Head-on collision of dust acoustic solitons in a nonextensive plasma with variable size dust grains of arbitrary charge

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The head-on collision of two dust acoustic solitons (DASs) in a nonextensive plasma with positive or negative dust grains fluid including the effect of dust size distribution (DSD) is studied. The phase shifts for the two solitons due to the collision are derived by applying the extended Poincaré-Lighthill-Kuo (PLK) method. The influences of the power law DSD and the nonextensivity of plasma particles on the characteristic properties of the head-on collision of DASs are analyzed. It is found that the phase shifts can vanish, only for the case of positive dust grains, for certain values and ranges of the dust grain radius and the entropic index of ions \( (q_i) \). Also, they undergo a cutoff in the range of \( q_i > 1 \) for the subextensive distribution. A brief discussion of possible applications in laboratory and space plasmas is included.

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I. INTRODUCTION

In the last two decades, the dynamics of solitary waves propagating in dusty plasma systems gained a great interest of research due to its important role in understanding many space and astrophysical phenomena, as well as many industrial and physical applications [1]. The actual observations show that the dust grain size ranges from nanometers to millimeters unless they are manmade. Thus, the radii, \( r_{dj} \), of the \( j \)th dust grains aren’t the same for all dust particles, but they vary within the range \([r_{\text{min}}, r_{\text{max}}]\) where \( r_{\text{min}} \) (\( r_{\text{max}} \)) is the lower (upper) limit [2]. The most widely applicable dust size distribution (DSD) is the power law distribution, because of its various applications in space plasmas [3–6]. It is remarked that the DSD may be discrete or continuous; however, the continuous model is the most reasonable [7].

It was usual to consider the dusty plasma model with negatively charged dust only. However, there are many cases in which dust particles have positive charges, e.g., Jupiter’s magnetosphere [8], cometary tails [9], and Earth’s mesosphere [10]. Many researchers [11–13] focused on the propagation and stability of dust acoustic solitons (DASs) in a plasma having positive or negative dust particles, to interpret the features of two different layered structures known as noctilucent clouds (NLCs) and polar mesosphere summer echoes (PMSEs) in Earth’s mesosphere. In low-temperature plasmas, dust particles are considered to have negative charge only due to the fact that the collection of plasma particles (electrons and ions) is the only important charging process. However, there are other important charging processes by which dust grains become positively charged [14–16]. The principal mechanisms of such processes are photoemission in the presence of a flux of ultraviolet photons [15], thermionic emission induced by radiative heating [16], secondary emission of electrons from the surface of the dust grains [14], etc. Moreover, Chow et al. [14] have shown that due to the dust grains’ size effect on secondary emission, insulated dust grains with different sizes can produce positive or negative polarity dust particles, smaller ones being positive and larger ones being negative. The opposite situation, i.e., massive positive and lighter negative dust grains, is also possible by triboelectric charging [16,17]. This is predicted from the observations of dipolar electric fields perpendicular to the ground, with the negative pole at higher altitudes, generated by dust devils [18] and thunderstorms [19]. The formation of these dipolar electric fields means that negatively charged dust particles are blown upward in the convection, while positively charged dust particles remain at the surface due to gravity. It may be noted here that the existence of same-sized dust particles of positive or negative polarity may also occur by photoemission if the photoemission yields of the dust material are very different [20].

The interaction between two solitons is an interesting and important nonlinear phenomenon during the process of soliton propagation in dusty plasmas. Zabusky [21] remarked on the important property that the solitons have an asymptotic preservation of form when they undergo a collision. In a one- (or quasi-one-) dimensional system, there are two distinct soliton interactions. One is the overtaking collision [22] (for copropagating solitons), and the other is the head-on collision (for counterpropagating solitons) [23]. The general features of the collision between solitons are summarized in the following: when two solitons approach closely, they interact, exchange their energies and positions with each other and then separate off, regaining their original wave forms [21]. Throughout the whole process of the collision, the solitons are remarkably stable entities, preserving their identities through interaction; the unique effect due to the collision is their phase shift and the trajectories [21], though many researchers have investigated the head-on collision between two solitons in different plasma systems using the extended Poincaré-Lighthill-Kuo (PLK) method [24–29]. Xue [24] investigated the head-on collision of DASs in a dusty plasma. He studied the effect of dust charge variation on the phase shift due to the collision. The head-on collision of DASs in a dusty plasma with Boltzmann distributed electrons, nonthermal ions, and dust grains with opposite polarity charge is studied by Ghosh et al. [26]. Recently, Zhang et al. [30] studied the head-on collision and the overtaking process between a Korteweg-de Vries (KdV) solitary wave and an envelope solitary wave in a dusty plasma.
plasma using the particle-in-cell simulation method. In plasma experiments, two experimental observations [31,32] have been reported on the head-on collision of two counterpropagating solitons in plasma.

