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Non-Linear Heat and Mass Transfer of Second Grade Fluid Flow with Hall Currents and Thermophoresis Effects

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Abstract: A mathematical model analysis has been developed to investigate the effect of thermophoresis on unsteady flow of non-Newtonian fluid with heat and mass transfer through a porous medium over a permeable infinite vertical plate. The considered non-Newtonian fluid follows a second grade model and is stressed by a uniform strong magnetic field; so the Hall currents are taken into consideration. The problem is modulated mathematically by a system of coupled non-linear partial differential equations which pertaining to describe the continuity, momentum, energy and concentration. These equations involve the effects of thermal radiation, heat generation, thermal diffusion(Soret), viscous dissipation and chemical reaction. The numerical solutions of the dimensionless equations are found as a functions of the physical parameters of this problem. The numerical formulas of the velocity components (u) and (w), temperature (θ) and concentration (C) as well as Nusselt number(Nu) and Sherwood number(Sh) are computed. The physical parameters effects of the problem on these formulas are described and illustrated graphically through some figures and tables.

Keywords: Second grade fluid, Heat and mass transfer, Porous medium, Thermophoresis.

1 Introduction

Viscoelastic second grade fluids which flow through a porous medium with heat and mass transfer have a broad and diverse applications which actually exist not only in the nature and every day life but also in vast industrialization and human body, In addition, they have currently admitted wide range of purposes needed such as the extraction of energy from geothermal region, soil sciences, permeation of drugs in human skin. So, most of researchers and scholars specially mathematicians, physicists, geologists, chemists and others have given the attention to these important issues especially under the influences of heat generation /absorption, thermal radiation, thermal diffusion, chemical reaction and Hall current. In the light of the brief review of the preceding studies concerned with flow over a permeable infinite vertical plate embedded in a porous medium furthermore, showing important external influences acts on flow motion; we found that Piazza [1] has displayed thermophoresis which mean moving particles with thermal gradients, consequently particle thermophoresis is a non-equilibrium cross-flow effect between mass and heat transport. In addition it is already exploited as a novel tool in macromolecular fractionation, micro-fluidic manipulation and selective tuning of colloidal structures. Chamkha and Issa [2] have investigated the effects of heat generation/absorption and thermophoresis on hydromagnetic flow with heat and mass transfer over a flat surface. After that, Alam et al [3] have analyzed the effects of thermophoresis and chemical reaction on unsteady free convection and mass transfer flow past an impulsively started infinite inclined porous plate in the presence of heat generation/absorption. lately, Bhuvanavijaya [4] has studied thermophoretic effect on convective heat and mass transfer flow over a vertical porous plate in a rotating system with suction/injection. whats more, Animasaun [5] has carried out free convective heat and mass transfer investigating the effects

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of thermophoresis, variable viscosity and thermal conductivity on non-darcian MHD dissipative Casson fluid flow with suction and nth order of chemical reaction.Analysis of thermophoretic MHD slip flow over a permeable surface has been presented by Das et al [6]. Fagbade et al [7] has investigated darcy-forcheimer mixed convection flow in a porous medium in the presence of magnetic field, viscous dissipation and thermopherisis. Flow over a permeable plate having momentousness in geophysics and most of industrials, besides equally important to interest with the unsteady motion under strong magnetic field which has important applications in nuclear engineering and industries. Influence of chemical reaction through heat and mass transfer plays the cornerstone in every chemical process and industries such as food processing and polymer production. El-Dabe et al [8] have investigated the Hall effect on third order fluid flow in a porous medium with heat and mass transfer. The problem of an unsteady free convective heat and mass transfer of a Walters-B viscoelastic fluid non-Darcy flow over a vertical cone with the influence of thermal radiation, thermophoresis and higher order chemical reaction has been studied by Kumar et al [9]. El-Dabe [10] and Kim [11] have studied unsteady MHD free convective flow over an infinite vertical porous plate with variable suction. Afterward, Chamkha et al [12] continued the previous work but with the addition of Hall current, thermal radiation and chemical reaction effects on heat and mass transfer. The problem of Soret and Dufour effects on unsteady MHD flow past an infinite vertical porous plate with thermal radiation has been analyzed by Vempati et al [13]. In a similar manner, Reddy et al [14] have investigated unsteady MHD free convection flow of a Kuvshinski fluid past a vertical porous plate in the presence of chemical reaction and heat source/sink. El-Dabe et al [15] have discussed thermal-diffusion and diffusion-thermo effects on mass transfer boundary layer flow for non-Newtonian fluid. Problems of the effects of Hall current and thermal radiation on heat and mass transfer of unsteady flow of a chemically reacting micropolar fluid past a vertical plate embedded in a porous medium have been indicated in [16,17] so did Olajuwon et al [18], but they have taken viscoelastic model into consideration. Pandya et al [19] have studied effects of thermophoresis, dufour, Hall and radiation on an unsteady MHD flow past an inclined plate with viscous dissipation, chemical reaction and heat absorption and generation.

Without a doubt to importance of viscoelastic second grade model with external effects of flow. Hence Hayat, Abbas, Sajid and Asghar [20] have studied the influence of thermal radiation on MHD flow of a second grade fluid. Moreover the first and third authors in addition to Nadeem and Siddiqui [21] have dealt with a rotating system for unsteady flow of second grade fluid under the influence of Hall current. Also, Hayat and Nawaz [22] have analyzed second grade fluid flow in three dimensional under the Hall and ion slip effects. Cortell [23] has been concerned with mass transfer of second grade fluid flow over a stretching sheet in a porous medium with chemically reactive species and after that, the problems of heat and mass transfer of viscoelastic fluid over vertical stretching sheet with Soret and Dufour effects are displayed in [24,25]. Chaudhary and Jain [26] have elucidated the effect of Hall current on MHD mixed convection flow of a viscoelastic fluid past an infinite vertical porous plate with mass transfer and radiation. Similarly, Kumar and Chand [27] have investigated the effect of slip conditions and Hall current on unsteady MHD flow of a viscoelastic fluid past an infinite porous plate embedded in a porous medium. Sahoo [28] has studied heat and mass transfer effect on MHD flow of a viscoelastic fluid through a porous medium bounded by an oscillating porous plate in slip flow regime. The analysis of viscoelastic fluid flow on unsteady two-dimensional in a porous channel with radiative heat transfer and mass transfer has been studied by Das [29]. Furthermore Jha et al [30] have examined the effect of Soret and Hall current on MHD mixed convection flow of visco-elastic fluid past a vertical surface. Recently(in 2016),K.Das et al [31] have studied influences of thermophoresis and thermal radiation on heat and mass transfer of second grade MHD fluid flow past a semi-infinite stretching sheet with convective surface heat flux.Also,K. Das [32] has investigated the combined effects of thermophoresis and thermal radiation on MHD mixed convective heat and mass transfer flow of a second grade fluid past a semi-infinite stretching sheet in the presence of viscous dissipation and Joule heating. M.VeeraKrishna and B.V.Swarnalathamma [33] have studied Hall effects on unsteady MHD free convection flow of an incompressible electrically conducting Second grade fluid through a porous medium over an infinite rotating vertical plate fluctuating with Heat Source/Sink and chemical reaction. To be more closer to the subject of research, Sudhakar et al [34] have studied heat and mass transfer on unsteady free convection of a Walters-B viscoelastic fluid flow past a semi infinite plate under thermophoresis effect. As can be seen more important related researches to our issue in the final analysis, Nayak and Panda [35] have presented an interesting result on mixed convective MHD flow of second grade fluid past a vertical infinite plate with mass transfer, Joule heating and viscous dissipation. The problem of Hall current effect on unsteady MHD viscoelastic fluid flow with radiative heat flux and heat source over a porous medium has been studied by Ahmed [36].

