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Citation: Physics of Plasmas (1994-present) **20**, 122114 (2013); doi: 10.1063/1.4853555
View online: http://dx.doi.org/10.1063/1.4853555
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/20/12?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF PLASMAS 20, 122114 (2013)

# Propagation and oblique collision of ion-acoustic solitary waves in a magnetized dusty electronegative plasma

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(Received 30 September 2013; accepted 9 December 2013; published online 26 December 2013)

The propagation and oblique collision of ion-acoustic (IA) solitary waves in a magnetized dusty electronegative plasma consisting of cold mobile positive ions, Boltzmann negative ions, Boltzmann electrons, and stationary positive/negative dust particles are studied. The extended Poincaré-Lighthill-Kuo perturbation method is employed to derive the Korteweg-de Vries equations and the corresponding expressions for the phase shifts after collision between two IA solitary waves. It turns out that the angle of collision, the temperature and density of negative ions, and the dust density of opposite polarity have reasonable effects on the phase shift. Clearly, the numerical results demonstrated that the IA solitary waves are delayed after the oblique collision. The current finding of this work is applicable in many plasma environments having negative ion species, such as D- and F-regions of the Earth's ionosphere and some laboratory plasma experiments. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4853555]

## I. INTRODUCTION

Electronegative plasma is a plasma which contains negative ion species with a significant amount that its contribution can not be ignored in any way. The study of electronegative plasma has gained much interest of research because of its potential applications in both space environment (e.g., Earth's ionosphere,[1,2] cometary comae,[3] upper region of Titan's atmosphere[4]) and laboratory devices (e.g., low-pressure discharge plasma,[5] Q-machine,[6] plasma ignition,[7] dc multidipole chamber[8]). In most cases,[9–13] electronegative plasma contains solid impurities (dust) which are easily charged positive or negative by the surrounding electronegative plasma species, i.e., electrons, positive ions, and negative ions. Kim and Merlino[10] reported the conditions under which dust grains could be positively charged in an electronegative plasma. Rosenberg and Merlino[11] investigated the polarity effect of dust grains on the instability of ion-acoustic waves in an electronegative plasma. Moslem *et al.*[13] studied the electrostatic structures associated with dusty electronegative magnetoplasmas with the polarity effect.

Since the last decade, it has been frequently considered that negative ions in electronegative plasma obey Boltzmann distribution.[8,12,14,15] On one hand, Bogdanov and Kudryavtsev[14] reported the conditions for realization of the Boltzmann distribution of negative ions in a plasma. They[14] illustrated that the situation of realizing Boltzmann distribution for both electrons and negative ions takes place for small role of attachment in comparison to the ambipolar diffusion of negative ions. On the other hand, Ghim and Hershowitz[8] investigated weakly collisional Ar-$O_2$ electronegative plasmas in a dc multidipole chamber. They verified that negative ions are in Boltzmann equilibrium with a temperature of $0.06 \pm 0.02$ eV. Mamun *et al.*[12] considered a case more general than the experiment of Ghim and Hershowitz,[8] and they examined theoretically the possibility for formation of solitary waves and double layers in a dusty electronegative plasma (DENP) with Boltzmann distributed negative ions. Recently, Zobaer *et al.*[15] have considered DENP with Boltzmann distributed negative ions, and they investigated the effects of nonplanar geometry on shock and solitary waves in DENP.

Actually, wave-wave interactions are very interesting and important nonlinear phenomena which occur during the propagation of solitary waves in plasmas. The interesting features of the collision between solitary waves have been revealed: when two solitary waves approach closely, they interact, exchange their energies and positions with each other, and then separate off, regaining their original wave forms.[16] Throughout the whole process of the collision, the solitary waves are remarkably stable entities, preserving their identities through interaction; the unique effect due to the collision is their phase shift and the trajectories.[16] In a one- (or quasi-one-) dimensional system, there are two distinct soliton interactions. One is the overtaking collision[17] and the other is the head-on collision.[18] In three-dimensional (3D) systems, the general case is $0 < \theta < \pi$, where $\theta$ is the angle between two propagation directions of the two solitary waves, so the head-on collision and the overtaking collision are only two special cases for $\theta = \pi$ and $\theta = 0$, respectively. In general, for the interaction (i.e., the oblique collision) between two solitary waves in 3D systems, we must search for the evolution of the solitary waves propagating in two different directions, and hence we need to employ a suitable asymptotic expansion to solve the original hydrodynamic equations, e.g., using the extended Poincaré-Lighthill-Kuo (PLK) perturbation method.[18–20] Indeed, the reality which cannot be ignored is that the one-dimensional geometry may

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not be a realistic situation in laboratory devices and in space. However, the oblique collision of solitary waves in a three dimensional geometry is more realistic in a magnetized dusty electronegative plasma, especially, of D- and F-regions of the Earth's ionosphere.

However, there are few investigations of the oblique collision of solitary waves in 3D geometry for different plasma systems.[21–24] For example, Xue[21] discussed how the magnetic field significantly modifies the solitons collision property. Han *et al.*[22] discussed the existence of ion-acoustic solitary waves and their interaction in a weakly relativistic two-dimensional thermal plasma. They[22] found that the relativistic factor has significant influence on the amplitude, the width of the newly formed nonlinear wave. Liang *et al.*[23] obtained the phase shifts after collision of two solitary waves with an arbitrary angle. They[23] illuminated that the value of phase shift increases as the angle between the propagation directions of two solitary waves increase. El-Labany *et al.*[24] found that the magnitude of the phase shift of the dust-acoustic solitary waves depends directly on the angle of the oblique collision and the concentrations of the two types of isothermal ions.

