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Propagating of Dust-Acoustic Radiation in Cosmic Dust-Laden Plasmas

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ABSTRACT

Obliquely dust-acoustic radiation in a collisional, magnetized dusty plasmas having cold dust grains, isothermal electrons, two temperature isothermal ions and stationary neutrals are studied via a reductive perturbation method. It is found that the effects of two temperature ions, collisions, magnetic field and directional cosine of the waves vector *k* along the x-axis have vital roles in the behavior of the dust acoustic radiation. The present investigation can be relevance to the electrostatic radiation structures observed in various cosmic dust-laden plasmas, such as Saturn's E-ring, noctilucent clouds, Halley's comet and interstellar molecular clouds.

INTRODUCTION

The study of dusty plasmas represents one of the most rapidly growing branches of plasma physics. Interest in dusty plasmas ranges since the Voyager observation in the early 1980s, that showed phenomena in the rings of Saturn, which could not really be explained on, purely gravitational ground alone. Telltales were the spokes in the B ring and the braids in the F ring, the later ring itself being discovered by these missions. Other examples in the solar system are circumsolar dust rings, noctilucent clouds in the arctic troposphere as the closest natural dusty plasmas, or even in the flame of a humble candle. Other dusty plasmas occur in the asteroid belt, in cometary comae and tails, in the rings of all the Jovian planets and in interstellar dust clouds, to name but a few [1]. On the other hand, the growing interest in physics of dusty plasmas not only because of dust being omnipresent ingredient of our universe, but also because of its vital role in understanding collective processes in astrophysical and space environments, such as mode modification, new eigenmodes, coherent structures, etc [2].

The consideration of charged dust grains in a plasma does not only modify the existing plasma wave spectra but also introduces a number of new novel eigenmodes, such as dust-acoustic waves that was reported theoretically first by Rao [3] and verified experimentally by Barkan [4]. Later, extensive work has been done to study the features of the dust-acoustic waves, e.g. Mamun [5] used the quasipotential analysis to investigate the nonlinear dust-acoustic waves. They found that, dusty plasma with inertial dust fluid and Boltzmann distributed ions admits only negative solitary potential. Tagare [6] extended the model of Mamun [5] to study plasma consisting of cold dust particles and two temperature isothermal ions. He found that, both compressive and rarefactive solitary waves as well as compressive and rarefactive double layers exist. Xie [7] investigated the small and large amplitude dust-acoustic solitary waves in dusty plasma with variable dust charge and two temperature ions.

They noticed that, both compressive and rarefactive solitary waves as well as double layers exist. Also the small, but finite, number density of two temperature ions provides the possibility of the coexistence of large amplitude rarefactive with compressive dust-density solitary waves. They also examined a necessary condition must satisfy to achieve the validity of the two temperature ion assumption. This condition is, the energy rate E_R between the two type ions must be much smaller than the characteristic frequency of the system $\omega_{pd} = \left(4\pi n_{d0} e^2 Z_d^2 / m_d\right)^{1/2}$, i.e., $E_R / \omega_{pd} << 1$, here ω_{pd} is the dust plasma frequency, n_{d0} is the unperturbed dust density, Z_d is the dust charge number, m_d is the dust mass. The two type ions are assumed to have the same mass m_i and charge number $Z_i = 1$, but one requires lower temperature $T_{i\ell}$ and another higher temperature T_{ih}. On the other hand, these temperatures satisfy $T_{i\ell} << T_{ih} \sim \; < T_e$, then $E_R = \dfrac{\Gamma}{v_{ith}^2}$, where $v_{ith} = \sqrt{T_{ih} / m_i}$ and $\Gamma = (4\pi n_{i\ell 0} e^4 \ln\Lambda) / m_i^2$, with a Coulomb logarithm $\ln\Lambda \sim 10 - 15$ and unperturbed low temperature ion number density $n_{i\ell 0}$ [7, 8]. Table 1 showst he validity of the two-temperature ions assumption is examined for various cosmic dust-laden plasma systems and the ratio E_R / ω_{pd} .