It is believed that the Maxwellian distribution, in Boltzmann-Gibbs (BG) statistics, is universally valid for the macroscopic ergodic equilibrium systems. However, for systems with long-range interactions, such as plasma and gravitational systems, BG statistics might be inadequate for describing the system. This failure is due to the fact that BG statistics are extensive and additive in its formalism. Therefore, another statistical approach, nonextensive statistics or Tsallis statistics, is introduced to treat this defect by the generalization of BG statistics [33,34]. The physical meaning of nonextensivity is that the entropy of the composition (A + B) of two independent systems, A and B, is equal to $S_A(A + B) = S_A(A) + S_B(B) - (1 - q)S_A(A)S_B(B)$. The degree of nonextensivity of the system is characterized by the nonextensive parameter (or the entropic index) $q$. It is noted that the $q$-nonextensive distribution tends to the Maxwellian distribution in the limiting case ($q \to 1$). Moreover, the $q$-nonextensive distribution leads to two different cases depending on the value of the entropic index $q$. Case 1, the superextensive distribution, is defined when $q < 1$, which behaves like the $\kappa$-distribution for superthermal particles in plasma. Case 2, the subextensive distribution, is presented when $q > 1$, which is suitable for describing systems of low-speed particles. In Case 2, the subextensive distribution, there is a thermal cutoff which limits the velocity of the particles [35]. Recently the $q$-nonextensive distribution was used to describe electrons and/or ions in several plasma models [6,29,36].

The motivation of the present article is to study the influence of the power law DSD on head-on collision of two DASs in a plasma with nonextensively distributed electrons and ions. Then the charge neutrality condition reads

$$n_{eo} = n_{io} + \delta \sum_{j=1}^{N} Z_{dj} n_{dj0}, \quad (1)$$

where $n_{eo}$, $n_{io}$, and $n_{dj0}$ are the particles number density at equilibrium for electrons, ions, and dust grains, respectively. $Z_{dj}$ is the charge number of the $j$th dust grain, and $\delta$ equals $+1(-1)$ for positive (negative) dust particles. Electrons and ions obey the nonextensive velocity distribution as follows [35]:

$$n_e = n_{eo} \left[ 1 + (q_e - 1) \frac{e_{e\phi}}{T_e} \right]^{\frac{(q_e - 1)}{q_e - 1}}, \quad (2)$$

$$n_i = n_{io} \left[ 1 - (q_i - 1) \frac{e_{i\phi}}{T_i} \right]^{\frac{(q_i - 1)}{q_i - 1}}, \quad (3)$$

where $q_e$ ($q_i$) is the nonextensivity index for electrons (ions). Now the set of governing equations in a dimensional form is introduced as

$$\frac{\partial n_{dj}}{\partial t} + u_{dj} \frac{\partial n_{dj}}{\partial x} = 0, \quad (4)$$

$$\frac{\partial u_{dj}}{\partial t} + u_{dj} \frac{\partial u_{dj}}{\partial x} = -e_{dj} \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi}{\partial x}, \quad (5)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\epsilon_0} \left( n_e - n_i - \delta \sum_{j=1}^{N} Z_{dj} n_{dj} \right). \quad (6)$$