Based on the above literature survey, a number of questions in our research can be arisen; questions that were overlooked in the previous papers such as what about the influences of thermophoresis with Soret, Hall current, thermal radiation, heat generation and chemical reaction on second grade fluid. And also, what about the flow with these effects through a porous medium and viscous dissipation.The focus of this study is extending the work of [35,36] and answering the above questions which have not been investigated so far. Thus, the present problem is concerned with the flow of second grade fluid past an permeable infinite vertical plate immersed in a porous medium and by including influences of thermophorsis, Soret, Hall, thermal radiation, heat generation and chemical reaction on the flow and in the presence of viscous dissipation.

2 Problem Formulation:

Consider unsteady flow with the oscillatory suction of an incompressible and electrically conducting viscoelastic second grade fluid over an infinite permeable vertical plate which located at the plane $y^* = 0$. The x^* axis is assumed to be vertically oriented upward direction along the plate and y^* axis is taken perpendicular to the plane of the plate as shown in fig.1. The plate is subjected to a variable suction velocity v_o. Since the plate is of infinite length, all physical quantities in this problem are functions of y* and t* only. All the fluid properties are assumed to be isotropic, absorbing and emitting heat but not scattering.

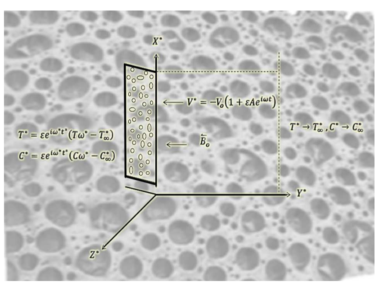

Fig. 1: Coordinate system and physical configuration of present study.

From Maxwell's equations, the divergence equation of magnetic field $\nabla.\mathbf{B} = 0$ gives $B_{y^*} = B_o$ which consider uniform magnetic field and by assuming a very small magnetic Reynolds number. This mean that $(R_{em} = \mu_m \sigma v L \ll 1)$ the induced magnetic field is neglected in comparison to the applied magnetic field so that $B_{x^*} = B_{z^*} = 0$ hence, $\mathbf{B} = (0, B_o, 0)$.Here L is the characteristic length and μ_m is the magnetic permeability. For the description of the thermodynamical or mechanical behavior of non-Newtonian fluids, much work has been devoted to the study of second grade fluids since they are of interest for theory and experiments. For these fluids the Cauchy stress T and the fluid motion are related by Truesdell and Noll (1965):

$$T = -PI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \quad , \tag{1}$$

where, $-PI$ is the indeterminate spherical stress due to the constraint of incompressibility while, P is the hydrostatic pressure, μ is is the dynamic viscosity, α_1 and α_2 are material constants and A_1 and A_2 are the kinematic Rivlin-Ericksen tensors,

$$A_1 = (\nabla V) + (\nabla V)^T \ , A_2 = \frac{dA_1}{dt} + A_1 (\nabla V) + (\nabla V)^T A_1 , \tag{2}$$

here, d/dt is the material time derivative. If the fluid of second grade modeled by Eq.(1) is to be compatible with thermodynamics and is to satisfy the inequality of Clausius–Duhem for all motions and the assumption that the specific Helmholtz free energy of the fluid is a minimum when it is locally at rest, then the coefficients μ, α_1 and α_2 describe the viscosity, visco-elasticity and cross-viscosity respectively and must satisfy:

$$\mu \geq 0, \quad \alpha_1 \geq 0, \quad \alpha_1 + \alpha_2 = 0 \quad . \tag{3}$$

It is necessary to be noted that when $\alpha_1 = \alpha_2 = 0$ then, the constitutive equation of second grade fluid is respond to that of viscous fluid. If $\mathbf{E}^* = \left(E_x^*, E_y^*, E_z^*\right)$ is the electric field vector then, it equals to zero as not only no polarization voltage is imposed on the flow field but also no energy is added or extracted from the fluid by the electric field. $\mathbf{V}$ is velocity vector of the fluid which it's components are u^*, v^*, w^*. $\mathbf{J}$ is the current density vector whose components are J_x^*, J_y^*, J_z^*, then the equation of conservation of charge $\nabla \cdot \mathbf{J} = 0$ gives, $J_y^* = constant$, this constant equal to zero since, the plate is not conducted. Then from equations of Maxwell, we have:

$$\nabla \wedge \mathbf{E} = 0, \nabla \wedge \mathbf{H} = \mathbf{J}, \nabla \cdot \mathbf{B} = 0, \tag{4}$$

where, $\mathbf{H}$ is the strength of the magnetic field vector. The Generalized Ohm's law is

$$\mathbf{J} = \frac{\sigma}{ene} \left[en_e \left(\mathbf{E} + \mathbf{V} \wedge \mathbf{B} \right) - \left(\mathbf{J} \wedge \mathbf{B} \right) + \nabla P_e \right], \tag{5}$$

consider strength of the magnetic field to be very large, furthermore, the generalized Ohm's law can be modified to include Hall current in the absence of electric field and neglecting the ion-slip and thermoelectric pressure for weakly ionized gases, so eq.(5) becomes of the form:

$$\mathbf{J} = \sigma \left(\mathbf{V} \wedge \mathbf{B} + \frac{1}{en_e} \nabla P_e \right) + \frac{\omega_e \tau_e}{B_o} \left(\mathbf{B} \wedge \mathbf{J} \right), \tag{6}$$

where, P_e and ω_e are electron pressure and frequency of the electron respectively while, τ_e is collision time of it and n_e is the number density of electron, σ is electrical-conductivity of the fluid. Hence, Hall parameter m define as $m = \omega_e \tau_e$ and it gives rise to the Lorentz force in z-direction which induces a cross flow in that direction. Consequently, the flow field becomes three dimensional, so from above eq.(6):

$$J_x^* = \frac{\sigma B_o}{(1+m^2)} \left(m u^* - w^* \right) \ , \ J_z^* = \frac{\sigma B_o}{(1+m^2)} \left(u^* + m w^* \right). \tag{7}$$