Table (1):

Environment	$n_{i\ell 0}$ (cm^{-3})	T_{ih} (eV)	n_{d0} (cm^{-3})	$\lvert Z_d \rvert$	n_N (cm^{-3})	E_R / ω_{pd}
Saturn's E-ring	1-120	10-100	10^{-7}	$\sim 10^4$	1	$\sim 10^{-4}$-10^{-7}
Noctilucent clouds	10^2-10^3	0.013	10-10^3	8-80	10^{14}	$\sim 10^{-1}$-10^{-3}
Interstellar Molecular clouds	$<3\times 10^{-4}$	0.001	10^{-7}	~1	10^4	$\sim 10^{-1}$
Halley's comet	$< 10^3$	< 0.1	10^{-3}	10^3	10^{10}	$\sim 10^{-1}$
Inside ionopause	10^2-10^3	~1	10^{-8}-10^{-7}	20-2×10^4	-	$\sim 10^{-1}$-10^{-3}
Outside ionopause	1-100	10-100	< 10	10-100	-	$\sim 10^{-5}$-10^{-8}
Saturn's F-ring	0.1-100	2	1	~ 10	-	$\sim 10^{-3}$-10^{-7}
Saturn's spokes	0.5-50	10	10^{-12}	10^4	-	$\sim 10^{-1}$-10^{-3}
Zodiacal dust disc (1AU)	10^2-10^4	0.2	10	20	-	$\sim 10^{-1}$-10^{-3}
Supernovae shells						

The values of the parameters T_{ih}, n_{d0}, Z_d and n_N corresponding to Mendis and Rosenberg [9] and Mendis [10], where n_N is the neutral gas density. However, he did not give the value of $n_{i\ell 0}$, so it is supposed to have wide range to confirm that the ratio E_R / ω_{pd} is valid for any possible change of $n_{i\ell 0}$.

It will be known that, any analytical study of dusty plasma being limited by complexity of the problem. Therefore, the plasma system cannot be described in its totality. On the other hand, last treatments of the dust-acoustic waves were limited to a simple unmagnetized dusty plasma model, although dusty plasma invariably occurs in the presence of an ambient magnetic field, both in space and in laboratory (see e.g. Ref. [11] and references therein). Besides the effect of neutrals was ignored since it increases the complexity of the problem. Though, neutral gas might be representative for stellar material in galactic dynamics and also most laboratory dusty magnetoplasmas are partially ionized, the effect of dust-neutral collisions cannot be ignored. Therefore, the effect of collisions between dust grains and neutrals should be taken into account. So, it is the purpose of the present paper to study, qualitatively, the effects of collisions, magnetic field, directional cosine of the wave vector *k* a long the x-axis and parameters of two temperature ions on the dust-acoustic solitary waves in collisional, magnetized dusty plasmas. This paper is organized in the following fashion; In Section 2, we write down the basic fluid equations describing the model. Using the reductive perturbation method the nonlinear dust-acoustic solitary waves are investigated through derivation of the damped Zakharov-Kuznetsov (DZK) equation. Section 3 is devoted to conclusions.

THEORY

We study electrostatic perturbation in a four-component collisional, magnetized dusty plasma consisting of negatively charged dust particles, isothermal electrons, two temperature isothermal ions and stationary neutrals (i.e., the neutrals are taken as immobile) in the presence of an external static magnetic field $\mathbf{B_0} = B_0\hat{x}$. The dust particles are assumed to be extremely massive point charges with much smaller than the plasma Debye length and the collisional mean free path. The dust particles dynamics are governed by the following system of nondimensional equations:

$$\frac{\partial n_d}{\partial t} + \nabla n_d u_d = 0, \quad \textbf{(1)}$$

$$\frac{\partial u_d}{\partial t} + u_d.\nabla u_d - \nabla\phi + u_d \times \Omega\hat{x} + \nu_{dn} u_d = 0, \quad \textbf{(2)}$$

$$\nabla^2\phi + n_{i\ell} + n_{ih} - n_d - n_e = 0. \quad \textbf{(3)}$$

The dimensionless number densities of electrons and ions are expressed as

$$n_{il} = \mu_{il} \exp(-\Delta_{il}\phi), \quad \textbf{(4)}$$

$$n_{ih} = \mu_{ih} \exp(-\Delta_{ih}\phi), \quad \textbf{(5)}$$

$$n_e = \mu_e \exp(\Delta_e\phi). \quad \textbf{(6)}$$

The conditions under which Eqs. (4) – (6) in magnetized dusty plasma are valid (i) the phase velocity of the dust acoustic solitary waves is much less than the electron and ion thermal speeds, and (ii) electrons and ions are thermalized along the external magnetic field direction.