To the best of our knowledge, there are no previous investigations for the interaction of ion-acoustic (IA) solitary waves in DENP in the presence of an external magnetic field. Though, the aim of this study is to demonstrate the existence of phase shift after the oblique collision between two IA solitary waves using the extended PLK method in 3D geometry, in DENP with Boltzmann distributed negative ions and dust polarity effect. It should be mentioned here that this work is applicable in many plasma environments having negative ion species, such as D- and F-regions of the Earth's ionosphere[1–4] as well as some laboratory plasma experiments.[5–8] This paper is organized as follows: In Sec. II, we apply the extended PLK method to the magnetized DENP system considering the dust polarity effect. Analytically, two Korteweg-de Vries (KdV) equations are derived for describing the dynamics of the two IA solitary waves. Furthermore, the phase shifts and the trajectories are estimated. Section III is devoted for numerical investigations and the discussion.

## II. THE OBLIQUE COLLISION BETWEEN TWO ION-ACOUSTIC SOLITARY WAVES

We consider a three-dimensional, magnetized, and collisionless four-component plasma consisting of cold mobile positive ions, Boltzmann distributed negative ions, Boltzmann distributed electrons, and stationary dust particles with positive or negative charge. The external magnetic field is directed along the $z$ axis, i.e., $B = B_0\hat{z}$, where $\hat{z}$ is the unit vector along the $z$ axis. At equilibrium, the charge neutrality condition reads as $n_{i0} = n_{e0} + n_{n0} - \delta Z_d n_{do}$, where $n_{i0}$, $n_{e0}$, $n_{n0}$, and $n_{d0}$ are, respectively, positive ion, electron, negative ion, and dust number densities at equilibrium. $Z_d$ is the number of electrons residing onto the surface of a stationary dust and $\delta = +1(-1)$ for positive (negative) dust.[13] The dynamics of the positive ion fluid are governed by the hydrodynamic equations, namely,[12]

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + (u_i.\nabla)u_i = -\nabla\phi + \Omega(u_i \times \hat{z}), \tag{2}$$

$$\nabla^2\phi = \mu_e e^{\phi} + \mu_n e^{\sigma\phi} - n_i - \delta\mu_d, \tag{3}$$

where $n_i$ and $u_i$ are the number density and velocity of the positive ion fluid. $\mu_e(= n_{eo}/n_{io})$, $\mu_n(= n_{no}/n_{io})$ and $\mu_d(= Z_d n_{do}/n_{io})$ are the ratio of the electron, the negative ion and the dust number densities to the ion number density at equillibrium, respectively. $\sigma(= T_e/T_n)$ is the ratio between the electron temperature to the negative ion temperature. $\Omega\left(= eB_o/m_i c\omega_{pi}\right)$ is the ion cyclotron frequency normalized by the ion plasma frequency $\omega_{pi}(= \sqrt{4\pi e^2 n_{io}/m_i})$. The following normalizations are used

$$n_i \to n_i/n_{io}, \quad u_i \to u_i/C_i,$$
$$\phi \to e\phi/KT_e, \quad t \to t\omega_{pi}, \quad \nabla \to \lambda_D\nabla,$$

where $C_i(= \sqrt{KT_e/m_i})$ is the ion-acoustic speed and $\lambda_D(= \sqrt{KT_e/4\pi e^2 n_{io}})$ is the Debye length.

Now, in order to analyze the effects of quasielastic oblique collision of two solitons S1 and S2 in magnetized dusty electronegative plasmas, we assume that they are, asymptotically, far apart in the initial state and travel toward each other. After some time they interact, and the amplitude of overlapping waves is greater than the algebraic sum of the individual solitons before the collision. Moreover, the amplitude slightly dips immediately after the collision and returns to its value before the collision at a later time. Accordingly, we employ the extended PLK perturbation method[18–20] to study the oblique collision between S1 and S2. According to this method, we introduce the stretched coordinates[21,22]

$$\left.\begin{aligned} \xi &= \epsilon(l_{x1}x + l_{y1}y + l_{z1}z - c_1 t) + \epsilon^2 P_0(\eta,\tau) + \cdots \\ \eta &= \epsilon(l_{x2}x + l_{y2}y + l_{z2}z + c_2 t) + \epsilon^2 Q_0(\xi,\tau) + \cdots \\ \tau &= \epsilon^3 t, \end{aligned}\right\}, \tag{4}$$