Equations (4)–(6) will be normalized using the formal setting; $x \to L_0 \hat{x}$, $t \to T_0 \hat{t}$, $n_{dj} \to N_{tot} n_{dj}$, $m_{dj} \to m_d n_{dj}$, and $Z_{dj} \to Z_d Z_{dj}$, where the tilde quantities are dimensionless. The dust number density, mass, and charge are normalized in terms of their average quantities according to the power law DSD: $N_{tot} = \sum_{j=1}^{N} n_{dj}$, $m_d = \sum_{j=1}^{N} m_{dj}$, and $Z_d = \sum_{j=1}^{N} Z_{dj}$. For simplicity, the tildes will be omitted after carrying out the transformation. The scaling quantities adopted above were chosen appropriately as

$$L_0 = \lambda_d = \left( \frac{e_0 T_e}{e^2 Z_d N_{tot}} \right)^{1/2}, \quad V_0 = L_0 \frac{T_0}{T_e} = \left( \frac{Z_d T_e}{m_d} \right)^{1/2},$$

$$T_0 = \omega_{pd}^{-1} = \left( \frac{e_0 m_d}{e^2 Z_d^2 N_{tot}} \right)^{1/2} \quad \text{and} \quad \varphi_0 = \frac{T_e}{e}. \quad (7)$$

The following set of normalized (dimensionless) equations is obtained:

$$\frac{\partial n_{dj}}{\partial \hat{t}} + \frac{\partial}{\partial \hat{x}} (n_{dj} u_{dj}) = 0, \quad (8)$$

$$\frac{\partial u_{dj}}{\partial \hat{t}} + u_{dj} \frac{\partial u_{dj}}{\partial \hat{x}} = -e_{dj} \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi}{\partial \hat{x}}, \quad (9)$$

$$\frac{\partial^2 \phi}{\partial \hat{x}^2} = v_e [1 + (q_e - 1)\phi]^{\frac{(q_e - 1)}{q_e}} - v_i [1 - (q_i - 1)\sigma \phi]^{\frac{(q_i - 1)}{q_i}} - \delta \sum_{j=1}^{N} Z_{dj} n_{dj},$$

where $v_e = \frac{n_{eo}}{Z_d N_{tot}}, v_i = \frac{n_{io}}{Z_d N_{tot}},$ and $\sigma = \frac{T_e}{T_i}. \quad (10)$

III. HEAD-ON COLLISION

Now the extended PLK perturbation method [21–24] is employed to study the effects of two weakly interacting DASs, colliding each other with an angle $\theta = \pi$, and propagating in the present nonextensive plasma. In details, the two DASs, $S_1$ and $S_2$, are assumed to be far apart in the initial states and propagate toward each other. After some time, they collide,
and then depart. For weakly interacting solitons with small amplitudes, the collision is likely to be quasielastic. Hence, the postcollision trajectories are shifted (i.e., phase shift). The extended PLK method is based on the combination of the standard reductive perturbation method [37] with the technique of strained coordinates. In this perturbation method, in the limit of the long wavelength approximation, asymptotic expansions for both the physical variables and the spatial or time coordinates are used. Then a uniformly valid asymptotic expansion is produced. The corresponding KdV equations are obtained after eliminating the secular terms, and at the same time the phase shifts of the two solitons due to the collision are derived. According to this method, we introduce the following stretched coordinates [24]:

\[ \xi = \epsilon(x - \lambda t) + \epsilon^2 P_0(\eta, \tau) + \epsilon^3 P_1(\xi, \eta, \tau) + \cdots, \]  
\[ \eta = \epsilon(x + \lambda t) + \epsilon^2 Q_0(\xi, \tau) + \epsilon^3 Q_1(\eta, \xi, \tau) + \cdots, \]
\[ \tau = \epsilon^3 \tau. \]

Here \( \xi \) and \( \eta \) denote the trajectories of the colliding solitons. The soliton velocity \( \lambda \) and the unknown phase functions \( P_0, P_1, Q_0, \) and \( Q_1 \) are to be determined later. The extended PLK method anticipates that the collision will result in phase shifts in the postcollision trajectories of the solitons. Therefore, the phase functions for the trajectories are expanded into asymptotic series of the smallness and ordering parameter \( \epsilon \) [24]. Accordingly, the dependent variables are expanded as the following:

\[ n_{dj} = n_{dj0} + \epsilon^2 n_{dj1} + \epsilon^3 n_{dj2} + \cdots, \]
\[ u_{dj} = \epsilon^2 u_{dj1} + \epsilon^3 u_{dj2} + \cdots, \]
\[ \phi = \epsilon^2 \phi_1 + \epsilon^3 \phi_2 + \cdots. \]