Here n_d and u_d refer to the number density and fluid velocity of the dust grains, respectively. ϕ, Ω and ν_{dn} are, respectively, the electrostatic potential, dust cyclotron frequency and dust neutral collision frequency. Z_d is the dust grain charge number. The densities n_d and n_q ($q = il, ih, e$) are normalized by n_{d0} and $Z_d n_{d0}$, respectively. The space coordinates x, y and z are normalized by the dust Debye length $\lambda_{Dd} = \left(T_{eff}/4\pi n_{d0}e^2Z_d^2\right)^{1/2}$. The time t is normalized by the inverse dust plasma frequency $\omega_{pd}^{-1} = \left(m_d/4\pi n_{d0}e^2Z_d^2\right)^{1/2}$. ϕ is normalized by (T_{eff}/eZ_d). Ω and ν_{dn} are normalized by dust plasma frequency $\omega_{pd} = \left(4\pi n_{d0}e^2Z_d^2/m_d\right)^{1/2}$. Now we will introduce the following notations:

$$\mu_{i\ell} = \frac{n_{i\ell 0}}{Z_d n_{d0}}, \qquad \mu_{ih} = \frac{n_{ih0}}{Z_d n_{d0}}, \qquad \mu_e = \frac{n_{e0}}{Z_d n_{d0}},$$

$$\Delta_{il} = T_{eff}/Z_d T_{il}, \qquad \Delta_{ih} = T_{eff}/Z_d T_{ih}, \qquad \Delta_e = T_{eff}/Z_d T_e,$$

$$\frac{1}{T_{eff}} = \frac{1}{Z_d^2 n_{d0}}\left[\frac{n_{i\ell 0}}{T_{i\ell}} + \frac{n_{ih0}}{T_{ih}} + \frac{n_{e0}}{T_e}\right],$$

with temperature T_e for electrons, temperature T_{il} for cold ions and temperature T_{ih} for hot ions in units of energy, respectively. $\Omega = eZ_d B_0 / m_d$ and $\nu_{dn} \approx 4r_d^2 m_N n_N V_{th,N} / m_d$, where m_d, r_d, m_N, n_N and $V_{th,N}$ are the dust grain mass, dust grain radius, neutral gas mass, neutral gas density and neutral gas thermal velocity, respectively.

In order to study the small amplitude nonlinear dust acoustic solitary waves in dusty plasma, we use the reductive perturbation theory [12]. According to this method, we introduce the stretched space-coordinates

$$X = \varepsilon^{1/2}(x - \lambda t), Y = \varepsilon^{1/2} y, Z = \varepsilon^{1/2} z, \textbf{ and } T = \varepsilon^{3/2} t, \tag{7}$$

where λ is an unknown phase velocity to be determined later and ε measures the size of the perturbation amplitude. The physical quantities appearing in Eqs. (1) — (6) are expanded as,

$$n_d = 1 + \varepsilon n_d^{(1)} + \varepsilon^2 n_d^{(2)} + \varepsilon^3 n_d^{(3)} + \dots\dots\dots\dots, \tag{8a}$$

$$u_{dx} = \varepsilon u_{dx}^{(1)} + \varepsilon^2 u_{dx}^{(2)} + \varepsilon^3 u_{dx}^{(3)} + \dots\dots\dots\dots\dots, \tag{8b}$$

$$u_{dy} = \varepsilon^{3/2} u_{dy}^{(1)} + \varepsilon^2 u_{dy}^{(2)} + \varepsilon^{5/2} u_{dy}^{(3)} + \dots\dots\dots\dots\dots, \tag{8c}$$

$$u_{dz} = \varepsilon^{3/2} u_{dz}^{(1)} + \varepsilon^2 u_{dz}^{(2)} + \varepsilon^{5/2} u_{dz}^{(3)} + \dots\dots\dots\dots\dots, \tag{8d}$$

$$\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \varepsilon^3 \phi^{(3)} + \dots\dots\dots\dots\dots . \tag{8e}$$

The charge-neutrality condition in the dusty plasma is always maintained through the relation,

$$\mu_{il} + \mu_{ih} - \mu_e - 1 = 0. \tag{9}$$

Substituting Eqs. (7) and (8) into Eqs. (1) — (6), then collecting terms of different powers of ε, in the lowest order we obtain

$$n_d^{(1)} = u_{dx}^{(1)} = -\phi^{(1)}, u_{dy}^{(1)} = \Omega^{-1} \frac{\partial \phi^{(1)}}{\partial Z}, u_{dz}^{(1)} = -\Omega^{-1} \frac{\partial \phi^{(1)}}{\partial Y}, \tag{10}$$

and the linear dispersion relation

$$\mu_{il}\Delta_{il} + \mu_{ih}\Delta_{ih} + \mu_e \Delta_e = \frac{1}{\lambda^2}, \tag{11}$$

where we have assumed that $\nu_{dn} \approx \varepsilon^{3/2} \nu_o$. Using the expression of $\Delta_{i\ell}, \Delta_{ih}$ and Δ_e in Eq. (11) we get $\lambda = 1$.

