}}{\partial Z}, \tag{25}$$

and Poisson's equation gives the following dispersion relation:

$$\frac{\alpha}{S}+\frac{\delta Z_d^{(0)2}}{G}+\frac{\delta Z_d^{(0)}}{F}\left(R_1+\frac{\alpha Q}{S}\right)=\mu_{e\ell}\beta_{e\ell}+\mu_{eh}\beta_{eh}, \tag{26}$$

where

$$R_1=R_\ell\mu_{e\ell}\beta_{e\ell}+R_h\mu_{eh}\beta_{eh},$$

$$R_{\ell,h}=-L_{1\ell,h}-L_{1\ell,h}L_{3\ell,h}-\frac{1}{2}L_{1\ell,h}L_{3\ell,h}^2,$$

$$Q=L_2(F_1-F_2L_4),$$

$$F=-\mu_{e\ell}L_{1\ell}(L_{3\ell}+L_{3\ell}^2)-\mu_{eh}L_{1h}(L_{3h}+L_{3h}^2)-\alpha F_2L_2L_4,$$

$$S=\lambda_1^2-\frac{5}{3}\sigma_i\alpha^{2/3},\quad G=\lambda^2\mu_d-\frac{5}{3}\sigma_d\delta^{2/3},\quad \lambda_1=\lambda-u_{ixo}.$$

If we consider the next order in ε, we obtain a system of equations in the second-order perturbed quantities. Solving this system with the aid of (23)–(26), we finally obtain the damped modified Kadomtsev-Petviashivili (DMKP) equation

$$\frac{\partial}{\partial X}\left[\frac{\partial\phi^{(1)}}{\partial T}+AB\sqrt{\phi^{(1)}}\frac{\partial\phi^{(1)}}{\partial X}+\frac{1}{2}\alpha A\frac{\partial^3\phi^{(1)}}{\partial X^3}+\frac{1}{2}AC\phi^{(1)}\right]+\frac{1}{2}AD\left[\frac{\partial^2\phi^{(1)}}{\partial Y^2}+\frac{\partial^2\phi^{(1)}}{\partial Z^2}\right]=0, \tag{27}$$

where

$$A=\left[\frac{\lambda^2\delta\mu_dZ_d^{(0)2}}{G^2}+\frac{\lambda\lambda_1\alpha\delta QZ_d^{(0)}}{FS^2}+\frac{\lambda\lambda_1\alpha}{S^2}\right]^{-1},$$

$$B=\mu_{e\ell}b_\ell\beta_{e\ell}^{3/2}+\mu_{eh}b_h\beta_{eh}^{3/2}-\frac{R_2\delta Z_d^{(0)}}{F},$$

$$C=\left[1+\frac{\delta QZ_d^{(0)}}{F}\right]\left[\frac{\lambda_2\alpha\nu_{cho}}{S^2}-\frac{\lambda_2\nu_{io}}{S}(\mu_{e\ell}\beta_{e\ell}+\mu_{eh}\beta_{eh})+\frac{\lambda\alpha\nu_o}{S^2}\right],$$

$$D=\left[\frac{\lambda^2\delta\mu_dZ_d^{(0)2}}{G^2}+\frac{\lambda_1^2\alpha}{S^2}\left(1+\frac{\delta QZ_d^{(0)}}{F}\right)\right],$$

$$R_2=R_\ell\mu_{e\ell}b_\ell\beta_{e\ell}^{3/2}+R_h\mu_{eh}b_h\beta_{eh}^{3/2},$$

$$\lambda_2=\lambda-2u_{ixo}.$$

III. DISCUSSION

To obtain the solution of Eq. (27) we introduce the variable

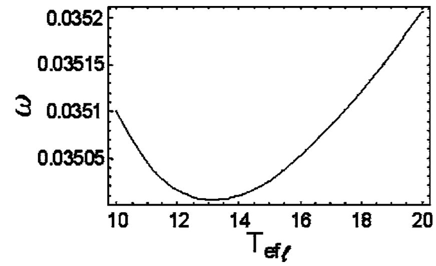

FIG. 1. ω is plotted against $T_{\text{ef}\ell}$ for $\alpha=2$, $\delta=0.1$, $\mu_{\text{e}\ell}=0.7$, $\mu_{\text{eh}}=0.3$, $n_d=1$, $Z_d=10$, $\ell=0.5$, $\lambda=2$, $r_d=6\times10^{-13}$, $T_i=1$, $T_{\text{efh}}=50$, $T_{\text{et}\ell}=40$, $T_{\text{eth}}=100$, $T_d=0.01$, $\nu_{cho}=6.6$, $\nu_{io}=7$, $\nu_o=8.8$, $\tau=2$, $\mu_d=10^{12}$, and $u_{xo}=0.4$.