where $\xi$ and $\eta$ denote the trajectories of the two solitons S1 and S2 propagating, respectively, in two different directions of $R_1(= l_{x1}x + l_{y1}y + l_{z1}z)$ and $R_2(= l_{x2}x + l_{y2}y + l_{z2}z)$ at $P_0(\eta,\tau) = Q_0(\xi,\tau) = 0$. After interaction, the trajectories will be changed and hence $P_0(\eta,\tau) \neq 0$ and $Q_0(\xi,\tau) \neq 0$. Here, $c_1$ and $c_2$ are the unknown phase velocities of the two IA solitary waves (to be determined later). Before going into details, let us determine the angle $\theta$ between the directions of propagation of the two waves, which is given by $\theta = \cos^{-1}((l_{x1}l_{x2} + l_{y1}l_{y2} + l_{z1}l_{z2})/[(l_{x1}^2 + l_{y1}^2 + l_{z1}^2)^{1/2} (l_{x2}^2 + l_{y2}^2 + l_{z2}^2)^{1/2}])$, where $l_{x1}$, $l_{y1}$, $l_{z1}$ ($l_{x2}$, $l_{y2}$, $l_{z2}$) are the directional cosines of S1 (S2) wave vector along the $x$-, $y$-, and $z$- axes, respectively. Also, the functions $P_0(\eta,\tau)$ and $Q_0(\xi,\tau)$ are to be determined later. The dependent variables are expanded as

$$\left.\begin{aligned} n_i &= 1+\epsilon^2 n_{i1}+\epsilon^3 n_{i2}+\epsilon^4 n_{i3}+\cdots \\ \phi &= \epsilon^2\phi_1+\epsilon^3\phi_2+\epsilon^4\phi_3+\cdots \\ u_x &= \epsilon^3 u_{x1}+\epsilon^4 u_{x2}+\cdots \\ u_y &= \epsilon^3 u_{y1}+\epsilon^4 u_{y2}+\cdots \\ u_z &= \epsilon^2 u_{z1}+\epsilon^3 u_{z2}+\epsilon^4 u_{z3}+\cdots \end{aligned}\right\}. \tag{5}$$

Substituting Eqs. (4) and (5) into the set of Eqs. (1)–(3), then collecting terms of the same powers of $\epsilon$, for the lowest nonzero order, we get

$$\phi_1=\phi_1^{\xi}(\xi,\tau)+\phi_1^{\eta}(\eta,\tau), \tag{6}$$

$$n_{i1}=\left(\frac{l_{z1}}{c_1}\right)^2\phi_1^{\xi}(\xi,\tau)+\left(\frac{l_{z2}}{c_2}\right)^2\phi_1^{\eta}(\eta,\tau), \tag{7}$$

$$u_{z1}=\frac{l_{z1}}{c_1}\phi_1^{\xi}(\xi,\tau)-\frac{l_{z2}}{c_2}\phi_1^{\eta}(\eta,\tau), \tag{8}$$

$$u_{y1}=\frac{l_{x1}}{\Omega}\frac{\partial\phi_1^{\xi}(\xi,\tau)}{\partial\xi}+\frac{l_{x2}}{\Omega}\frac{\partial\phi_1^{\eta}(\eta,\tau)}{\partial\eta}, \tag{9}$$

$$u_{x1}=\frac{-l_{y1}}{\Omega}\frac{\partial\phi_1^{\xi}(\xi,\tau)}{\partial\xi}-\frac{l_{y2}}{\Omega}\frac{\partial\phi_1^{\eta}(\eta,\tau)}{\partial\eta}. \tag{10}$$

Using the solvability condition, the phase velocities $c_1$ and $c_2$ are obtained as

$$c_1=\sqrt{\frac{l_{z1}^2}{1+\delta\mu_d+(\sigma-1)\mu_n}}, \tag{11}$$

$$c_2=\sqrt{\frac{l_{z2}^2}{1+\delta\mu_d+(\sigma-1)\mu_n}}. \tag{12}$$

The unknown functions $\phi_1^{\xi}(\xi,\tau)$ and $\phi_1^{\eta}(\eta,\tau)$ will be determined at higher orders. Equations (6)–(10) imply that, at the lowest nonzero order, we have two waves, one of them is traveling toward each other. For the next-order, we obtain

$$\phi_2=\phi_2^{\xi}(\xi,\tau)+\phi_2^{\eta}(\eta,\tau), \tag{13}$$

$$n_{i2}=\left(\frac{l_{z1}}{c_1}\right)^2\phi_2^{\xi}(\xi,\tau)+\left(\frac{l_{z2}}{c_2}\right)^2\phi_2^{\eta}(\eta,\tau), \tag{14}$$

$$u_{z2}=\frac{l_{z1}}{c_1}\phi_2^{\xi}(\xi,\tau)-\frac{l_{z2}}{c_2}\phi_2^{\eta}(\eta,\tau), \tag{15}$$

$$u_{y2}=\frac{1}{\Omega}\left[\begin{aligned}&\frac{c_1 l_{y1}}{\Omega}\frac{\partial^2\phi_1^{\xi}(\xi,\tau)}{\partial\xi^2}-\frac{c_2 l_{y2}}{\Omega}\frac{\partial^2\phi_1^{\eta}(\eta,\tau)}{\partial\eta^2}\\ &+l_{x1}\frac{\partial\phi_2^{\xi}(\xi,\tau)}{\partial\xi}+l_{x2}\frac{\partial\phi_2^{\eta}(\eta,\tau)}{\partial\eta}\end{aligned}\right], \tag{16}$$

$$u_{x2}=\frac{-1}{\Omega}\left[\begin{aligned}&\frac{-c_1 l_{x1}}{\Omega}\frac{\partial^2\phi_1^{\xi}(\xi,\tau)}{\partial\xi^2}+\frac{c_2 l_{x2}}{\Omega}\frac{\partial^2\phi_1^{\eta}(\eta,\tau)}{\partial\eta^2}\\ &+l_{y1}\frac{\partial\phi_2^{\xi}(\xi,\tau)}{\partial\xi}+l_{y2}\frac{\partial\phi_2^{\eta}(\eta,\tau)}{\partial\eta}\end{aligned}\right]. \tag{17}$$