The next-order in ε yields a system of equations in the second-order perturbed quantities. Eliminating the second-order perturbed quantities; we get the damped Zakharov Kuznetsov (DZK) equation,

$$\frac{\partial\phi^{(1)}}{\partial T}+B\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial X}+\frac{1}{2}\frac{\partial^3\phi^{(1)}}{\partial X^3}+\frac{1}{2}\left(1+\frac{1}{\Omega^2}\right)\left\{\frac{\partial^3\phi^{(1)}}{\partial X\partial Y^2}+\frac{\partial^3\phi^{(1)}}{\partial X\partial Z^2}\right\}+\frac{1}{2}\nu_o\phi^{(1)}=0, \quad \textbf{(12)}$$

where

$$B=\frac{1}{2}\left\{-3+\frac{\mu_{il}(1-\beta_1^2)+\mu_{ih}\beta^2(1-\beta_2^2)+\beta_1^2}{\left(\mu_{il}(1+\beta_1)+\mu_{ih}\beta(1+\beta_2)-\beta_1\right)^2}\right\},$$

$\beta_1=T_{il}/T_e, \beta_2=T_{ih}/T_e$ and $\beta=T_{il}/T_{ih}$.

To obtain a solitary wave solution of Eq. (12) we introduce the variable,

$$\eta=lX+mY+nZ-MT, \quad \textbf{(13)}$$

where η is the transformed coordinates with respect to a frame moving with velocity M. l, m and n are the directional cosines of the wave vector k along the X, Y and Z axes, respectively, so that $l^2+m^2+n^2=1$. Integrating Eq. (12) with respect to the variable η and using the vanishing boundary conditions for $\phi^{(1)}$ and its derivatives up to the second order for $|\eta|\to\infty$, we obtain the time evolution solitary waveform approximate solution as

$$\phi^{(1)}=Ae^{-\gamma T}\sec h^2\sqrt{\frac{AB}{12\rho}e^{-\gamma T}}\eta, \quad \textbf{(14)}$$

where $\rho=\frac{1}{2}l^2+\rho_1(m^2+n^2)$, $\rho_1=\frac{1}{2}\left(1+\Omega^{-2}\right)$, $\gamma=\frac{1}{2}\nu_o$ and $M=\frac{lAB}{3}e^{-\gamma T}$. To get the value of A, we let $\gamma=0$ in Eq. (14) and then its soliton solution given by

$$\phi^{(1)}=\frac{3\overline{M}}{lB}\sec h^2\sqrt{\frac{\overline{M}}{4l\rho}}\overline{\eta}, \quad \textbf{(15)}$$

where $\overline{\eta}$ is the transformed coordinates with respect to a frame moving with velocity $\overline{M}$ at $\gamma=0$ [i.e., for $\gamma=0$; $\eta\to\overline{\eta}$ and $M\to\overline{M}$]. From Eqs. (14) and (15) it is clear that $A=3\overline{M}/lB$. Therefore, Eq. (14) can be rewritten as

$$\phi^{(1)}=\phi_0\sec h^2(\eta/\delta), \quad \textbf{(16)}$$

where the amplitude ϕ_0 and the width δ are given by $\frac{3\overline{M}}{lB}e^{-\gamma T}$ and $\sqrt{\frac{4l\rho}{\overline{M}}e^{\gamma T}}$, respectively.