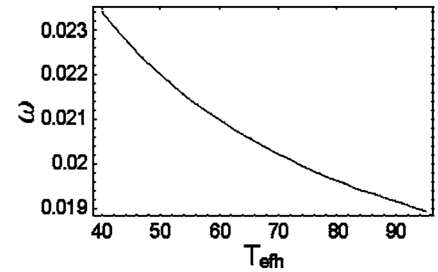

FIG. 2. ω is plotted against T_{efh}. The parameters are the same as Fig. 1 and $T_{\text{ef}\ell}=10$.

$$\chi=\ell X+mY+nZ-U\tau,$$

where χ is the transformed coordinates with respect to a frame moving with velocity U. ℓ, m, and n are the directional cosine of the wave vector k along the X, Y, and Z axes, respectively, so that $\ell^2+m^2+n^2=1$. Equation (27) can be integrated with respect to the variable χ and using the vanishing boundary condition for $\phi^{(1)}$ and their derivatives up to second order for $|\chi|\to\infty$, we obtain the time evolution solitary wave form approximate solution as

$$\phi=He^{-(1/2)AC\tau}\operatorname{sech}^4\sqrt{\frac{B\sqrt{He^{-(1/2)AC\tau}}}{15}}\chi, \tag{28}$$

where $\phi\equiv\phi^{(1)}$. To obtain the value of H, let $C=0$ in Eq. (27) and then its solution is given by

$$\phi=\left(\frac{15\bar{h}}{8AB\ell^2}\right)^2\operatorname{sech}^4\sqrt{\frac{\bar{h}}{8\alpha A\ell^4}}\bar{\chi}, \tag{29}$$

where $\bar{\chi}$ is the transformed coordinates with respect to a frame moving with velocity $\bar{U}$ at $C=0$ (i.e., for $C=0$; $\chi\to\bar{\chi}$ and $U\to\bar{U}$). $\bar{h}=\bar{U}\ell-\frac{1}{2}AD(1-\ell^2)$. From (28) and (29) it is clear that $H=(15\bar{h}/8AB\ell^2)^2$. Thus Eq. (29) can be rewritten as

$$\phi=\phi_o\operatorname{sech}^4(\chi/\omega), \tag{30}$$

where the amplitude ϕ_o and the width ω are given by $(15\bar{h}/8AB\ell^2)^2e^{(-1/2)AC\tau}$ and $\sqrt{(8\alpha A\ell^4/\bar{h})\sqrt{e^{(1/2)AC\tau}}}$, respectively.

To determine the stability or the properties of the instability associated with a given plasma equilibrium, we will use a method based on energy considerations. According to this method it is necessary to calculate the change in potential energy of the plasma as a result of a given perturbation.[33] From Eq. (30), it is clear that the coefficient C is responsible for damping the wave. So, for simplicity we put $C=0$ and integrate Eq. (27) to yield the nonlinear equation of motion as

$$\frac{1}{2}\left(\frac{d\phi^{(1)}}{d\chi}\right)^2+V(\phi^{(1)})=0, \tag{31}$$

where the Sagdeev potential $V(\phi^{(1)})$ is given by

$$V(\phi^{(1)})=\frac{8B}{15\alpha\ell^2}(\phi^{(1)})^{5/2}-\frac{\bar{h}}{A\alpha\ell^4}(\phi^{(1)})^2. \tag{32}$$

A necessary condition for the existence of solitary waves is

$$d^2V(\phi^{(1)})/d\phi^{(1)2}<0 \quad \text{for } \phi^{(1)}=0. \tag{33}$$

A value of $d^2V(\phi^{(1)})/d\phi^{(1)2}$ greater than zero predicts the formation of unstable soliton in the plasma. From (32) and (33) we have

$$d^2V(\phi^{(1)})/d\phi^{(1)2}=-\frac{2\bar{h}}{A\alpha\ell^4}. \tag{34}$$

Equation (34) shows that stable solitons will exist when $2\bar{h}/A\alpha\ell^4>0$; otherwise stable solitons do not exist in the plasma. It is clear that A, α, and ℓ are always positive but $\bar{h}$ may be negative. For positive $\bar{h}$, the following condition must be satisfied:

$$\ell^2+\frac{2\bar{U}}{AD}\ell-1>0. \tag{35}$$

From this condition, it is clear that the existence of solitary waves requires a necessary condition depending on ℓ, A, and D. Also, for $T_{\text{eff}}=T_{\text{effc}}$ [$=5T_i\alpha^{2/3}/3\lambda_1^2$ or $5T_d\delta^{2/3}/3\lambda^2\mu_d$] the value of $A=0$ and the soliton cannot exist.