Furthermore, for the next order of $\epsilon$, after some algebraic manipulation, we obtain the following equation:

$$\begin{aligned}-2(c_1 l_{z2}+c_2 l_{z1})u_{z3}=&\frac{2l_{z1}^2}{c_1}\int\left[\frac{\partial\phi_1^{\xi}}{\partial\tau}+A_1\phi_1^{\xi}\frac{\partial\phi_1^{\xi}}{\partial\xi}+B_1\frac{\partial^3\phi_1^{\xi}}{\partial\xi^3}\right]d\eta\\ &+\frac{2l_{z2}^2}{c_2}\int\left[\frac{\partial\phi_1^{\eta}}{\partial\tau}-A_2\phi_1^{\eta}\frac{\partial\phi_1^{\eta}}{\partial\eta}-B_2\frac{\partial^3\phi_1^{\eta}}{\partial\eta^3}\right]d\xi\\ &+\iint\left[D_1\frac{\partial P_0}{\partial\eta}-E_1\phi_1^{\eta}\right]d\xi d\eta\\ &-\iint\left[D_2\frac{\partial Q_0}{\partial\xi}+E_2\phi_1^{\xi}\right]d\xi d\eta,\end{aligned} \tag{18}$$

where

$$\begin{aligned} A_1&=\frac{1}{2}c_1\left(\frac{3l_{z1}^2}{c_1^2}-1\right), \quad B_1=\frac{c_1^3}{2l_{z1}^2}\left(\frac{l_{x1}^2+l_{y1}^2}{\Omega^2}+1\right),\\ D_1&=2\left(\frac{c_2 l_{z1}^2}{c_1}+l_{z1}l_{z2}\right), \quad E_1=l_{z1}^2\left(1+\frac{l_{z1}l_{z2}}{c_1c_2}\right),\\ A_2&=\frac{1}{2}c_2\left(\frac{3l_{z2}^2}{c_2^2}-1\right), \quad B_2=\frac{c_2^3}{2l_{z2}^2}\left(\frac{l_{x2}^2+l_{y2}^2}{\Omega^2}+1\right),\\ D_2&=2\left(\frac{c_1 l_{z2}^2}{c_2}+l_{z1}l_{z2}\right), \quad E_2=l_{z2}^2\left(1+\frac{l_{z1}l_{z2}}{c_1c_2}\right).\end{aligned} \tag{19}$$

The first (second) term on the right hand side of Eq. (18) will be proportional to $\eta(\xi)$ because the integrated function is independent of $\eta(\xi)$. Thus, the first two terms of Eq. (18) are all secular terms, which must be eliminated in order to avoid spurious resonances. Hence, we have

$$\frac{\partial\phi_1^{\xi}}{\partial\tau}+A_1\phi_1^{\xi}\frac{\partial\phi_1^{\xi}}{\partial\xi}+B_1\frac{\partial^3\phi_1^{\xi}}{\partial\xi^3}=0, \tag{20}$$

$$\frac{\partial\phi_1^{\eta}}{\partial\tau}-A_2\phi_1^{\eta}\frac{\partial\phi_1^{\eta}}{\partial\eta}-B_2\frac{\partial^3\phi_1^{\eta}}{\partial\eta^3}=0. \tag{21}$$

The third and the fourth terms in Eq. (18) are not secular terms, but they will generate secular contributions at the next higher order, so they must vanish and the leading equations for the phase shifts read

$$\frac{\partial P_0}{\partial\eta}=\frac{-E_1}{D_1}\phi_1^{\eta}, \tag{22}$$

$$\frac{\partial Q_0}{\partial\xi}=\frac{-E_2}{D_2}\phi_1^{\xi}. \tag{23}$$

It is well known that the solitary wave solutions of KdV equations (20) and (21) are the "sech squared" solitons, given as

$$\phi_1^{\xi}=\phi_{1m}^{\xi}\mathrm{sech}^2\left[\left(\frac{A_1\phi_{1m}^{\xi}}{12B_1}\right)^{1/2}\left(\xi+\frac{1}{3}A_1\phi_{1m}^{\xi}\tau\right)\right], \tag{24}$$

$$\phi_1^{\eta}=\phi_{1m}^{\eta}\mathrm{sech}^2\left[\left(\frac{A_2\phi_{1m}^{\eta}}{12B_2}\right)^{1/2}\left(\eta-\frac{1}{3}A_2\phi_{1m}^{\eta}\tau\right)\right], \tag{25}$$

where $\phi_{1m}^{\xi}$ and $\phi_{1m}^{\eta}$ are the amplitudes of solitons S1 and S2 in their initial positions. The leading phase changes due to the collision can be calculated from Eqs. (22) and (23) as

$$P_0(\eta,\tau) = \frac{-E_1}{D_1}\left(\frac{12B_1\phi_{1m}^{\eta}}{A_1}\right)^{1/2} \times \tanh\left[\left(\frac{A_2\phi_{1m}^{\eta}}{12B_2}\right)^{1/2}\left(\eta - \frac{1}{3}A_2\phi_{1m}^{\eta}\tau\right) - 1\right], \tag{26}$$

$$Q_0(\xi,\tau) = \frac{-E_2}{D_2}\left(\frac{12B_2\phi_{1m}^{\xi}}{A_2}\right)^{1/2} \times \tanh\left[\left(\frac{A_1\phi_{1m}^{\xi}}{12B_1}\right)^{1/2}\left(\xi + \frac{1}{3}A_1\phi_{1m}^{\xi}\tau\right) + 1\right]. \tag{27}$$