The behavior of the dust-acoustic solitary wave amplitude vs. μ_{il} is displayed in Fig. 1. It is clear that the amplitude of the compressive (rarefactive) solitons increases (decreases) with μ_{il}. There is a special value of μ_{il}, which may be called critical density of low temperature ions, at which the soliton does not exist. In this case the dust-acoustic solitary waves may be described by using a new stretched variable. This situation is discussed below. Fig. 2 (3) indicates that the compressive (rarefactive) soliton increases (decreases) with μ_{ih}. The relation between the compressive and rarefactive soliton amplitude with β_1 and β_2 are plotted in Fig. 4 and 5. From Fig. 4, it is clear that compressive soliton amplitude increases with β_1 and decreases with β_2. From Fig 5, it is obvious that rarefactive soliton amplitude decreases with β_1 and increases with β_2. From Eq. (16) it is seen that the width of the solitons depends only on ℓ, Ω, ν_o and T but other plasma parameters have not any effect on the behavior of the width. Fig. 6 shows how the width varies when l changes between 0 and 1, i.e. the width increases as l increases for its lower range (from 0 to ~ 0.6) but decreases from its higher range (from ~ 0.6 to 1). Also the width decreases with Ω. It is clear that from the expression of the width, the width increases as ν_o and T increase.

CONCLUSION

In the above analysis we have investigated the properties of nonlinear dust-acoustic solitary waves in a collisional, magnetized dusty plasmas comprising negatively charged cold dust grains, isothermal electrons, two temperature isothermal ions and stationary neutrals. The reductive perturbation method is used to reduce the basic set of fluid equations to a DZK equation (12). It is found that, the present system admits coexistence of both compressive and rarefactive solitons.

The compressive (rarefactive) soliton amplitude increases (decreases) with μ_{il}, μ_{ih} and β_1. However, β_2 decreases the compressive soliton amplitude and increases the rarefactive soliton amplitude.

The width of the soliton decreases with Ω, ν_o and T while it increases for $l = 0 \sim 0.6$ and decreases for $l = 0.6 \sim 1$. At critical density of low temperature ion, the DZK fails to describe the system that forced us to apply a new stretching and then we get a DMZK equation (19).

The feature of the soliton that describes by Eq. (19) differs from the soliton that described by Eq. (12). On the other hand, the compressive and rarefactive soliton amplitude decrease with μ_{il} and β_1 while the compressive and rarefactive soliton amplitude increase as β_2 increases.

There exist also some critical points that make the nonlinear coefficients of the DZK and DMZK equations become zero, then they also fail to describe the system and we replace the previous two stretchings with a newer one.

Applying this stretching leads us to get the ZK type Eq. (23), which covers and solves this problem and allows only rarefactive solitons.

In conclusion, the model and the results presented here may be applicable to some dusty space plasma environments, such as Saturn's E-ring, noctilucent clouds, interstellar molecular clouds and inside ionopause of Halley's comet.

Also the present analysis can be applied for collisionless (i.e. $\nu_o = 0$) dusty plasma systems, such as outside ionopause of Halley's comet, Saturn's F-ring, Saturn's spokes, zodiacal dust disc (1AU) and supernovae shells, but one has to be careful about the choice of plasma parameters.

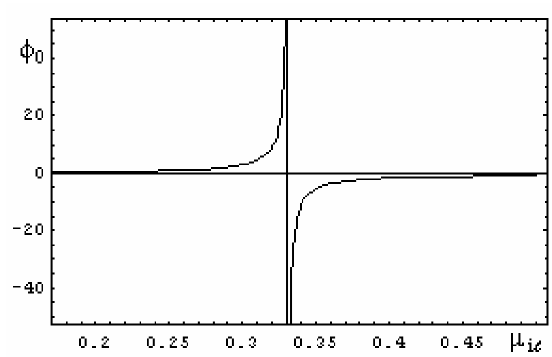

Fig. 1. Graph of the amplitude ϕ_0 **vs.** $\mu_{i\ell}$ **for** $\mu_{ih} = 1.2$, $\beta_1 = 0.0001$, $\beta_2 = 0.1$, $M = 1.5$, $\nu_0 = 2$, $t = 3$ **and** $\ell = 0.5$.

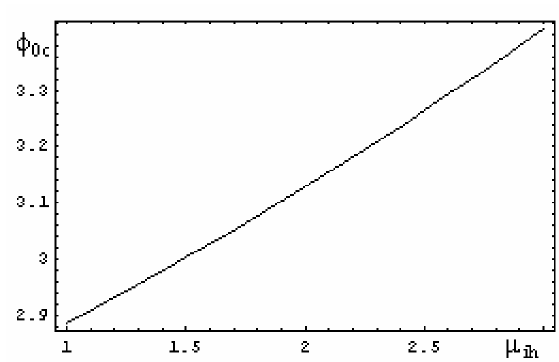

Fig. 2. Graph of the compressive amplitude ϕ_{0c} **vs.** μ_{ih} **for** $\mu_{i\ell} = 0.3$, $\beta_1 = 0.0001$, $\beta_2 = 0.1$, $M = 1.5$, $\nu_0 = 2$, $t = 3$ **and** $\ell = 0.5$.