Before going to the numerical analysis, it is necessary to clarify the condition under which the solitons or shocks can propagate in dusty plasma. This condition could be derived from Eqs. (11) and (16) but in dimensional forms. When the soliton wave structure is formed, the soliton width $\Delta\xi$ is described by the following theoretical estimate:

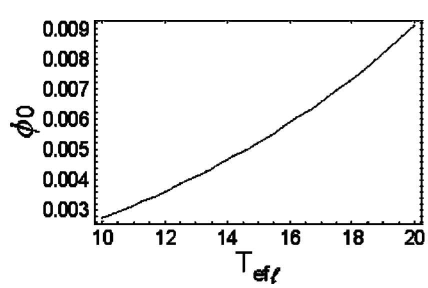

FIG. 3. ϕ_0 is plotted against $T_{\text{ef}\ell}$. The parameters are the same as Fig. 1.

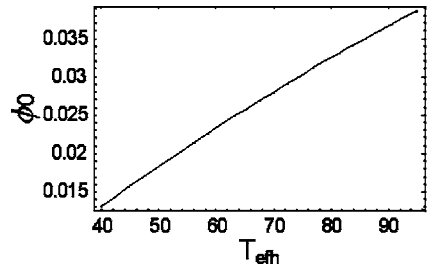

FIG. 4. ϕ_0 is plotted against $T_{\rm efh}$. The parameters are the same as Fig. 2.

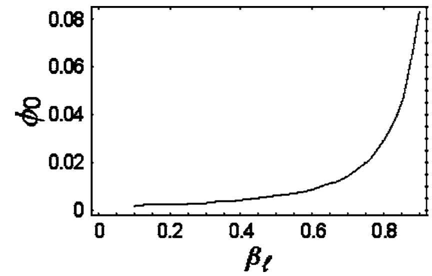

FIG. 5. ϕ_0 is plotted against β_ℓ. For $\alpha=2$, $\delta=0.1$, $\mu_{e\ell}=0.7$, $\mu_{\rm eh}=0.3$, $n_d=1$, $Z_d=10$, $\ell=0.5$, $\lambda=2$, $r_d=6\times10^{-13}$, $T_i=1$, $T_{\rm efh}=50$, $T_{\rm eth}=100$, $T_d=0.01$, $\nu_{cho}=6.6$, $\nu_{io}=7$, $\nu_o=8.8$, $\tau=2$, $\mu_d=10^{12}$, and $u_{xo}=0.4$.

$$\frac{\Delta\xi}{\Sigma} \ll 1, \tag{36}$$

where $\Sigma=(M|\phi_0|/4\pi n_d\nu_d q_d)^{1/3}$, $\xi=x-Mt$, $M[=V/C_s]$ is the Mach number, V is the soliton speed, ϕ_0 is the soliton amplitude, and ν_d is the grain charging rate given in Ref. 12. When one uses inequality (36), it is important to determine which terms are more important, i.e., if one considers some nonlinear structure and its characteristic width $\Delta\xi \ll \Sigma$ then this nonlinear structure is soliton. Otherwise, if the characteristic scale of the change of the parameters of the structure satisfies the inequality $\Delta\xi \gg \Sigma$, then this nonlinear structure is expected to be shock wave. It is important to mention here that the soliton wave is formed due to ion motion, but the presence of dust could make the soliton wave disappears and converts to shock wave or at least modifies its features. This is clear from condition (36), which indicates that the dust parameters n_d, ν_d, and Q_d have an important role in the existence of soliton.

Now, one may ask to what extent the fluid equations that we used are applicable to experimental situations or space plasma observations? At the beginning, we assumed that the system under investigation is a fully ionized, weakly coupled, three-component dusty plasma consisting of warm positive ions and warm negatively charged dust particles and trapped electrons. Also, we neglected the effect of gravity. Fully ionized means that there are no neutrals in the plasma, i.e., collision between ions and neutrals does not found. The term dusty plasma means that $d/\lambda_{\rm D}<1$, where d is the intergrain distance between dust particles and $\lambda_{\rm D}$ is the dust plasma Debye radius.[1] Weakly coupled means that the coupling parameter Γ is less than 1. To neglect the effect of gravity the dust particle sizes (r_d) should be not more than 1 μm. Actually, we have a lot of examples for plasma in space, but which one of them could be described by our model. Table 2 in Ref. 34 has some typical values of cosmic dust-laden plasmas. Actually, there is an example in that table that achieves the conditions under which our basic equations are valid, i.e., Saturn's F ring is one of the space plasma systems that satisfies our conditions: (i) there are no neutrals, (ii) $d/\lambda_{\rm D}<1$, (iii) $\Gamma \ll 1$, and (iv) $r \leqslant 1$ μm. The plasma parameters of the Saturn's F ring have the typical values:[34] $n_e=10$ cm^{-3}, $T_e=10-100$ eV, $n_d<10$ cm^{-3}, $r_d=1$ μm, $Z_d\sim 10-100$. It is obvious that the electron temperature has a wide range. So, the electrons may be found in low and high temperatures. These values are supposed to investigate their effects on the soliton behavior. Using this plasma parameters, it is found that $\Delta\xi/\Sigma$ is of the order 10^{-1}. Therefore, condition (36) is well satisfied and the nonlinear structure in the Saturn's F ring is expected to be soliton.