Substituting Eqs. (10)–(15) into Eqs. (7)–(9) and then collecting terms of the same powers of \( \epsilon \), the lowest order perturbed quantities are obtained as the following:

\[ \phi_1 = \phi_1^\xi(\xi, \tau) + \phi_1^\eta(\eta, \tau), \]
\[ n_{dj1} = \frac{\delta n_{dj0} Z_{dij}}{\lambda^2 m_{dij}} (\phi_1^\xi + \phi_1^\eta), \]
\[ u_{dj1} = \frac{\delta Z_{dij}}{\lambda m_{dij}} (\phi_1^\xi - \phi_1^\eta). \]

The unknown functions \( \phi_1^\xi(\xi, \tau) \) and \( \phi_1^\eta(\eta, \tau) \) are the supposed solutions corresponding to the two solitons, and they will be determined at a higher order [24]. A compatibility condition [i.e., the condition to obtain a uniquely defined \( n_{dj1} \) and \( u_{dj1} \) from Eqs. (17)–(18) when \( \phi_1 \) is given by Eq. (16)] is imposed, in the form

\[ \lambda^2 = \chi_1^{-1} \sum_{j=1}^{N} \frac{n_{dijo} Z_{dij}^2}{m_{dij}}, \]

where

\[ \chi_1 = \left[ v_c \left( \frac{q_c + 1}{2} \right) + \sigma v_i \left( \frac{q_i + 1}{2} \right) \right]. \]

To the next order of \( \epsilon \), we obtain

\[ \phi_2 = \phi_2^\xi(\xi, \tau) + \phi_2^\eta(\eta, \tau), \]
\[ n_{dij2} = \frac{\delta n_{dij} Z_{dij}}{\lambda^2 m_{dij}} (\phi_2^\xi + \phi_2^\eta), \]
\[ u_{dij2} = \frac{\delta Z_{dij}}{\lambda m_{dij}} (\phi_2^\xi - \phi_2^\eta). \]

Furthermore, for the next order of \( \epsilon \) and after some algebraic manipulations, we obtain the following equation:

\[ -2u_{dij3} = \frac{\delta Z_{dij}}{\lambda^2 m_{dij}} \int \left( \frac{\partial \phi_1^\xi}{\partial \tau} + A \phi_1^\xi \frac{\partial \phi_1^\xi}{\partial \xi} + B \frac{\partial^3 \phi_1^\xi}{\partial \xi^3} \right) d\eta \]
\[ + \frac{\delta Z_{dij}}{\lambda^2 m_{dij}} \int \left( \frac{\partial \phi_1^\eta}{\partial \eta} - A \phi_1^\eta \frac{\partial \phi_1^\eta}{\partial \eta} - B \frac{\partial^3 \phi_1^\eta}{\partial \eta^3} \right) d\xi \]
\[ + \int \int \left( \frac{\partial P_0}{\partial \eta} + D \phi_1 \frac{\partial^2 \phi_1}{\partial \xi^2} \right) d\xi d\eta, \]

where

\[ A = \left( \frac{3 \delta Z_{dij}}{2 \lambda m_{dij}} + \lambda \chi_2, \right), \]
\[ B = \frac{\lambda}{2}, \]
\[ C = \frac{2 \delta Z_{dij}}{\lambda m_{dij}}, \]
\[ D = \frac{Z_{dij}}{\lambda m_{dij}} \left( \delta \chi_2 - \frac{Z_{dij}}{2 \lambda^2 m_{dij}} \right), \]

and

\[ \chi_2 = \left[ v_c \left( \frac{q_c + 1}{8} \right) (q_c - 3) - \sigma^2 v_i \left( \frac{q_i + 3}{8} \right) (q_i - 3) \right]. \]

The first (second) term on the right-hand side of equation (24) will be proportional to \( \eta(\xi) \) because the integrated function is independent of \( n(\xi) \). Thus, the first two terms of Eq. (24) are all secular terms, which must be eliminated in order to avoid spurious resonances. Hence, we have

\[ \frac{\partial \phi_1^\xi}{\partial \tau} + A \phi_1^\xi \frac{\partial \phi_1^\xi}{\partial \xi} + B \frac{\partial^3 \phi_1^\xi}{\partial \xi^3} = 0, \]
\[ \frac{\partial \phi_1^\eta}{\partial \tau} - A \phi_1^\eta \frac{\partial \phi_1^\eta}{\partial \eta} - B \frac{\partial^3 \phi_1^\eta}{\partial \eta^3} = 0. \]