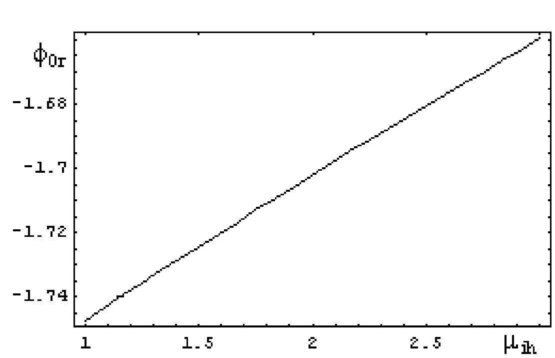

Fig. 3. Graph of the rarefactive amplitude ϕ_{0r} **vs.** μ_{ih} **for** $\mu_{i\ell} = 0.4$, $\beta_1 = 0.0001$, $\beta_2 = 0.1$, $M = 1.5$, $\nu_0 = 2$, $t = 3$ **and** $\ell = 0.5$.

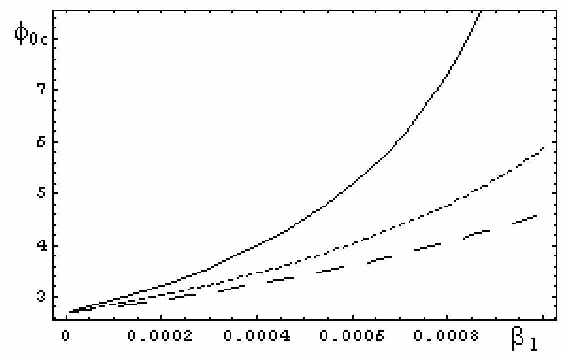

Fig. 4. Graph of the compressive amplitude ϕ_{0c} **vs.** β_1 **for** $\mu_{i\ell} = 0.3$, $\mu_{ih} = 1.2$, $M = 1.5$, $\nu_0 = 2$, $t = 3$, $\ell = 0.5$, $\beta_2 = 0.1$ **(solid line),** $\beta_2 = 0.15$ **(dotted line) and** $\beta_2 = 0.2$ **(dashed line).**

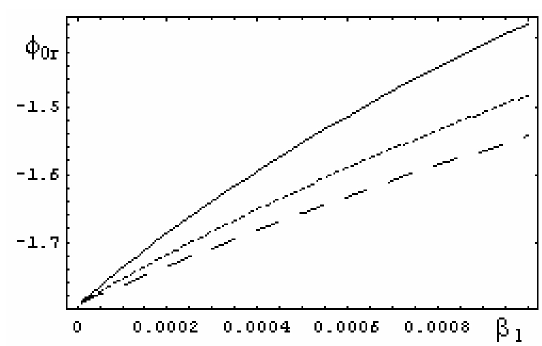

Fig. 5. Graph of the rarefactive amplitude ϕ_{0r} **vs.** β_1 **for** $\mu_{i\ell} = 0.4$, $\mu_{ih} = 1.2$, $M = 1.5$, $\nu_0 = 2$, $t = 3$, $\ell = 0.5$, $\beta_2 = 0.1$ **(solid line),** $\beta_2 = 0.15$ **(dotted line) and** $\beta_2 = 0.2$ **(dashed line).**

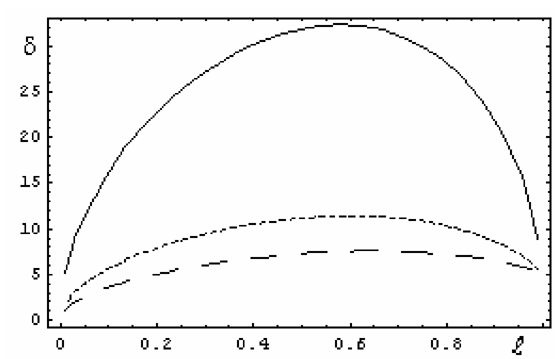

Fig. 6. Graph of the width δ vs. ℓ for $\nu_0 = 2$, $t = 3$, $\Omega = 0.1$ (solid line), $\Omega = 0.3$ (dotted line) and $\Omega = 0.5$ (dashed line).

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