Substituting Eqs. (26) and (27) into Eq. (4), we obtain the trajectories of the two solitary waves for oblique collision as

$$\xi = \epsilon(l_{x1}x + l_{y1}y + l_{z1}z - c_1t) - \epsilon^2\frac{E_1}{D_1}\left(\frac{12B_1\phi_{1m}^{\eta}}{A_1}\right)^{1/2} \times \tanh\left[\left(\frac{A_2\phi_{1m}^{\eta}}{12B_2}\right)^{1/2}\left(\eta - \frac{1}{3}A_2\phi_{1m}^{\eta}\tau\right) - 1\right] + \cdots, \tag{28}$$

$$\eta = \epsilon(l_{x2}x + l_{y2}y + l_{z2}z + c_2t) - \epsilon^2\frac{E_2}{D_2}\left(\frac{12B_2\phi_{1m}^{\xi}}{A_2}\right)^{1/2} \times \tanh\left[\left(\frac{A_1\phi_{1m}^{\xi}}{12B_1}\right)^{1/2}\left(\xi + \frac{1}{3}A_1\phi_{1m}^{\xi}\tau\right) + 1\right] + \cdots. \tag{29}$$

To obtain the phase shifts after the oblique collision of the two solitons, we assumed that the two solitons S1 and S2 are, asymptotically, far from each other at the initial time ($\tau = -\infty$), i.e., soliton S1 is at $\xi = 0$, $\eta = -\infty$ and soliton S2 is at $\eta = 0$, $\xi = +\infty$. After the collision ($\tau = +\infty$), the soliton S1 is far to the right of soliton S2, i.e., soliton S1 is at $\xi = 0$, $\eta = +\infty$ and soliton S2 is at $\eta = 0$, $\xi = -\infty$. Using Eqs. (28) and (29) we obtain the corresponding phase shifts $\Delta P_0$ and $\Delta Q_0$ as follows:

$$\Delta P_0 = 2\epsilon^2\frac{E_1}{D_1}\left(\frac{12B_2\phi_{1m}^{\eta}}{A_2}\right)^{1/2}, \tag{30}$$

$$\Delta Q_0 = -2\epsilon^2\frac{E_2}{D_2}\left(\frac{12B_1\phi_{1m}^{\xi}}{A_1}\right)^{1/2}. \tag{31}$$

## III. NUMERICAL INVESTIGATIONS AND DISCUSSION

We consider a magnetized dusty electronegative plasma consisting of cold positive ions fluid, isothermal electrons, isothermal negative ions with immobile positively/negatively charged dust particles. The oblique collision between two IA solitary waves propagating in the plasma is studied. Though, the extended PLK perturbation method is employed to derive the expressions for the phase shifts $\Delta P_0$ and $\Delta Q_0$. It is clear from Eqs. (11), (12), (19), (30), and (31) that the phase shifts depend upon the physical parameters of the plasma system. For instance, the magnitude of the phase shift depends on the initial amplitudes of IA solitary waves with large amplitudes causing large phase shifts.[25] Accordingly, the effects of the oblique collision (i.e., $\theta \neq 0$ or $\pi$), the dust density, the dust polarity, the temperature and the density of negative ions, and the static magnetic field on the phase shifts and trajectories of the two IA solitons after the oblique collision are discussed as follows.

In Figure 1, the electrostatic potential $\phi_1^{\xi}$ is plotted against $\xi$, showing the presence of only the bell-shaped electrostatic potential of one wave. It is clear from Figure 1 that the width of the electrostatic potential decreases as the temperature of negative ions $\sigma$ increases. Moreover, the dust polarity effect is shown by taking $\delta = +1$ for positive dust or $\delta = -1$ for negative dust; the width of the potential in case of positive dust is larger than the case of negative dust. Figures 2(a) and 2(b) demonstrate the variation of the positive ion density ($n_i$) against the space coordinate $R_1 (= l_{x1}x + l_{y1}y + l_{z1}z)$ and the time variable $t$ for $\delta = +1$ (Figure 2(a)) and $\delta = -1$ (Figure 2(b)). Figure 2 shows that two propagated IA solitons approach to each other, collide, and asymptotically separate away. We observe that during oblique collision one practically motionless composite structure is formed for some time interval. In other words, it is shown that when two same amplitude IA solitary waves interact obliquely, a new nonlinear wave is formed during their interaction which moves ahead of the colliding IA solitary waves; both its amplitude and width are larger than those of colliding solitary waves. Therefore, as a result to create a new nonlinear wave, the IA solitary waves after the oblique collision are delayed. That is to say, the phase shift depends directly on a new created nonlinear wave. After the oblique collision, it is easy to find the propagation of the two solitons along the deviated trajectories from the initial trajectories. In fact, these deviations are just the phase shifts for the two IA solitons. Accordingly, one can see that the new