The third and the fourth terms in Eq. (24) are not secular terms now, but they will generate secular contributions at the next higher order, so they must vanish, and the leading equations for the phase shifts read

\[ \frac{\partial P_0}{\partial \eta} = -D \phi_1^\xi, \]
\[ \frac{\partial Q_0}{\partial \xi} = -D \phi_1^\xi. \]
The solitary wave solutions of KdV equations (30) and (31) are the well-known “squared sech” solitons, which are given as

$$\phi_1^x = \phi_{1m}^x \text{sech}^2 \left[ \left( \frac{A\phi_{1m}^x}{12B} \right)^{1/2} \left( \xi + \frac{1}{3} A\phi_{1m}^x \tau \right) \right]$$ (34)

and

$$\phi_1^n = \phi_{1m}^n \text{sech}^2 \left[ \left( \frac{A\phi_{1m}^n}{12B} \right)^{1/2} \left( \eta - \frac{1}{3} A\phi_{1m}^n \tau \right) \right],$$ (35)

where $\phi_{1m}^x$ and $\phi_{1m}^n$ are the amplitudes of solitons $S_1$ and $S_2$, respectively. Using Eqs. (32)–(35), the leading phase changes due to the collision are derived as

$$P_0(\eta, \tau) = -\frac{D}{C} \left( \frac{12B\phi_{1m}^n}{A} \right)^{1/2} \times \left[ \tanh \left[ \left( \frac{A\phi_{1m}^n}{12B} \right)^{1/2} \left( \eta - \frac{1}{3} A\phi_{1m}^n \tau \right) \right] + 1 \right],$$ (36)

$$Q_0(\xi, \tau) = -\frac{D}{C} \left( \frac{12B\phi_{1m}^x}{A} \right)^{1/2} \times \left[ \tanh \left[ \left( \frac{A\phi_{1m}^x}{12B} \right)^{1/2} \left( \xi - \frac{1}{3} A\phi_{1m}^x \tau \right) \right] - 1 \right].$$ (37)

Substituting Eqs. (36)–(37) into Eqs. (10)–(11), and assuming that the two solitons $S_1$ and $S_2$ are, asymptotically far from each other at the initial time ($\tau = -\infty$), i.e., soliton $S_1$ is at $\xi = 0$, $\eta = -\infty$ and soliton $S_2$ is at $\eta = 0$, $\xi = +\infty$. After the collision ($\tau = +\infty$), the soliton $S_1$ goes far to the right of soliton $S_2$, i.e., soliton $S_1$ is at $\xi = 0$, $\eta = +\infty$ and soliton $S_2$ is at $\eta = 0$, $\xi = -\infty$, the corresponding phase shifts $\Delta P_0$ and $\Delta Q_0$ are obtained as

$$\Delta P_0 = -2\epsilon^2 D \left( \frac{12B\phi_{1m}^n}{A} \right)^{1/2},$$ (38)

$$\Delta Q_0 = 2\epsilon^2 D \left( \frac{12B\phi_{1m}^x}{A} \right)^{1/2}.$$(39)

Since the soliton $S_1$ is traveling to the right and the soliton $S_2$ is traveling to the left, Eqs. (38) and (39) indicate that each soliton has a negative phase shift in its traveling direction. The meaning of negative phase shift after the collision is that the trajectories of the propagated solitons are behind what would be expected if they just passed through each other with no interaction. In other words, each soliton is lagging its corresponding case of no collision [24,25,32].