The dependence of soliton width and amplitude on the electron temperatures is displayed in Figs. 1–6. It is found that the effect of low-electron temperature reduces (increases) the width for $T_{\rm ef\ell}<13(T_{\rm ef\ell}>13)$ (cf. Fig. 1). The high-electron temperature makes the solitons more spiky for all values of $T_{\rm efh}$ that is due to the decrease of the width or the increase of the amplitude (cf. Figs. 2 and 4). From Figs. 3 and 4, it is clear that the amplitude increases with $T_{\rm ef\ell}$ and $T_{\rm efh}$. The trapped electron temperature has no effect on the soliton width but it is responsible for increasing the soliton amplitude (cf. Figs. 5 and 6). For $\beta_\ell<0.7$ the amplitude increases gradually but for $\beta_\ell>0.7$ the amplitude rises sharply (cf. Fig. 5). From Fig. 6, it is obvious that the amplitude increases with β_h.

It is interesting to compare our results with that of Moslem *et al.*[20] They considered one-temperature trapped electrons and investigated the effect of β (the ratio of free to trapped electron temperatures) on the soliton amplitude. They found that the amplitude increases with β. While the present study clears that free low and free high electron temperatures have different effects on both the width and the amplitude. Also, this study indicates that β_ℓ increases the amplitude slowly for $\beta_\ell<0.7$ and rapidly for $\beta_\ell>0.7$. The amplitude increases for all values of β_h. These results could not be obtained for one-temperature trapped electron case. Thus, we can consider this study as a modification and generalization of the previous work.

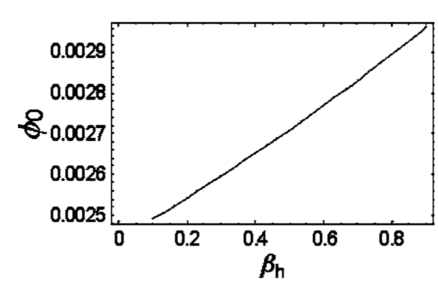

FIG. 6. ϕ_0 is plotted against β_h. For $\alpha=2$, $\delta=0.1$, $\mu_{e\ell}=0.7$, $\mu_{\rm eh}=0.3$, $n_d=1$, $Z_d=10$, $\ell=0.5$, $\lambda=2$, $r_d=6\times10^{-13}$, $T_i=1$, $T_{\rm ef\ell}=10$, $T_{\rm et\ell}=40$, $T_d=0.01$, $\nu_{cho}=6.6$, $\nu_{io}=7$, $\nu_o=8.8$, $\tau=2$, $\mu_d=10^{12}$, and $u_{xo}=0.4$.

IV. CONCLUSIONS

In this paper, we have investigated the propagation of nonlinear DIASs in a dusty plasma consisting of a mixture of warm positive ions, warm negatively charged dust particles, and two-temperature trapped electrons. The basic set of fluid equations is reduced to DMKP equation. From the present analysis the following interesting features are seen.

(1) The wave amplitude is exponentially decaying with time due to ion recombination frequency on dust particles, plasma ionization frequency, and frequency characterizing a loss in ion momentum due to recombination on dust particles and Coulomb elastic collisions between ions and dusts.
(2) The wave amplitude admits only a positive potential, i.e., it has a compressive solitons only.
(3) From stability analysis, it is found that the wave cannot propagate for any directions but it propagates only at the directions which satisfy inequality (35). This result could not be obtained for one-dimensional case ($\ell=1$).
(4) Either soliton or shock structures could propagate in dusty plasma. But solitons can propagate if the inequality (36) is satisfied. On the other hand, the dust parameters n_d, ν_d, and Q_d have an important role to specify the type of the waves (solitons or shocks).
(5) The soliton cannot exist when $T_{\text{eff}}=5T_i\alpha^{2/3}/3\lambda_1^2$ or $5T_d\delta^{2/3}/3\lambda^2\mu_d$.
(6) The soliton width decreases to its minimum at some critical value of $T_{\text{ef}\ell}\approx 13$ but for $T_{\text{efl}}>13$ it increases. The width decreases with T_{efh}. The width is independent of trapped electron temperature.
(7) The soliton amplitude increases gradually with $T_{\text{ef}\ell}$, T_{efh}, and β_h. However, it increases with lower values of β_ℓ and it rises sharply for higher values of β_ℓ.

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