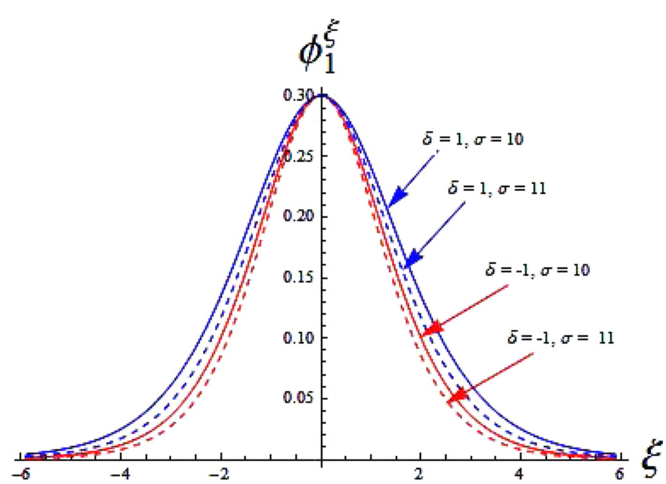


FIG. 1. The bell-shaped electrostatic potential $\phi_1^{\xi}$ of one IA solitary wave for the following cases: $\delta = +1(-1)$ for blue (red) curve, $\sigma = 10(11)$ for solid (dashed) curve. The parameters are set as $\epsilon = 0.05, l_{x1} = l_{y1} = l_{z1} = 0.1, t = 0, \Omega = 0.2, \mu_n = 0.4, \mu_d = 0.4, \phi_{1m}^{\xi} = \phi_{1m}^{\eta} = 0.3$.

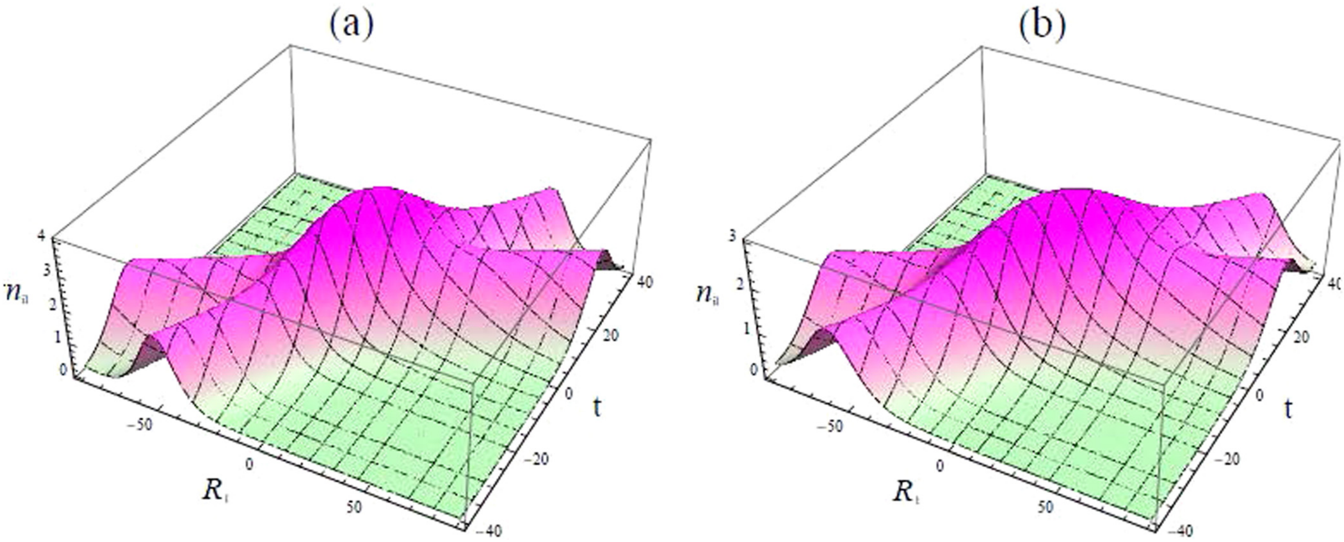


FIG. 2. Profile of interaction between the two IA solitary waves of the same amplitude, (a) for $\delta = +1$, (b) for $\delta = -1$, where $\sigma = 10, \epsilon = 0.1, \theta = \pi/1.172$ (i.e., $l_{x1} = 0.1, l_{y1} = 0.1, l_{z1} = 0.6, l_{x2} = 0.1, l_{y2} = 0.1, l_{z2} = -0.6$). The other parameters are the same as in Figure 1.

nonlinear wave width for concentration of negatively charged dust particles is greater than the width of the new nonlinear wave for concentration of positively charged dust particles. Therefore, the deviation of the trajectories (phase shifts) in case of negative polarity dust is larger than the case of positive polarity dust. Figures 3(a)–3(d) represent the space-time coordinates plots of the two colliding IA solitons in order to show the existence of the phase shifts clearly. In these coordinates wave phenomena look as sloped straight lines. The two colliding IA solitary waves trajectories are then represented by two crossing straight lines. It is clear that there is a small temporal delay between converging (left part of the plots) and diverging (right part of the plots) waves. To measure the propagation delay of the waves we fitted their trajectories with straight lines