where, J_x^* and J_z^* are electric current density along x^*-axis and z^* - axis respectively. Wu and Greif [39] have determined the thermophoretic velocity V_T as

$$V_T = \frac{-k_t \nu}{T_r} \nabla T = \frac{-k_t \nu}{T_r} \frac{\partial T}{\partial y}, \tag{8}$$

where, T_r is some reference temperature, $k_t \nu$ represents the thermophoretic diffusivity while, k_t is coefficient of thermophoresis which ranges in value from 0.2 to 1.2 as indicated by Batchelor and Shen [37], Also by Talbot et al. [38]:

$$k_t = \frac{2C_s\left(\frac{\lambda_g}{\lambda_p} + C_t K_n\right)\left[K_n\left(C_1 + C_2 \exp^{\frac{-C_3}{K_n}}\right)\right]}{2\left(1+3C_m K_n\right)\left(C_t K_n + \frac{\lambda_g}{\lambda_p} + \frac{1}{2}\right)}, \tag{9}$$

where, $C_s, C_t, C_1, C_2, C_3, C_m$ are constants and λ_g, λ_p, K_n are the thermal conductivities of the fluid and diffused particles, Knudsen number respectively. A thermophoretic parameter τ can be defined as follows;

$$\tau = \frac{k_t\left(T_w - T_\infty\right)}{T_r}, \tag{10}$$

typical values of τ are 0.01, 0.05 and 0.1 corresponding to approximate values of $(T_w - T_\infty)$ equal to 3.15 and 30k for a reference temperature of $T_r = 300k$.Within the above framework, the governing partial differential equations of the flow under the usual Boussinesq approximation are:

The continuity equation:

$$\frac{\partial v^*}{\partial y^*} = 0, \tag{11}$$

The momentum equation in x direction:

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\alpha_1}{\rho}\left(\frac{\partial^3 u^*}{\partial t^* \partial y^{*2}} + v^* \frac{\partial^3 u^*}{\partial y^{*3}}\right) + g\beta_T(T^* - T_\infty^*) + g\beta_C(C^* - C_\infty^*) - \frac{\sigma B_o^2(u^* + m w^*)}{\rho(1+m^2)} - \frac{\nu}{k} u^*, \tag{12}$$

The momentum equation in z direction:

$$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} = \nu \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\alpha_1}{\rho}\left(\frac{\partial^3 w^*}{\partial t^* \partial y^{*2}} + v^* \frac{\partial^3 w^*}{\partial y^{*3}}\right) - \frac{\sigma B_o^2(w^* - m u^*)}{\rho(1+m^2)} - \frac{\nu}{k} w^*, \tag{13}$$

The energy equation:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\alpha_1}{\rho c_p}\left(v^* \frac{\partial u^*}{\partial y^*} \frac{\partial^2 u^*}{\partial y^{*2}} + v^* \frac{\partial w^*}{\partial y^*} \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \frac{\mu}{\rho c_p}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right] + \frac{Q_o}{\rho c_p}(T^* - T_\infty^*) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y^*}, \tag{14}$$

Species concentration equation

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D_m \frac{\partial^2 C^*}{\partial y^{*2}} - \frac{\partial V_T (C^* - C_\infty^*)}{\partial y^*} + \frac{D_m K_T}{T_m} \frac{\partial^2 T^*}{\partial y^{*2}} - k_c(C^* - C_\infty^*). \tag{15}$$

The appropriate boundary conditions for the problem are:

$$\left.\begin{array}{l} \text{at } y^* = 0 \; u^* = 0, w^* = 0, T^* = T_\infty^* + \left(1 + \varepsilon e^{i\omega^* t^*}\right)(T_w^* - T_\infty^*) \\ \quad C^* = C_\infty^* + \left(1 + \varepsilon e^{i\omega^* t^*}\right)(C_w^* - C_\infty^*) \\ \text{as } y^* \to \infty \; u^* \to 0, \frac{\partial u^*}{\partial y^*} \to 0, w^* \to 0, \frac{\partial w^*}{\partial y^*} \to 0, T^* \to T_\infty^*, \\ \quad C^* \to C_\infty^* \end{array}\right\} \tag{16}$$

where, nomenclature in the above equations are: ν is the kinematic viscosity, ρ is the fluid density, g is the gravitational acceleration, β_T and β_c are thermal expansion volumetric coefficient and volumetric coefficient of concentration expansion respectively. T^*, T_w^* and T_∞^* are denote the dimensional temperature of the fluid, the temperature at the plate and temperature far away from the plate respectively. C^* is the dimensional concentration of the solute, the concentration of the solute at the plate and the concentration of the solute far from the plate are C_w^* and C_∞^* respectively. k is the permeability of the porous medium, c_p is the specific heat at constant pressure, κ is the thermal conductivity of the medium, m is the Hall current parameter, D_m is chemical molecular diffusivity.

The Rosseland approximation for radiation is given by:

$$q_r = \frac{-4\sigma^*}{3k^*} \frac{\partial T^{*4}}{\partial y^*}, \tag{17}$$

where, q_r is the radiative heat flux, k^* is the mean absorption coefficient and σ^* is the Stefan Boltzmann constant, using Taylor series T^{*4} can be expressed as linear function of temperature in addition to expanding T^{*4} about T_∞ and neglecting higher terms, thus

$$T^{*4} \cong 4T^* T_\infty^{*3} - 3T_\infty^{*4}. \tag{18}$$

by substitution from eq.(18) into eq.(17), then eq.(14) becomes:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\alpha_1}{\rho c_p}\left(v^* \frac{\partial u^*}{\partial y^*} \frac{\partial^2 u^*}{\partial y^{*2}} + v^* \frac{\partial w^*}{\partial y^*} \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \frac{\mu}{\rho c_p}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right] + \frac{Q_o}{\rho c_p}(T^* - T_\infty^*) + \frac{16\sigma^* T_\infty^{*3}}{3\rho c_p k^*} \frac{\partial^2 T^*}{\partial y^{*2}}. \tag{19}$$

From eq.(11), it can be seen that v^* is either a constant or a function of time. Thus, assuming that the suction velocity to be oscillatory about a non-zero constant mean, one can assume that:

$$v^* = -v_o\left(1 + \varepsilon A e^{i\omega^* t^*}\right) \tag{20}$$

where, v_o is a scale of mean suction velocity $(v_o > 0)$, A is the suction parameter which can be positive real constant while, ε and εA are small less than unity, ω^* is the frequency of the of the oscillation suction velocity. Let us obtain the dimensionless partial differential equations, by using the following non-dimensional variables:

$$\left.\begin{aligned} u=\frac{u^*}{v_o}, w=\frac{w^*}{v_o}, y=\frac{v_o y^*}{\nu}, t=\frac{t^* v_o^2}{4\nu} \\ \omega=\frac{4\nu\omega^*}{v_o^2}, \theta=\frac{T^*-T^*_\infty}{T^*_w-T^*_\infty}, C=\frac{C^*-C^*_\infty}{C^*_w-C^*_\infty} \end{aligned}\right\} \quad (21)$$