IV. THE POWER LAW DSD

Here the effect of power law DSD on head-on collision of DAWs in the proposed nonextensive plasma is examined. We begin with the differential form of the power law DSD, given as [3,5]

$$n_{dj}(r_j)dr_j = K \ r_j^{-\beta} \ dr_j,$$ (40)

where $n_e(r_j)dr_j$ is the number density of the dust grains per unit volume with radii in the range from $r_j$ to $r_j + dr_j$, $\beta$ is the power law index, and $K$ is the normalization constant. The total number density of all grains is given by

$$N_{tot} = \int_{r_{min}}^{r_{max}} n_{dj}(r_j)dr_j = \frac{K r_{min}^{1-\beta}}{1-\beta} (R^{1-\beta} - 1).$$ (41)

The mass and the charge of the $j$th dust grain can be approximated as [3]

$$m_{dj} = K_m r_j^3,$$ (42)

$$Z_{dj} = K_z r_j,$$ (43)

where $K_m (\sim 4\pi\rho_d j)$ is a constant with the assumption that the mass density of the dust grains $\rho_d$ is the same for all grains, and $K_z (\sim 4\pi\rho_d j)$ is a constant when we take the electric surface potential of the dust grains $V_d$ to be constant at equilibrium. $\epsilon_o$ is the permittivity of free space. The physical quantities will be rewritten in terms of the power law DSD as

$$\lambda^2 = \frac{K K_z^2}{\beta K_m r_{min}^{1-\beta}} (1 - R^{-\beta}),$$ (44)

$$n_{dj1} = \frac{\delta K K_z}{\lambda^2 K_m r_{min}^{1-\beta}} (1 - R^{-\beta+1}) (\phi_1^x + \phi_1^n),$$ (45)

$$u_{dj1} = \frac{\delta K K_z}{\lambda^2 K_m r_{min}^{1-\beta}} (1 - R^{-1}) (\phi_1^x - \phi_1^n),$$ (46)

$$A = \frac{3\delta K_z}{2\lambda K_m r_{min}^{1-\beta}} \left( 1 - \frac{1}{R} \right) + \lambda \chi_2,$$ (47)

$$B = \left[ \frac{K K_z^2}{4\beta K_m r_{min}^{1-\beta}} \right] (1 - R^{-\beta}),$$ (48)

$$C = \frac{2\delta K_z}{\lambda K_m r_{min}^{1-\beta}} \left( 1 - \frac{1}{R} \right),$$ (49)

$$D = \frac{\delta \chi_2 K_z}{\lambda K_m r_{min}^{1-\beta}} \left( 1 - \frac{1}{R} \right) - \frac{K_z^2}{6\lambda^3 K_m r_{min}^{1-\beta}} (1 - R^{-3}),$$ (50)

where $R = \frac{r_{min}}{r_{max}}$. Now, from Eqs. (38)–(39) and Eqs. (44)–(50), it is clear that the phase shifts depend on the power law DSD parameters $R$, $\beta$, and $r_{min}$ as well as the nonextensive parameters $\epsilon_o$ and $q_i$.

V. NUMERICAL INVESTIGATIONS AND DISCUSSION

In this paper, the head-on collision phenomenon between two DASs in an unmagnetized dusty plasma consisting of nonextensively distributed electrons and ions, and positive or negative dust fluid with variable size dust grains has been investigated theoretically. The phase shifts due to the collision are obtained using the extended PLK method. Then the power law DSD is applied to study the effect of DSD on the collision process and phase shift. In numerical investigations, the following numerical values are used [6,14,31,32]: $K = 10^{-5}$, $K_z = 1$, $K_m = 4$, $r_{min} = 8 \mu m$, $q_e = 0.2 - 0.5$, $\epsilon = 0.1$, $\nu_e = 1.3$, $\nu_i = 0.8$, $\sigma = 1.5$. The various effects of $R$, $\beta$, $r_{min}$, and $q_i$ on the phase shifts and the collision process are illustrated.
FIG. 1. The head-on collision profile between two DASs is depicted via the first order perturbed quantities \( n_{dj1} \) [from Eq. (45), on the left] and \( u_{dj1} \) [from Eq. (46), on the right]. Two different dust size cases are presented here; the dust dust radius ratio \( R = r_{\text{max}}/r_{\text{min}} = 1.5 \) for the solid line and \( R = 2.5 \) for the dashed line. Note that the time \( t \) is scaled by \( T_0 \), and the collision takes place at \( t = 0 \). The following parameters are used: \( \phi_{\xi 1m} = 0.3, \phi_{\eta 1m} = 0.6, q_e = 0.5, q_i = 1.5, \beta = 3.5, \epsilon = 0.1, r_{\text{min}} = 8 \mu m. \)