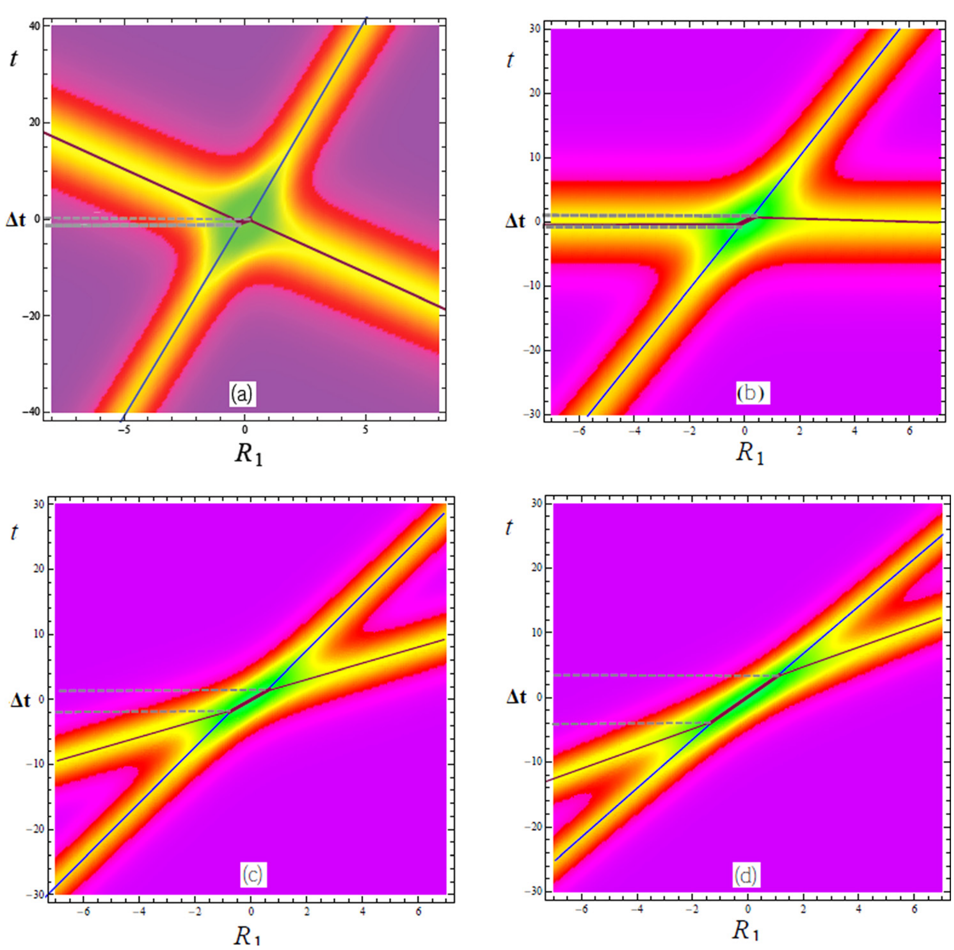


FIG. 3. Space-time plots of two colliding IA solitons at different values of the oblique angle $\theta$. Degree of brightness indicates $n_{i1}$. Each line marks the soliton path before and after the collision. The values of the angle $\theta$ are as the following: (a) plotted for $\theta = \pi/2.552$ (i.e., $l_{x1} = 0.1, l_{y1} = 0.1, l_{z1} = 0.1, l_{x2} = 0.1, l_{y2} = 0.1, l_{z2} = -0.1$). (b) plotted for $\theta = \pi/1.644$ (i.e., $l_{x1} = 0.1, l_{y1} = 0.1, l_{z1} = 0.2, l_{x2} = 0.1, l_{y2} = 0.1, l_{z2} = -0.2$). (c) plotted for $\theta = \pi/1.276$ (i.e., $l_{x1} = 0.1, l_{y1} = 0.1, l_{z1} = 0.4, l_{x2} = 0.1, l_{y2} = 0.1, l_{z2} = -0.4$). (d) plotted for $\theta = \pi/1.172$ (i.e., $l_{x1} = 0.1, l_{y1} = 0.1, l_{z1} = 0.6, l_{x2} = 0.1, l_{y2} = 0.1, l_{z2} = -0.6$).

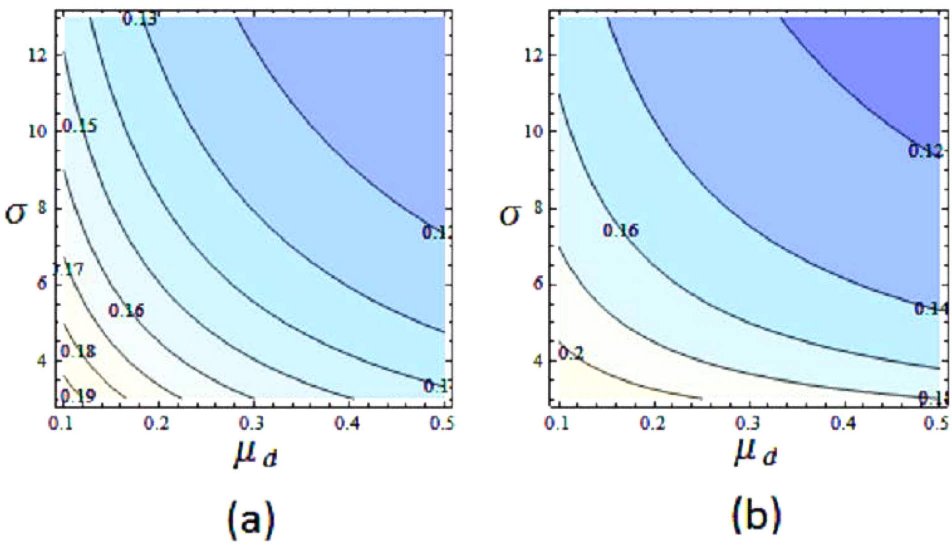


FIG. 4. The contour regions for the phase shift $\Delta P_0$ versus $\mu_d$ and $\sigma$ at $R = 1,\ t = 10, \Omega = 0.4,\ \mu_n = 0.4,\ \theta = \pi/1.276$ (i.e., $l_{x1} = 0.1, l_{y1} = 0.1, l_{z1} = 0.4, l_{x2} = 0.1, l_{y2} = 0.1, l_{z2} = -0.4$). (a) for $\delta = +1$, (b) for $\delta = -1$.