Using the above dimensionless quantities, the system of equations (12, 13, 19) and (15) with boundary conditions (16) can be written in dimensionless form as:

$$\begin{aligned} \frac{1}{4}\frac{\partial u}{\partial t} - \left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} &= \frac{\partial^2 u}{\partial y^2} \\ &+ \alpha\left[\frac{1}{4}\frac{\partial^3 u}{\partial t \partial y^2} - \left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial^3 u}{\partial y^3}\right] \\ &+ Gr\theta + GcC - \frac{M}{1+m^2}(u+mw) - \frac{u}{K} \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{1}{4}\frac{\partial w}{\partial t} - \left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial w}{\partial y} &= \frac{\partial^2 w}{\partial y^2} \\ &+ \alpha\left[\frac{1}{4}\frac{\partial^3 w}{\partial t \partial y^2} - \left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial^3 w}{\partial y^3}\right] \\ &- \frac{M}{1+m^2}(w-mu) - \frac{w}{K} \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{1}{4}\frac{\partial \theta}{\partial t} - \left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial \theta}{\partial y} &= \frac{1}{Pr}(1+Nr)\frac{\partial^2 \theta}{\partial y^2} \\ &- Ec\alpha\left(1+\varepsilon A e^{i\omega t}\right)\left[\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + \frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}\right] \\ &+ Ec\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \phi\theta \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{1}{4}\frac{\partial C}{\partial t} - \left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial C}{\partial y} &= \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - \tau\left(\frac{\partial^2 \theta}{\partial y^2}C + \frac{\partial C}{\partial y}\frac{\partial \theta}{\partial y}\right) \\ &+ Sr\frac{\partial^2 \theta}{\partial y^2} - \gamma C \end{aligned} \quad (25)$$

The relevant corresponding boundary conditions in view of dimensionless form are reduce to:

$$\left.\begin{aligned} &\text{at} \quad y=0 && u=0, w=0, \theta=1+\varepsilon e^{i\omega t} \\ & && C=1+\varepsilon e^{i\omega t} \\ &\text{as} \quad y\to\infty && u\to 0, \tfrac{\partial u}{\partial y}\to 0, w\to 0, \tfrac{\partial w}{\partial y}\to 0, \theta\to 0, C\to 0 \end{aligned}\right\} \quad (26)$$

The dimensionless numbers are: $\alpha = \dfrac{\alpha_1 v_o^2}{\rho\nu^2}$ is the viscoelastic parameter, $Gr = \dfrac{g\beta_T(T^*_w - T^*_\infty)\nu}{v_o^3}$ is the Grashof number, $Gc = \dfrac{g\beta_c(C^*_w - C^*_\infty)\nu}{v_o^3}$ is the modified Grashof number, $M = \dfrac{\sigma B_o^2 \nu}{\rho v_o^2}$ is the magnetic field parameter, $K = \dfrac{k v_o^2}{\nu^2}$ is the Darcy parameter, the Prandtl number $Pr = \dfrac{\mu c_p}{\kappa}$ that represents the ratio of momentum to thermal diffusivity, $Nr = \dfrac{16T^{*3}_\infty\sigma^*}{3\kappa k^*}$ is the thermal radiation parameter, $Ec = \dfrac{v_o^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $\phi = \dfrac{\nu Q_o}{v_o^2 c_p}$ is the heat source parameter, the Schmidt number $Sc = \dfrac{\nu}{D_m}$ that represents the ratio of momentum to mass diffusivity, $Sr = \dfrac{D_m k_T}{T_m \nu}\left(\dfrac{T^*_w - T^*_\infty}{C^*_w - C^*_\infty}\right)$ is Soret (thermal diffusion) parameter, $\gamma = \dfrac{k_c \nu}{v_o^2}$ is the non-dimensional chemical reaction parameter and $\tau = \dfrac{k_t}{T_r}(T^*_w - T^*_\infty)$ is the thermophoretic parameter.

3 Method of solution:

In order to obtain solutions of Eqs.(22, 23, 24, 25) together with boundary conditions (26), we superimposed the unsteady flow on the mean steady flow [29], so Let:

$$\left.\begin{aligned} u(t,y) &= u_o(y) + \varepsilon e^{i\omega t} u_1(y) \\ w(t,y) &= w_o(y) + \varepsilon e^{i\omega t} w_1(y) \\ \theta(t,y) &= (1-\theta_o(y)) + \varepsilon e^{i\omega t}(1-\theta_1(y)) \\ C(t,y) &= (1-C_o(y)) + \varepsilon e^{i\omega t}(1-C_1(y)) \end{aligned}\right\} \quad (27)$$

After substituting with the above assumptions (27), and equating the coefficient of the powers of ε, we obtain the following equations with the corresponding boundary conditions :

3.1 Zeroth order :

$$\begin{aligned} &\alpha u_o''' - u_o'' - u_o' + \left(\frac{M}{1+m^2} + \frac{1}{K}\right)u_o + \frac{mM}{1+m^2}w_o + Gr\theta_o \\ &+ GcC_o - Gr - Gc = 0 \end{aligned} \quad (28)$$

$$\alpha w_o''' - w_o'' - w_o' + \left(\frac{M}{1+m^2} + \frac{1}{K}\right) w_o - \frac{mM}{1+m^2} u_o = 0 \tag{29}$$

$$\frac{1}{Pr}(1+Nr)\theta_o'' + \theta_o' + \phi\theta_o - \phi + Ec\alpha\left(u_o'u_o'' + w_o'w_o''\right) - Ec\left(u_o'^2 + w_o'^2\right) = 0 \tag{30}$$

$$\frac{1}{Sc}C_o'' + C_o' - \gamma C_o - \gamma + \tau\left(C_o'\theta_o' - \theta_o''(1-C_o)\right) + Sr\theta_o'' = 0 \tag{31}$$

With boundary conditions :

$$\left.\begin{array}{lll} \text{at} & y=0 & u_o=0, w_o=0, \theta_o=0, C_o=0 \\ \text{as} & y\to\infty & u_o\to 0, u_o'\to 0, w_o\to 0, w_o'\to 0, \theta_o\to 1, C_o\to 1 \end{array}\right\} \tag{32}$$