Figure 1 shows some timing shut before and after the head-on collision between two DASs, via the first order perturbed quantities \( n_{dj1} \) and \( u_{dj1} \). Moreover, it is clear that the dust radius ratio \( R \) changes the amplitude of the two DASs before, after, and at collision time. The phase shifts \( \Delta P_0 \) and \( \Delta Q_0 \) of the solitons are drastically influenced by the dust radius ratio \( R \) and the power law index \( \beta \) as shown in Fig. 2. For positive dust case \( (\delta = +1) \), \( \Delta P_0 \) decreases rapidly as either \( R \) or \( \beta \) increases. On contrary, \( \Delta Q_0 \) increases rapidly as either \( R \) or \( \beta \) increases. In the case \( \delta = +1 \), both \( \Delta P_0 \) and \( \Delta Q_0 \) can vanish or even turned to the opposite polarity at smaller values of both \( R \) and \( \beta \). On the other side, for the negative dust case \( (\delta = -1) \), \( \Delta P_0 \) and \( \Delta Q_0 \) never reach the zero value, and \( \Delta P_0 \) \( (\Delta Q_0) \) increases (decreases) as either \( R \) or \( \beta \) increases.

Figures 3–5 illustrate the combined effect of the DSD (through \( R \)) and the nonextensive parameter \( q_i \) on the phase

FIG. 2. The dependence of the phase shifts \( \Delta P_0 \) (blue) and \( \Delta Q_0 \) (red) on the DSD parameters \( R \) and \( \beta \) (the power law index). The case of positive dust grains \( (\delta = 1) \) is shown above the case of negative dust grains \( (\delta = -1) \), where, \( \phi_{\xi 1m} = 0.8, \phi_{\eta 1m} = 0.5, q_e = 0.5, q_i = 1.5, \epsilon = 0.1, r_{\text{min}} = 8 \mu m. \)

FIG. 3. The variation of the phase shifts, \( \Delta Q_0 \) (red) and \( \Delta P_0 \) (blue), with \( q_i \) at different values of \( R \) for the positive dust case \( (\delta = 1) \). The criticality of \( \Delta Q_0, \Delta P_0 \rightarrow 0 \) is indicated, where \( \phi_{\xi 1m} = 0.8, \phi_{\eta 1m} = 0.6, q_e = 0.2, \beta = 3.5, \epsilon = 0.1, r_{\text{min}} = 8 \mu m. \)

FIG. 4. \( \Delta Q_0 \) is depicted versus \( q_i \) at different values of \( R \) for the negative dust case \( (\delta = -1) \). The cutoff in \( \Delta Q_0 \) is indicated. The system parameters are the same like those used in Fig. 3.
shifts. It is clear from Figs. 3–5 that the phase shifts have certain thermal cutoffs at \( q_i \approx 2.15 \) at which \( \Delta P_0 \) and \( \Delta Q_0 \) take zero value then change their polarity for the case of positive dust grains (\( \delta = +1 \)); see Fig. 3. But for the case of negative dust grains (\( \delta = -1 \)), \( \Delta P_0 \) and \( \Delta Q_0 \) at \( q_i \approx 2.15 \) change their trend without reaching the zero limit; see Figs. 4 and 5. For the superextensive distribution case (\( q_i < 1 \)), where particles behave like the \( \kappa \)-distribution for superthermal particles in plasma, \( \Delta Q_0 (\Delta P_0) \) increases (decreases) as \( q_i \) increases for both positive and negative dust cases. However, for the subextensive distribution case (\( q_i > 1 \)), which is suitable for describing systems of low-speed particles, \( \Delta Q_0 (\Delta P_0) \) decreases (increases) as \( q_i \) increases for both positive and negative dust cases. Away from the nonextensivity, it is found that \( \Delta P_0 \) or \( \Delta Q_0 \) can vanish depending on the minimum positive dust radius only and never vanish for negative dust grains; see Fig. 6. A critical value of \( r_{\text{min}} \approx 23 \text{ \mu m} \) is detected at which the phase shifts tend to zero for positive dust only.

Finally, the present results should be useful for the explanation of head-on collision of DASs in dusty plasmas, where the dust size distribution plays an important role, which are observed in some astrophysical plasma environments, e.g., Jupiter’s magnetosphere [8], cometary tails [9], and Earth’s mesosphere [10] as well as some recent plasma experiments [31,32].