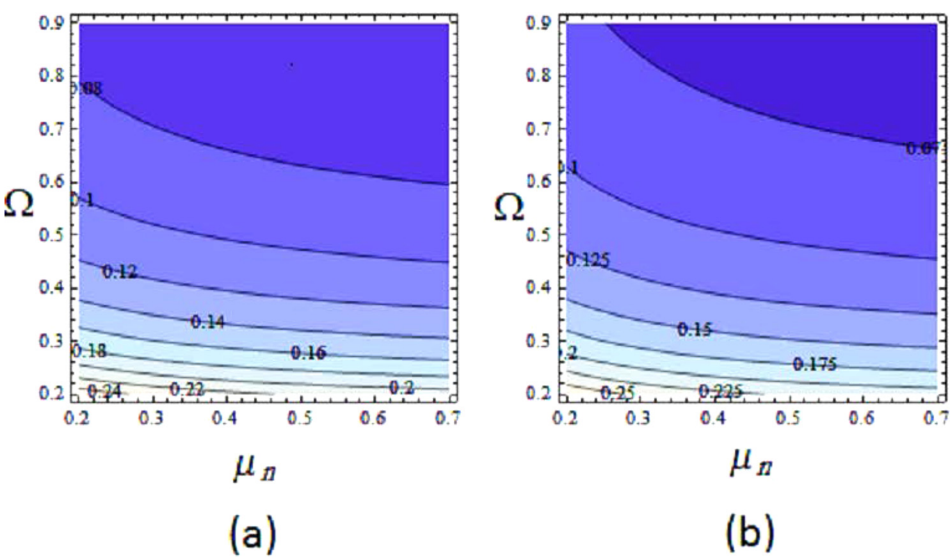


FIG. 5. The contour regions for the phase shift $\Delta P_0$ versus $\mu_n$ and $\Omega$ at $R = 1, t = 10, \sigma = 10, \mu_d = 0.3,\ \theta = \pi/1.276$ (i.e., $l_{x1}=0.1, l_{y1}=0.1, l_{z1}=0.4, l_{x2}=0.1, l_{y2}=0.1, l_{z2}=-0.4$). (a) for $\delta=+1$, (b) for $\delta=-1$.

separately before and after the oblique collision obtaining two pairs of lines. These lines cross at two different points. The delay time $\Delta t$ was determined from the offset between the crossing points. Owing to increase (decrease) $l_{z1}$ ($l_{z2}$) the angle between the propagation directions of IA solitary waves increases, which give rise to increase phase shifts (i.e., $\Delta t$) as depict in Figure 3. Crucially, the propagation delay in our numerical results increases with the increase of the angle between the propagation direction of IA solitons.

The variation of the phase shift $\Delta P_0$ with the dust number density $(\mu_d)$ and the temperature of negative ions $(\sigma)$ is shown in Figure 4 (for positive polarity dust, i.e., Figure 4(a) and for negative polarity dust, i.e., Figure 4(b)). We can see that $\Delta P_0$ decreases as both $\mu_d$ and $\sigma$ increase. So, the temperature of negative ions has a significant effect on the phase shift which can not be ignored specially when negative ions obey Boltzmann distribution. Apparently, $\Delta P_0$ for negative (positive) dust particles decreases smoothly (gradually) with increasing of the dust number density $(\mu_d)$ and the temperature of negative ions $(\sigma)$. Moreover, Figures 5(a) and 5(b) indicate that $\Delta P_0$ decreases as both the negative ion density $(\mu_n)$ and the ion cyclotron frequency $(\Omega)$ increase. For a given value of $\mu_n$, the phase shift curves when $\Omega < 0.3$ are fairly near to each other, but the phase shift curves when $\Omega > 0.3$ are far away from each other. Moreover, for a given value of the parameter $\Omega$, especially for $\Omega < 0.3\,(\Omega > 0.3)$, phase shift $\Delta P_0$ decreases gradually (smoothly) with increasing of $\mu_n$. Accordingly, it is worth to notice that $\Omega$ and $\mu_n$ have strong effects on the IA solitary waves collision.

At last, these findings should be very useful for explaining the interaction features of the nonlinear IA solitary waves in space environment[1–4] (e.g., D- and F-regions of the Earth's ionosphere) as well as laboratory devices[5–8] where dusty electronegative plasma with Boltzmann distributed negative ions and dust polarity effect exists.

## ACKNOWLEDGMENTS

The authors would like to express their gratitude to the referees for a number of valuable criticisms and comments that have led to improvement of the original manuscript. The authors also thank the editor and his staff for their kind cooperation.

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