3.2 First order :

$$\alpha u_1''' - \left(1+\frac{i\omega\alpha}{4}\right)u_1'' - u_1' + \left(\frac{M}{1+m^2} + \frac{i\omega}{4} + \frac{1}{K}\right)u_1 + \frac{mM}{1+m^2}w_1 + Gr\theta_1 + GcC_1 - Gr - Gc - Au_o' + \alpha A u_o''' = 0 \tag{33}$$

$$\alpha w_1''' - \left(1+\frac{i\omega\alpha}{4}\right)w_1'' - w_1' + \left(\frac{M}{1+m^2} + \frac{i\omega}{4} + \frac{1}{K}\right)w_1 - \frac{mM}{1+m^2}u_1 - Aw_o' + \alpha A w_o''' = 0 \tag{34}$$

$$\frac{1}{Pr}(1+Nr)\theta_1'' + \theta_1' + \theta_1\left(\phi - \frac{i\omega}{4}\right) + \left(\frac{i\omega}{4} - \phi\right) + A\theta_o' + Ec\alpha\left(u_o'u_1'' + w_o'w_1''\right) + Ec\alpha\left(u_1'u_o'' + w_1'w_o''\right) + Ec\alpha A\left(u_o'u_o'' + w_o'w_o''\right) - 2Ec\left(u_o'u_1' + w_o'w_1'\right) = 0 \tag{35}$$

$$\frac{1}{Sc}C_1'' + C_1' - \left(\gamma + \frac{i\omega}{4}\right)C_1 + \left(\frac{i\omega}{4} + \gamma\right) + AC_o' + \tau\left(C_o'\theta_1' - \theta_1''(1-C_o)\right) + \tau\left(C_1'\theta_o' - \theta_o''(1-C_1)\right) + Sr\theta_1'' = 0 \tag{36}$$

The associated boundary conditions are :

$$\left.\begin{array}{lll} \text{at} & y=0 & u_1=0, w_1=0, \theta_1=0, C_1=0 \\ \text{as} & y\to\infty & u_1\to 0, u_1'\to 0, w_1\to 0, w_1'\to 0, \theta_1\to 1, C_1\to 1 \end{array}\right\} \tag{37}$$

Nusselt Number

From the temperature field; the rate of heat transfer between plate and the fluid can be expressed in terms of non-dimensional Nusselt number as $q_w = -\kappa\frac{\partial\theta}{\partial y} = \kappa Nu$ where the Nusselt number is given by:

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)\bigg|_{y=0} \tag{38}$$

Sherwood Number

From the concentration field; the rate of mass transfer between plate and the fluid can be expressed in terms of non-dimensional Sherwood number as $j_m = -\rho D\frac{\partial C}{\partial y} = \rho D Sh$ where the Sherwood number is given by:

$$Sh = -\left(\frac{\partial C}{\partial y}\right)\bigg|_{y=0} \tag{39}$$

4 Results and discussions :

For the physical significance, a program was designed by using Mathematica 10 software including using of parametric ND solve package to simulate the numerical solutions of the system of the partial differential equations which describe our problem after separating time variable. The purpose of these numerical computations to illustrate the influence of various governing physical parameters such as: the viscoelastic parameter α, the Grashof number Gr, the modified Grashof number Gc, the magnetic field parameter M, Hall parameter m, the permeability of the porous medium parameter K, the Prandtl number Pr, the thermal radiation parameter Nr, the Eckert number Ec, the heat source parameter ϕ, the Schmidt number Sc, Soret (thermal diffusion) parameter Sr, the non-dimensional chemical reaction parameter γ and the thermophoretic parameter τ on the velocities, the temperature and the concentration fields which have been done at the following values: $\alpha = 0.2$, $m = 0.6$, $M = 1.5$, $Gr = 2$, $Gc = 2$, $K = 1$, $Pr = 0.72$, $Nr = 0.4$, $\gamma = 1$, $\phi = 0.4$, $Ec = 0.5$, $Sc = 0.62$ $\tau = 0.2$, $Sr = 0.5$, $\omega = 10$, $t = 1$, $A = 0.3$, $\omega t = \frac{\pi}{2}$. As well as, Nusselt number and Sherwood number have been computed and represented in tabulated forms. The previous influences of various parameters are graphically presented in Figures $(2-21)$.In Fig. 2, by increasing the viscoelastic second grade parameter α, the secondary velocity profile $w(t,y)$ decreases and the same effect happens in the primary velocity $u(t,y)$. Since the influence of the second grade parameter on the temperature and concentration fields is very small; then the results are not shown here. It is observed from Fig. 3 that an increase in Hall parameter m leads to a rise in the values of velocities $u(t,y)$ and $w(t,y)$. As well as,$u(t,y)$ and $w(t,y)$ increase with increasing of the magnetic field parameter M as shown in Fig. 4; the reason of this increasing is presence of magnetic field in an electrically conducting fluid that generates a force called the Lorentz force which acts against the flow if the magnetic field is applied in the normal direction. The primary velocity $u(t,y)$ curves increase when the heat source parameter ϕ increases as seen in Fig. 5. But in Fig. 6, we notice that $u(t,y)$ decreases with the increasing in thermophoretic parameter τ until $u(t,y)$ becomes convergent decreased curves whenever τ increases. The same simulation of effect τ occurs on $w(t,y)$. The increase in the suction parameter A results in the decrease in $u(t,y)$ as displayed in Fig. 7. The values of the Grashof number Gr are taken to be positive and negative as they respectively represent symmetric heating of the plate when $Gr < 0$ and symmetric cooling of the plate when $Gr > 0$. The primary and secondary velocities increase not only by increasing the Grashof number Gr but also by increasing the modified Grashof number Gc as shown in Fig. 8 and Fig. 9 respectively. It is seen from Fig. **??** that $w(t,y)$ increases with increasing of Darcy parameter K. Fig. 11 depicts that $u(t,y)$ increases as the chemical reaction parameter γ increases; this also happens with $w(t,y)$. Moreover, Fig. 12 indicates that primary velocity curves increase with increasing of Schmidt number . The effect of increasing Eckert number Ec is to enhance $u(t,y)$ as shown in Fig. 13. Higher Eckert number values lead to greater viscous dissipative heat. From figures (2–13)we have noticed that the secondary velocity $w(t,y)$ and primary velocity $u(t,y)$ increase when the fluid approaches to the plate but they decrease when the fluid is far way from the plate for different values of physical parameters of interest ; this is compatible with physical phenomena. Equally important here to display the effects of magnetic field parameter M on temperature distributions θ where these effects appear in linear forms and decreases with the increases in M as seen in Fig. 14. It was found that an increase in the value of heat source parameter ϕ leads to an increase in the temperature distributions $\theta(t,y)$ as seen in Fig. 15; by the continuous increasing of ϕ, the temperature $\theta(t,y)$ becomes quasi-linear curve. It is further observed from Fig. 16 that an increase in the Prandtl number Pr results in a decrease in the temperature distributions $\theta(t,y)$, where the linear behavior appear at $Pr = 0.2$. Small values of the Prandtl number, $Pr \ll 1$, means that the thermal diffusivity dominates. We choose the values of Prandtl number which physically correspond to gases and fluids which are commonly used in commerce and industry. Around 0.16-0.7 for mixtures of noble gases or noble gases with hydrogen, 0.63 for oxygen, around 0.7-0.8 for air and many other gases, 1.38 for gaseous ammonia. Whereas with large values of $Pr \gg 1$, the momentum diffusivity dominates the behavior. On the contrary, the reversed effect of Pr is observed on $\theta(t,y)$ when the thermal radiation parameter Nr increases as seen in Fig. 17.It is clear from Figs.(14–17) that the temperature is maximum at the plate but it is minimum far away from the plate for different values of M, ϕ,Pr and Nr. An increase in thermophoretic parameter τ leads to decrease in the curves of concentration $C(t,y)$ which tend to quasi linear curves via the continuous increase, especially at value of $\tau = 1.2$ as reflected in Fig. 18. Fig. 19 shows that the effect of Schmidt parameter Sc on concentration profile $C(t,y)$ to be a linear curve at value of 0.2 and after the increase of Sc, $C(t,y)$ become deceased curves. It is noticed from Fig. 20 that the influence of Soret parameter Sr at values of $Sr = 0, 0.5, 1$ and 4 on the concentration profiles $C(t,y)$ appears in two regions $0 \leq y < 0.6$ and $0.6 < y \leq 2.5$ where theses regions approximately intersect at point $y = 0.6$ at which the effect of Sr on $C(t,y)$ is negligible. The effect of Sr on the first region which locates at $0 \leq y < 0.6$ is clearly observed especially for curves $Sr = 0$ and $Sr = 4$; where there are two inflection points such that the critical point that lies at $y = 0.4$ for the curve $Sr = 0$ going from increasing to decreasing.While the reversed effect of Sr in that region happens. Within the second region which locates at $0.6 < y \leq 2.5$, it is noticed that the concentration curves $C(t,y)$ decrease with increasing y for all values of Sr then the curves become regressive. The increase in chemical reaction parameter γ at values $\gamma = -0.3, 0, 0.3, 0.5$ leads to an increase in concentration distributions $C(t,y)$. And after continuously increasing, the curve becomes quasi linear; then becomes linear at $\gamma = 0.5$ as shown in Fig. 21.

velocities profiles

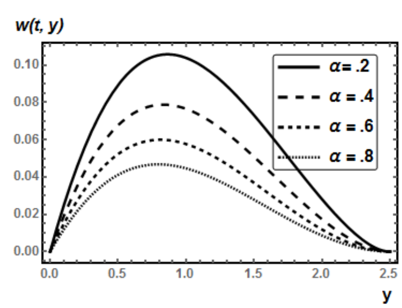

Fig. 2: Effect of viscoelastic second grade parameter α on secondary velocity profile $w(t,y)$.

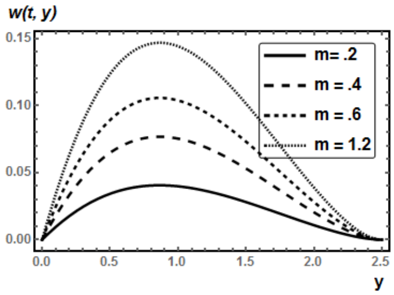

Fig. 3: Effect of Hall parameter m on secondary velocity profile $w(t,y)$.

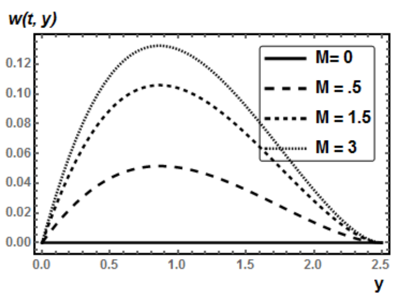

Fig. 4: Effect of the Hartmann number M on secondary velocity profile $w(t,y)$.

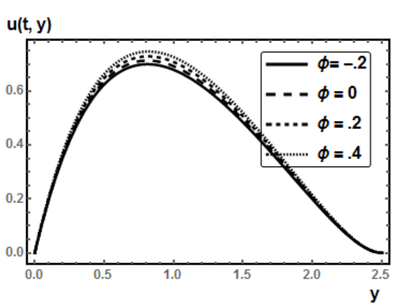

Fig. 5: Effect of heat source parameter ϕ on primary velocity profile $u(t,y)$.

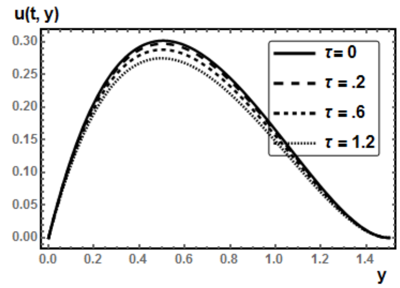

Fig. 6: Effect of thermophoretic parameter τ on primary velocity profile $u(t,y)$.

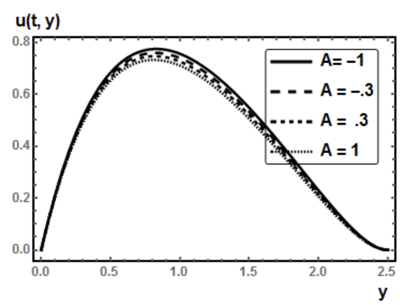

Fig. 7: Effect of suction parameter A on primary velocity profile $u(t,y)$.

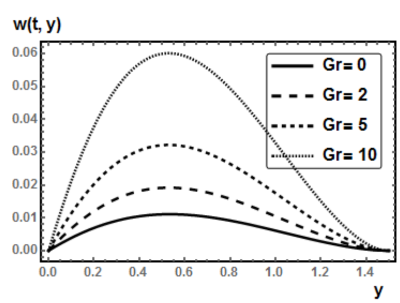

Fig. 8: Effect of Grashof number Gr on secondary velocity profile $w(t,y)$.

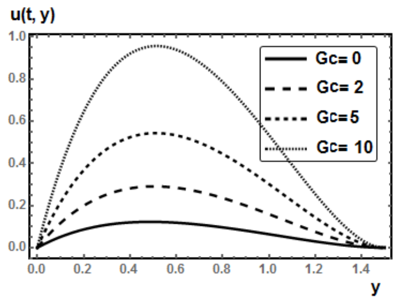

Fig. 9: Effect of modified Grashof number Gc on primary velocity profile $u(t,y)$.

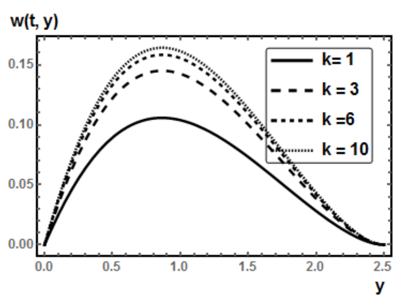

Fig. 10: Effect of Darcy parameter K on secondary velocity profile $w(t,y)$.

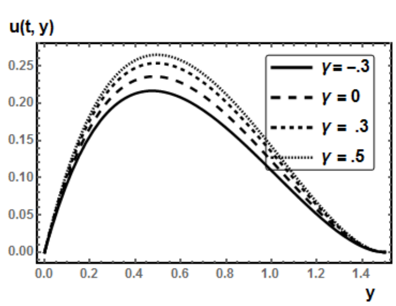

Fig. 11: Effect of chemical parameter γ on primary velocity profile $u(t,y)$.

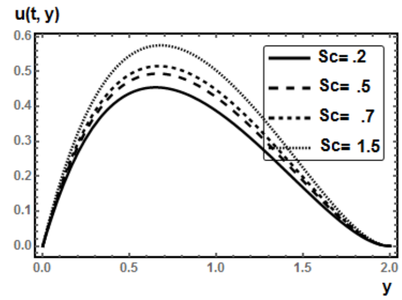

Fig. 12: Effect of Schmidt parameter Sc on primary velocity profile $u(t,y)$.

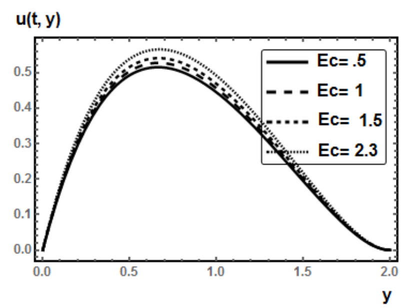

Fig. 13: Effect of Eckert parameter Ec on primary velocity profile $u(t,y)$.

Temperature profiles

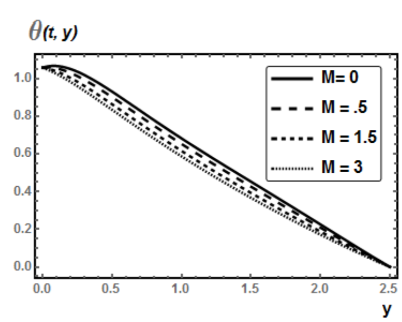

Fig. 14: Influence of the Hartmann number M on temperature profile $\theta(t,y)$.

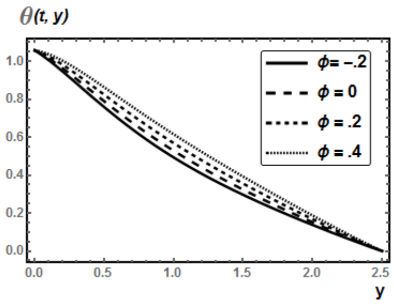

Fig. 15: Influence of heat generation parameter ϕ on temperature profile $\theta(t,y)$.

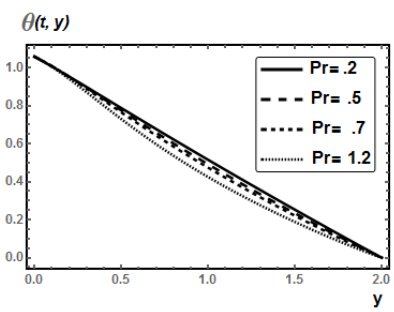

Fig. 16: Influence of Prandtl parameter Pr on temperature profile $\theta(t,y)$.

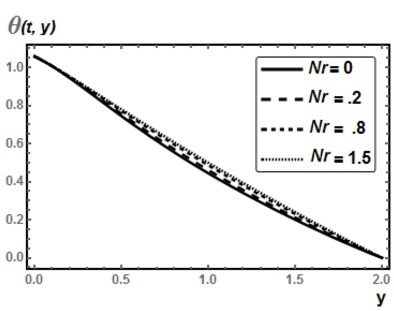

Fig. 17: Influence of thermal radiation parameter Nr on temperature profile $\theta(t,y)$.

Concentration profiles

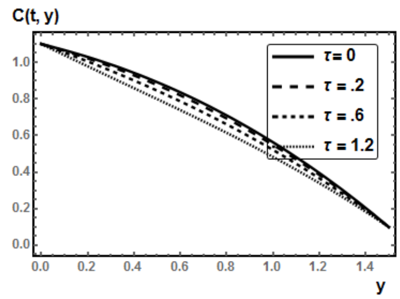

Fig. 18: Effect of thermophoretic parameter τ on concentration profile $C(t,y)$.

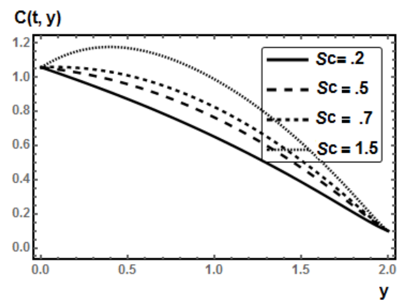

Fig. 19: Effect of Schmidt parameter Sc on concentration profile $C(t,y)$.

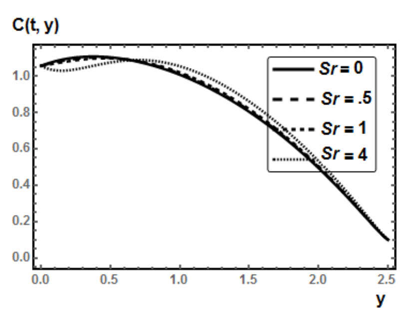

Fig. 20: Effect of Soret parameter Sr on concentration profile $C(t,y)$.

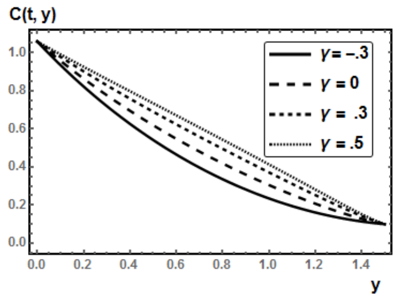

Fig. 21: Effect of chemical reaction parameter γ on concentration profile $C(t,y)$.

Table 1: Nusselt number data for $m = 0.6$, $\tau = 0.2$, $\gamma = 1$ $Sr = 0.5$, $Sc = 0.3$, $Gr = 5$, $Gc = 5$, $K = 3$, $\omega t = 0$, $\omega = 10$, $A = 0.5$, $\varepsilon = 0.01$

M	Pr	Nr	Ec	α	ϕ	Nu
1	0.7	0.3	0.001	0.1	0.4	0.53651
2	0.7	0.3	0.001	0.1	0.4	0.538758
5	0.7	0.3	0.001	0.1	0.4	0.541414
3	1	0.3	0.001	0.1	0.4	0.638245
3	2	0.3	0.001	0.1	0.4	1.14282
3	3	0.3	0.001	0.1	0.4	1.83907
3	0.72	0.5	0.001	0.2	0.4	0.519113
3	0.72	0.7	0.005	0.2	0.4	0.506675
3	0.72	0.7	0.008	0.2	0.4	0.497307
3	0.72	0.7	0.010	0.2	0.4	0.491043
3	0.72	0.7	0.001	0.1	0.4	0.500347
3	0.72	0.7	0.001	0.3	0.4	0.500351
3	0.72	0.7	0.001	0.8	0.4	0.50058
3	0.72	0.7	0.001	0.2	0.5	0.460023
3	0.72	0.7	0.001	0.2	0.7	0.37408
3	0.72	0.7	0.001	0.2	1.2	0.119283
3	1	0.8	0.001	0.5	0.4	0.546584
5	1	0.9	0.010	0.6	0.5	0.46878
6	1	1.2	0.07	0.9	0.6	0.30243

Nusselt numbers Nu at phase $\omega t = 0$ are given in table1. It can be clearly seen that the Nusselt number Nu increases with an increase of magnetic parameter M; this is due to the fact that the applied magnetic field induces a Lorenz force which is responsible for the increase of the rate of heat transfer. It is further observed that an increase in the Prandtl number Pr results in an increase in Nu. However, the reverse effect is observed when the Eckert number increases. Because of the increase in second grade parameter α, Nu increases. The Nusselt number Nu is decreasing not only by increasing the thermal radiation Nr but also by increasing the heat generation ϕ. By increasing of M, Nr, Ec, α and ϕ,at the same time, this implies a decreasing in Nusselt number.

Table 2: Sherwood number data for $\alpha = 0.2$, $m = 0.6$, $M = 4$, $Nr = 0.4$, $\phi = 0.3$, $Pr = 0.72$, $Ec = 0.001$, $Gr = 5$, $Gc = 5$, $K = 3$, $\omega t = 0$, $\omega = 10$, $A = 0.5$, $\varepsilon = 0.01$

Sc	τ	Sr	γ	Sh
0.3	0.3	0.3	0.3	0.415147
0.5	0.5	0.5	0.5	0.256865
0.8	0.8	0.8	0.8	0.206895
0.3	0.3	0.3	1	0.0712626
0.3	0.3	0.3	2	0.314289
0.4	0.3	0.3	1	0.0361691
0.9	0.3	0.3	1	0.557422
1.2	0.3	0.3	1	0.859563
1	1.5	0.3	1	0.373743
1.2	1.5	0.3	1	0.535102
1.3	1.5	0.3	1	0.615904
0.3	0.2	0.3	1	0.0626367
0.3	0.5	0.3	1	0.0887411
0.3	0.9	0.3	1	0.124596
0.3	1.2	0.3	1	0.152263
0.3	3	0.3	1	0.331625
0.3	0.3	0.2	1	0.0765422
0.3	0.3	0.4	1	0.0659834
0.3	0.3	0.9	1	0.0395943
0.3	0.3	1.5	1	0.00794458

Sherwood number Sh at phase $\omega t = 0$ are given in table 2. It is observed that the Sherwood number decrease at the same rate of increasing for Sc, τ, Sr andγ. Also, it is noticed that an increase in chemical reaction parameter γ leads to decrease in Sherwood number likewise, Sh decrease with the increase of Soret number Sr.On the contrary, the increase of Sherwood number occurs when increasing both thermophoretic parameter τ and Schmidt number Sc.

5 Conclusions

–There is no flow in Z axis-direction when the magnetic field does not exist.

–The increase in thermophoretic parameter τ leads to reduce primary and secondary velocities as well as temperature profile, while enhancing the concentration.

–The concentration $C(t,y)$ is independent of Soret parameter Sr at certain values of y that locate among $y = 0.6$ and $y = 0.7$. But,before and after this region,the effect of Sr has been observed.

–Increase in heat source parameter ϕ enhance and increase the primary and secondary velocities as in the case of temperature profile.

–Increasing in chemical reaction γ leads to increase in velocities and its important to enhance the concentration.

–Increase in magnetic parameter M, thermal radiation parameter Nr, Eckert number Ec, second grade parameter α and heat source parameter ϕ at the same time causes a decrease in Nusselt number.

– The Sherwood number decreases at the same rate of increasing for Sc, τ, Sr and γ, while it increases with increasing the thermophoretic parameter τ.

6 Application

It is hoped that the obtained results from thermophoresis phenomenon and soret effect, Flow over a permeable plate embedded in a porous medium, MHD, Hall current and chemical reaction will provide useful information for applications in information science.

1-In thermophoresis, the thermal analog to electrophoresis, molecules are moved along a microscopic temperature gradient. Its practical applications for analytics in biology show considerable potential. Here we measured the thermophoresis of highly diluted single stranded DNA using an all-optical capillary approach. Temperature gradients were created locally by an infrared laser. The thermal depletion of oligonucleotides of between 5 and 50 bases in length were investigated by fluorescence at various salt concentrations. To a good approximation, the previously tested capacitor model describes thermophoresis: the Soret coefficient linearly depends on the Debye length and is proportional to the DNA length to the power of 0.35, dictated by the conformation-based size scaling of the diffusion coefficient. The results form the basis for quantitative DNA analytics using thermophoresis.

2-Thermophoresis is of practical importance in many industrial applications, such as in aerosol collection (thermal precipitator), micro contamination control, removing small particles from gas streams, nuclear reactor safety, in studying the particulate material deposition on turbine blades, and also in determining exhaust gas particle trajectories from combustion devices.

3-The obtained results due to Flow over a permeable plate embedded in a porous medium have applications in many areas of science and technological fields, namely study of ground water resources in agricultural engineering, in petroleum technology to study the moment of ordinary gas, oil, and water through the oil reservoirs.

4-MHD provides a mean of cooling the turbine blade and keeping the structural integrity of the nose cone.

5-Hall current (strong magnetic field) effect with species concentration has many application in MHD power generations, in several astrophysical and metro-logical studies as well as in flow of plasma through MHD generators .

6-Furthermore, The results that have been obtained from our research can be used in the following applications: develops and manufactures wellhead control panels, and chemical injection systems. Including wellhead safety control systems, metallurgical process, polymer extrusion, glass blowing, crystal growing, oil recovery, food processing, paper making, Ultra-filtration, Transfer of fuels and lubricants.

It is known that Information science is an interdisciplinary field concerned with applied sciences, analysis, collection, classification, manipulation, storage, retrieval, movement, dissemination and protection of information.Using supercomputers, natural expectation by using a combination of observations of physical results from many sources such as mathematical representation of the behavior of the temperature and concentration. Information Science is a field that is changing rapidly, which it focuses on the following areas:

Artificial Intelligence applications, including computational modelling, evolutionary computing, machine learning and data mining. **Databases**, including geographic and spatial information systems. **Distributed information systems**, including multi-agent systems, and wireless and mobile applications. **Health informatics**. **Human-computer interfaces**, including multimedia systems and augmented reality. **Information assurance and security**. **Information systems** development and management. **Social networking** and **organisational computing**, including decision support systems. **Software engineering** and application software development. Information science is often (mistakenly) considered a branch of computer science; however, it predates computer science and is actually a broad, interdisciplinary field, incorporating not only aspects of computer science, but often diverse fields such as archival science, cognitive science, commerce, communications, law, library science, museology, management, mathematics, philosophy, public policy, and the social sciences.

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